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THE DISTRIBUTION OF EXCHANGE RATES UNDER A MINIMUM EXCHANGE RATE REGIME

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This paper adjusts the Chen and Giovannini (1992) methodology to estimate the unconditional distribution of exchange rates under a one-sided target zone regime, where a central bank commits itself to intervene on foreign exchange markets to prevent its currency to move beyond a previously announced target level vis-à-vis a specific foreign currency. An application of this methodology to the 2011–2015 EUR/CHF minimum exchange rate regime shows that the Swiss National Bank presumably intervened only at (or very close to) the floor level of EUR/CHF 1.20 and not at a level significantly above that boundary. Hence, contrary to previous studies, the reported results accord with the predictions of the Krugman (1991) target zone model about the behavior of exchange rates, allowing investors to gain insights about the central bank's policy function in extraordinary monetary situations and with important consequences for the descriptive validity of theoretical one-sided target zone models.

JEL classification codes: C18, E42, E52, E61, F31
Key words: central banking, euro/Swiss franc floor, exchange rate distribution, minimum exchange rate regime, exchange rate target zone model

I. Introduction

Inspired by the work of Chen and Giovannini (1992) who propose a methodology to estimate the unconditional distribution of exchange rates under

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a symmetric two-sided target zone regime, this paper proposes an adjustment of their methodology such that it can be applied to an asymmetric one-sided, unilateral exchange rate target zone, also called a minimum exchange rate regime.

Specifically, Chen and Giovannini (1992) propose to apply the Box-Cox transformation to a bounded exchange rate (i.e., under a two-sided target zone), a transformation that is based on the system of frequency curves analyzed in Johnson (1949).[1] As a consequence, their approach offers a methodology that allows a wide variety of shapes for the unconditional distribution of bounded exchange rates, making it possible to infer which intervention policies to defend a minimum exchange rate (i.e., intramarginal vs. marginal foreign exchange interventions) are actually more likely being used in practice. From an empirical perspective, the EUR/CHF 1.20 floor, set by the Swiss National Bank (SNB) on September 6, 2011 and finally abandoned on January 15, 2015, provides ample data to analyze the shape of such a distribution.[2] This asymmetric, one-sided, unilateral exchange rate target zone, where the central bank committed itself to intervene on foreign exchange markets to prevent its currency to move beyond a previously announced target level vis-à-vis either a specific foreign currency or a basket of foreign currencies, functioned as a strong-side convertibility commitment.[3]

This episode has important consequences both for the descriptive validity of theoretical exchange rate target zone models and for how to specify the currency risk associated with an investment in a bounded currency (i.e., a long position in a currency under a strong-side convertibility commitment).[4] The target zone model developed by Krugman (1991) or Sutherland (1994), for instance, implies a U-shaped (or bimodal) exchange rate distribution in the case of a two-sided exchange rate target zone (Svensson 1992), since it is assumed that foreign

[1] See, for instance, Sakia (1992) for a review of this transformation technique.

[2] Following the foreign exchange market convention (see Reiswich and Wystup 2010, among others), the exchange rate is referred to as "EUR/CHF" in the body of the text, although the exchange rate is analyzed from a CHF perspective in the empirical section of this paper, i.e., the number of units of Swiss francs needed to buy one euro.

[3] Under the SNB's recently abandoned strong-side convertibility commitment, the SNB promised to buy an unlimited amount of euros to prevent the EUR/CHF exchange rate from moving below the implemented EUR/CHF 1.20 floor level.

[4] In regard to currency risk, any conditional model for the dynamics of a given asset return series must be in accordance with its unconditional distribution of asset returns (Mittnik et al. 1998).

exchange interventions only occur at the edges of the implemented exchange rate band, whereby the exchange rate spends relatively more time close to the edges.[5] This implication contrasts with the implications in the target zone models developed by Bertola and Caballero (1992), Beetsma and van der Ploeg (1994), Lindberg and Söderlind (1994) or Bauer et al. (2009), where a hump-shaped (or unimodal) distribution of exchange rates is predicted, with most of the probability mass near the center of the implemented exchange rate band, as these models assume that in order to implement a minimum exchange rate regime central banks only intervene in the interior of the band (i.e., these models are characterized by intramarginal interventions). Consequently, these models imply a completely different distribution of exchange rates for the case of a minimum exchange rate regime: the former target zone models (i.e., the models in the spirit of Krugman 1991 or Sutherland 1994) imply a right-skewed distribution, whereas the latter models (i.e., the aforementioned enumeration of relevant publications) imply a hump-shaped distribution. To shed additional light on the descriptive validity of these two types of theoretical exchange rate target zone models, the present paper analyzes the recently abandoned EUR/CHF minimum exchange rate regime.

