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The New Keynesian Phillips Curve of rational  
expectations: A serial correlation extension



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## **THE NEW KEYNESIAN PHILLIPS CURVE OF RATIONAL EXPECTATIONS: A SERIAL CORRELATION EXTENSION**

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This paper evaluates the empirical validity of the New Keynesian Phillips Curve (NKPC) model of rational expectations. We employ an instrumental variable (IV) projection method to approximate inflation expectations, and show that the inference based on this approach can differ significantly from the one based on rational expectations. More importantly, using an IV test for serial correlation in the GMM context, we find that the error term in the stylized NKPC model is significantly serially correlated. To compensate for the serial correlation problem, we propose an extended framework which can be easily rationalized in terms of sticky price setting of backward-looking firms. Empirical results show that further lags of inflation are needed in the hybrid specification of the NKPC in order to rule out serial correlation in the Euler equation.

*JEL classification codes:* E31, E37, E52

*Key words:* NKPC, GMM, serial correlation, monetary policy

### **I. Introduction**

In recent years, the empirical validity of the microeconomic foundations of the New Keynesian Phillips Curve (NKPC) with rational expectations has attracted considerable attention from both policy makers and academic researchers; see Goodfriend and King (1997). In particular, Gali and Gertler (1999) (hereafter GG) estimate a hybrid version of the NKPC using the Generalized Method of Moments (GMM) estimator

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under rational inflation expectations proxies, and show that labor income share, rather than the more usual output gap explanation, is a valid inflation catalyst.

Subsequent studies by Linde (2005), Mavroeidis (2005), Rudd and Whelan (2006, 2007), and Dufour et al. (2006), among others, have all criticized GG's results citing estimation deficiencies such as weak identification, and model specification bias. Galí et al. (2005), on the other hand, argue that their "marginal cost-based" version of the NKPC is robust to a variety of estimation procedures. Nonetheless, the robustness of the results in GG must depend on an implicit assumption that the specification error in the stylized NKPC is free of serial correlation.<sup>1</sup> This assumption may be difficult to maintain, and can be highly implausible in empirical estimations. The presence of serial correlation casts large doubts on the validity of the commonly used (in the literature) instrumental variables, and the efficacy of the stylized specification for the NKPC.

Surprisingly, despite its conspicuous importance, the issue of serial correlation has been almost entirely ignored by the research community. In particular, a common practice in the literature involves using lagged dependent variables as instruments in the GMM estimation to solve the endogeneity problem, irrespective of the potential serial correlation problem this may induce.

In this paper, we reconsider the problem of estimating the NKPC, with a particular focus on the serial correlation problem in the empirical NKPC model. Following Pagan (1984), we first employ an instrumental variable (IV) projection approach to approximate inflation expectations, specifically, the projection of the realized future inflation on the underlying IV, and then estimate the NKPC model by GMM. This method can effectively isolate the model specification error from the composite error which consists of both the specification error and a rational inflation forecasts error. This makes a serial correlation test for the specification disturbance now feasible and facilitates comparisons of statistical inference under different data generating mechanisms.

To preview our empirical results, we find that the error term in the stylized NKPC model specified by GG manifests significant serial correlation. In addition, even under the setup in GG, assuming the absence of serial correlation, the inferences associated with labor income share differ significantly between the NKPC specifications with and without rational inflation expectations error.

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<sup>1</sup> The results also depend on the assumption that the error term induced by rational inflation expectations approximation plays a negligible role in the covariance estimates of the GMM estimator. We will discuss this issue in the following sections.

One of the reasons for the presence of serial correlation, as suggested (though not pursued) by Gali et al. (2001) and also by Henry and Pagan (2004), is the inefficacy of lagged inflation dynamics in the stylized NKPC. Therefore, we propose an extended NKPC model by assuming that the “backward-looking” firms in the sticky prices model set their prices based on an extended horizon of historical inflation. Our empirical results reveal that this extended model is indeed free of serial correlation, and additional lagged inflation is statistically significant, a finding strikingly different from that in GG. The results in the present research also suggest that the output gap based on the estimates of the real potential GDP from the U.S. Congressional Budget Office (CBOGAP) plays a significant role in driving inflation in the extended NKPC.