Therefore, this paper contributes to the target zone literature by first showing how the methodology in Chen and Giovannini (1992) can be adjusted to the case of a one-sided exchange rate target zone, making it possible to estimate the unconditional distribution of exchange rates under a minimum exchange rate regime. Second, by applying the proposed adjustment to the recent euro/Swiss franc episode, this paper finds that the estimated EUR/CHF exchange rate distribution accords with the predictions of the Krugman (1991) target zone model about the behavior of exchange rates in a target zone, thereby underlining the suitability of this model in monetary regimes that are characterized by the implementation of unconventional monetary policy measures, contrary to previous empirical studies that put into question the implications of this model (see the detailed survey in Duarte et al. 2013). Third, due to the limited number of empirical studies that analyze and estimate the density function of an exchange rate under a one-sided target zone (apart from, for instance, Chen

[5] For more details about the target zone literature (e.g., for an in-depth overview on relevant empirical results and alternative theoretical models), the interested reader is referred to the survey in Kempa and Nelles (1999) or Duarte et al. (2013).

et al. 2013 who analyze Hong Kong's currency board that was one-sided until May 2005), this paper adds new insights to the target zone literature.

The remainder of this paper is structured as follows: Section II presents the adjusted version of the methodology developed by Chen and Giovannini (1992) and shows the impact that the parameters have in general on the shape of the unconditional distribution of exchange rates in a generalized setting. Section III shows the estimated unconditional density function for the recent EUR/CHF episode and discusses the main results of the paper. Section IV provides the concluding remarks.

II. Methodology

A. Transformed distribution

First, define the log percentage deviation from the implemented exchange rate floor level $x_t = s_t - c_t$, where $s_t \equiv \ln S_t$ denotes the log exchange rate in EUR/CHF and c_t denotes the corresponding log floor level, namely $c \equiv c_t = \ln(1.20)$.[6] Second, transform the log percentage deviation from the implemented floor level x_t as follows:

$$y_t \equiv \gamma + \delta \cdot \ln x_t \sim N(0,1), \qquad x_t \geq 0. \tag{1}$$

As in Johnson and Kotz (1970), assume that y_t is standard normally distributed and therefore unconstrained (see Chen and Giovannini 1992 for arguments that support the use of transformations that lead to normality). Notice that the distribution of $\ln x_t$ has the same shape as the distribution of y_t and only depends on the two parameters δ and γ.

The density function of x_t (and herewith the density function of the scaled log exchange rate $\ln(S_t/1.20)$) is obtained by applying the change-of-variables rule to the density function of $y_t, \phi(y_t)$, where $\phi(*)$ denotes the density function of the standard normal distribution:

6 In the following, the minimum exchange rate of EUR/CHF 1.20 will be simply abbreviated by 1.20 for the sake of notational clarity.

$$f(x_t) = J_t \cdot \phi(\gamma + \delta \cdot \ln x_t). \tag{2}$$

In Equation 2, the Jacobian J_t is equal to:

$$J_t = \frac{\delta}{x_t}. \tag{3}$$

The corresponding log likelihood function then is equal to:

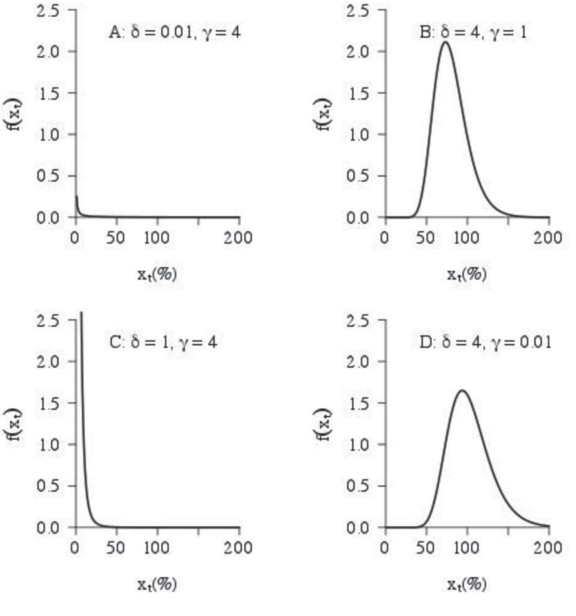
$$l \equiv \ln(\prod_{t=1}^{T} f(x_t)) = \sum_{t=1}^{T} \ln J_t + \sum_{t=1}^{T} \ln\big(\phi(\gamma + \delta \cdot \ln x_t)\big). \tag{4}$$

Hence, the parameter estimates of δ and γ are obtained by maximizing the log likelihood function l accordingly.

B. Impact of the parameters

To exemplify the impact that different parameter values of δ and γ have on both the location and the shape of the unconditional exchange rate distribution of x_t, Figure 1 displays the density function $f(x_t)$for different parameter combinations:

Figure 1. Density function for x_t and different parameter values



Note: The figure plots the density function $f(x_t)$ (Equation 2) for different parameter values δ and γ.

A value of γ close to zero (combined with a larger parameter δ) results in an unconditional density function for x_t that is approximately symmetric (see, e.g., panel D). Similarly, a value of δ close to zero results in an asymmetric U-shaped unconditional density function, with the major part of the probability mass concentrated at the implemented exchange rate floor level (see, e.g., panel A). For larger values of δ, the unconditional distribution exhibits a hump-shaped form and becomes more and more bell-shaped, the larger δ is (i.e., for $\delta \to \infty$).

III. Empirical results

To estimate the unconditional distribution of x_t, the daily EUR/CHF spot FX rate from Bloomberg is used, covering the period from September 6, 2011 to January 14, 2015. Hence, the total number of observations equals 877. Table 1 provides some descriptive statistics of the data sample:

Table 1. Descriptive statistics of the daily EUR/CHF exchange rate

Minimum	1st quantile	Median	Mean	3rd quantile	Maximum
1.20	1.21	1.22	1.22	1.23	1.26

Note: The table provides the minimum, the first quantile, the median, the mean, the third quantile and the maximum of the daily spot EUR/CHF exchange rate S_t from September 6, 2011 to January 14, 2015 (where the total number of observations is equal to 877). Data source: Bloomberg.

Evidently, the range of actual spot EUR/CHF exchange rates is relatively close to the defended floor level of EUR/CHF 1.20 in the period of interest, which is a first indication that the SNB presumably intervened only at an exchange rate close to the implemented 1.20-floor level.

The following Table 2 contains the estimated parameter values $\hat{\delta}$ and $\hat{\gamma}$ and the corresponding standard errors that are obtained by maximizing the log likelihood function in Equation 4, where all the computations are performed with the statistical software package R using the Newton-Raphson method and the wrapper maxLik for maximization routines, which is included in the package maxLik:

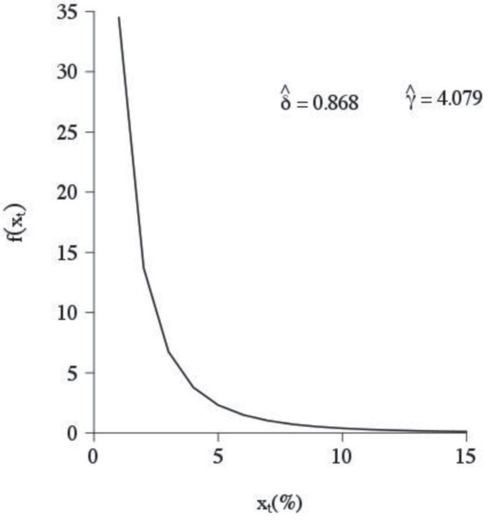
Table 2. Estimated parameters for the unconditional distribution of x_t

$\hat{\delta}$	$\hat{\gamma}$
0.8680 (0.0208)	4.0798 (0.1033)

Note: The table shows the estimated parameter values $\hat{\delta}$ and $\hat{\gamma}$ (with the corresponding standard errors in parentheses) of the distribution of the scaled daily log exchange rate $\ln(S_t/1.20)$ in the period from September 6, 2011 to January 14, 2015. Data source: Bloomberg.

For the recent EUR/CHF minimum exchange rate regime, the estimated parameters indicate that the unconditional exchange rate distribution is asymmetric and right-skewed, since both $\hat{\delta}$ is relatively small and $\hat{\gamma}$ is much larger than zero (similar to panel C in Figure 1). Finally, Figure 2 displays the unconditional density function of x_t (or the scaled log exchange rate $\ln(S_t/1.20)$) that results by using the estimated parameter values in Equation 2:

Figure 2. Estimated unconditional distribution of the EUR/CHF exchange rate



Note: The figure plots the estimated unconditional density function $f(x_t)$ for the scaled log exchange rate $\ln(S_t/1.20)$, with $\hat{\delta}$ = 0.8680 and $\hat{\gamma}$=4.0798.