The remainder of this paper is organized as follows. Section II discusses the motivation of the present research. Section III presents a replication of the results of GG with a focus on the serial correlation problem. Section IV extends the stylized hybrid NKPC model with richer dynamics and shows that additional lags of inflation can be easily rationalized in terms of price setting of backward-looking firms in the conventional sticky prices model. Section V summarizes the empirical results pertaining to the extended NKPC model. Section VI discusses the link between the contribution in the present research with those in the existing studies, and section VII concludes.

## II. Motivation

Recent advances in the NKPC model with rational expectations has evolved from earlier models of sticky prices based on Akerlof (1969), Taylor (1980), Rotemberg (1982), and Calvo (1983). Roberts (1995) shows that these models (of Taylor, Rotemberg, and Calvo) can be synthesized in a common formulation as:

$$\pi_t = c_0 + \alpha_f E_t \pi_{t+1} + \alpha_y y_t + \eta_t, \quad (1)$$

where  $c_0$  is a constant,  $\pi_t$  denotes the rate of inflation,  $E_t \pi_{t+1}$  is expected inflation for period  $t+1$  given information available up to period  $t$ ,  $y_t$  denotes the real driving variable, and  $\eta_t$  refers to the specification error. By construction,  $\alpha_f$  in (1) represents a subjective discount factor. Roberts (1995) sets the subjective discount factor to be one and refers to the equation as the New Keynesian Phillips Curve.

Equation (1) suggests that the inflation process is purely forward-looking, with current inflation a function of future inflation and a current real driving variable.

This model is theoretically appealing because it can be developed from the explicit and solid microeconomic foundations relating to the Dynamic Stochastic General Equilibrium (DSGE) models; see Taylor (1999).

Unfortunately, model (1) implies that central banks can reduce inflation without cost; that is,  $\pi_t$  can be moderated by adjusting next period's inflation expectations without having to depress the real economy. This appears to be in stark contrast with the costly disinflation experience in the United States as articulated by Ball (1994), Fuhrer and Moore (1995), Nelson (1998), and Fuhrer (2006).

Consequently, in their very influential work, GG provide a theoretical structure which nests the sticky prices model as a special case of a hybrid specification of the NKPC. The authors assert that the lack of inflation persistence implied by (1) can be resolved by employing a theoretically-based real marginal cost measure, namely labor income share, as the driving force for inflation. Specifically, in the spirit of Akerlof and Yellen (1985), GG develop their theory by assuming an environment of monopolistically competitive firms, in which a fraction of the firms behave optimally in their price setting, as in Calvo's (1983) model, while the remaining firms follow a rule of thumb of backward-looking price-setting behavior. Following a similar procedure, starting with equation (1), the hybrid specification of the NKPC can be obtained as

$$\pi_t = c_0 + \alpha_f E_t \pi_{t+1} + \alpha_b \pi_{t-1} + \alpha_y y_t + \eta_t. \quad (2)$$

GG stress that the distinct feature of (2), compared with the earlier models in Roberts (1995), Fuhrer and Moore (1995), and Fuhrer (1997), is that real marginal cost, rather than the real output gap, is a valid real driving force for inflation. Using the GMM estimator, GG also show that the specification of (2) with an appropriate amount of forward-looking behavior captures U.S. inflation dynamics very well.

The GMM estimation procedure in GG can be summarized as follows: the authors approximate the unobserved inflation expectations in (2) by the corresponding realized future inflation, i.e., instead of estimating the stylized NKPC model (2), GG estimate

$$\pi_t = c_0 + \alpha_f \pi_{t+1} + \alpha_b \pi_{t-1} + \alpha_y y_t + u_t, \quad (3)$$

where  $u_t = \eta_t - \alpha_f e_{t+1}$ , which is induced by the rational expectations approximation, viz.

$$E_t \pi_{t+1} = \pi_{t+1} - e_{t+1}, \quad (4)$$

where  $e_{t+1}$  denotes the rational prediction error.

This approach was first introduced in the 1950s by Mills and Muth (Young and Darity 2001), re-introduced by McCallum (1976) and has been used by Cumby et al. (1983), Hayashi and Sims (1983), and others to study the properties of efficient IV estimators. It has also been used by Roberts (1995) in estimating the sticky prices model of the NKPC with annual data.