The shape of the unconditional distribution of x_t indicates which theoretical exchange rate target zone model is more likely to be consistent with the data sample. From a theoretical perspective, a hump-shaped distribution is possible: to prevent arbitrage opportunities (see Ingersoll Jr. 1987: 270 and Bergman 1996, among others), the SNB might have intervened in the spot currency market at an EUR/CHF exchange rate level above the implemented EUR/CHF 1.20 floor level in the period of interest. In this respect, Lera and Sornette (2016), for instance, propose a testing procedure to assess the validity of the Krugman (1991) target zone model for the recently abandoned minimum exchange rate regime vis-à-vis the euro currency that indeed assumes that the SNB intervened at an exchange rate level slightly above the 1.20-floor.

Alternatively, if the SNB mainly intervened when the EUR/CHF exchange rate reached the EUR/CHF 1.20 floor level in the period of interest, then the distribution of the EUR/CHF exchange rate should be rather right-skewed. Interestingly, the reported empirical results suggest that previous target zone regimes (for instance, the Bretton Woods agreements (1946-1973) or the exchange rates under the European Monetary System (EMS) that prevailed from 1979 until 1999) are different from the recent Swiss experience, since most studies find that in the former periods the exchange rates instead exhibited a hump-shaped (see, for instance, Flood et al. 1991, Bertola and Caballero 1992, Delgado and Dumas 1992, Svensson 1992, Beetsma and van der Ploeg 1994, Rose and Svensson 1995, Kempa and Nelles 1999 or Duarte et al. 2013) or a twin-peaked distribution (Bessec 2003).

The recent EUR/CHF episode, however, which suggests that the SNB presumably intervened mainly at an exchange rate level close to the 1.20-floor level, accords with the empirical findings in Chen and Giovannini (1992) for some exchange rates during the Bretton Woods regime and under the EMS, where some exchange rates indeed exhibited a U-shaped distribution, indicating that the involved domestic central banks most likely intervened only at the target zone edges. Moreover, the documented findings for the EUR/CHF exchange rate in the period of interest is in line with the results reported in Feliz and Welch (1993) for the Chilean peso in the 1980s and 1990s and both the evidence in Honohan (1998) for the Belgian franc and in Lundbergh and Teräsvirta (2006) for the Norwegian krone under the EMS in the 1980s, where both studies conclude that for these currencies the empirical results suggest that the Belgian and Norwegian central bank both defended the target zone with marginal rather than with intramarginal interventions.

Moreover, the results of this paper accord with the findings in Studer-Suter and Janssen (2014) and Lera and Sornette (2016) for the EUR/CHF exchange rate in the period of interest, who both report empirical results that support the assumptions that underlie the Krugman (1991) target zone model, i.e., the assumption that central banks under a target zone regime only intervene at the target zone edge (or edges). Hence, for the EUR/CHF exchange rate, the implications of the models in the spirit of Krugman (1991) with regards to the exchange rate distribution under a minimum exchange rate regime accords with the real-world data. Therefore, the results in this paper indicate that the model of Krugman (1991) also applies to one-sided target zones and thus supports future empirical and theoretical studies that rely on this framework to analyze exchange rate target zones.

IV. Conclusion

This paper contributes to the existing target zone literature in the following way: First, it proposes an adjustment to the methodology developed by Chen and Giovannini (1992) to estimate the unconditional distribution of exchange rates under a one-sided target zone regime. Second, this paper applies this methodology to the recent EUR/CHF episode and finds that in accordance with the predictions of the Krugman (1991) target zone model about the behavior of exchange rates adjusted for the existence of only one exchange rate boundary, the distribution of the EUR/CHF exchange rate is right-skewed rather than hump-shaped. Hence, contrary to previous papers that mostly conclude that the insights from the Krugman (1991) model are rejected in empirical studies, this paper documents that the central bank in Switzerland most likely intervened only at the implemented floor level of EUR/CHF 1.20 and not at a level significantly above that boundary (e.g., to prevent arbitrage opportunities that otherwise might have arisen).

These results are important for at least three reasons: First, they indicate how new (or alternative) theoretical exchange rate target zone models must be structured in order to accord with the real-world data. Second, the documented results indicate how central banks promising to defend a minimum exchange rate might intervene in currency markets in the future. This allows both investors and researchers to gain insights on the central bank's policy function in specific monetary regimes. Third, for risk managers the reported results are relevant when it comes to model currency risk (e.g., in a value-at-risk or in an expected shortfall framework) in cases where an exchange rate is subject to a strong-side convertibility commitment, since the estimated distribution of exchange rates indicates which shape the distribution of currency returns might have for a currency under such a monetary regime. Modeling currency risk then requires approaches that are in accordance with the documented asymmetric and right-skewed exchange rate distribution.

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