In terms of this method, GG assume that both  $\eta_t$  and  $e_{t+1}$  are white noise. In turn, they argue that (2) can be consistently estimated through (3) by GMM with relevant variables dated  $t-1$  and earlier as instruments. A fundamental principle of rational expectations is that the forecasting error  $e_{t+1}$  is assumed to be orthogonal to agents' sets in forming their expectations. This principle combined with white noise assumption on  $\eta_t$  gives rise to moment conditions in the GMM estimation, viz.

$$E(Z'u_t | I_{t-1}) = 0, \quad (5)$$

where  $I_{t-1}$  denotes the information set at time  $t-1$ , and  $Z$  refers to a vector of instruments in the information set available at date  $t-1$  and earlier. Based on these assumptions, lagged values of inflation (and other relevant variables) are legitimate instruments, and the GMM estimator will yield consistent estimates for (2).

As suggested earlier, however, the potential problem of serial correlation may invalidate the GMM estimates in GG. Specifically, if the specification error  $\eta_t$  is serially correlated, while lagged values of the dependent variable (i.e. lagged inflation) are included in the IV set (as in the existing literature), then the moment conditions associated with lagged inflation are invalid. In effect, it is likely that lagged values of the real driving variable are also correlated with  $\eta_t$  when this error term exhibits serial correlation. As a consequence, the GMM estimates associated with the stylized NKPC may not only be biased, but also inconsistent.

In addition, even if the parameters in (2) were consistently estimated via (3) by GMM, it does not imply that the inference based on (3) is necessarily valid. In particular, the volatility of the prediction error  $e$  and its potential correlation with the specification error  $\eta_t$  are likely to inflate (or deflate) the variance of the error term associated with the stylized model (2). Consequently, the variance of the disturbance in (3) may differ substantively from that in the NKPC model (2).

These issues are crucial in the GMM estimation for the NKPC model and yet they are under-investigated in the literature. Gali et al. (2001) come close to the

issue but leave the consideration for “future research”. The current study, therefore, embarks on an investigation of serial correlation and the nature of these error terms, based on the differing constructions of (2) and (3).

To this end, we follow Pagan (1984) and approximate the unobserved inflation expectations in (2) by projecting realized future inflation (i.e.  $\pi_{t+1}$ ) on the underlying IV set  $Z$ , viz.

$$E_t \pi_{t+1} = P_Z \pi_{t+1}, \quad (6)$$

where  $P_Z = Z(Z'Z)^{-1}Z'$  is the projection matrix in terms of the IV set.

It follows that this procedure will yield precisely the same coefficient estimates as those obtained through the GMM estimation for (3), while the standard errors will be different since the former ignores uncertainty in the estimation of the projection matrix (Pagan 1984).

In addition, to provide valid tests for serial correlation in the GMM estimation of the dynamic model with a lagged dependent variable, we employ the IV serial correlation test proposed by Godfrey (1994) under the null hypothesis of no serial correlation in the underlying GMM estimation. It is an instrumental variable test for serial correlation implemented by adding appropriate lagged residuals from the initial estimation to the regressors of the underlying model, and checking their joint significance by applying the Lagrange Multiplier principle.

### III. Replication of the results of Gali and Gertler (1999)

Based on the foregoing construction, Table 1 summarizes the replication results of GG, with S1 and S2 denoting standard errors corresponding to the specifications (3) and (2) respectively. Note that the IV set in GG includes four lags of inflation, the output gap, labor income share (denoted NFB-LS), term spread of long-short interest rates, commodity price inflation, and wage inflation (non-farm business sector).<sup>2</sup>

Notice that the coefficient estimates reported in Table 1 generally coincide with those in GG: the GMM point estimates on future inflation are generally higher than 0.55 while those on lagged inflation are smaller than 0.45, either with or without

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<sup>2</sup> It should be noted that the data used in this paper are not exactly the same as used in Gali and Gertler (1999). In particular, the data for the output gap in the replication exercise is different from that used in Gali and Gertler (1999).

**Table 1. Replication results of Gali and Gertler (1999)**

	$\tilde{\alpha}_f$	$\tilde{\alpha}_b$	$\tilde{\alpha}_y$	S.E.	$p$ -auto	$\bar{R}^2$	$p$ -over
CBOGAP	0.556	0.427	-0.003				
S1	(0.084)	(0.082)	(0.005)	0.224	0.000	0.87	0.812
S2	(0.086)	(0.070)	(0.006)	0.253	0.000	0.83	0.888
NFB-LS	0.529	0.384	0.136				
S1	(0.072)	(0.060)	(0.077)*	0.222	0.000	0.87	0.738
S2	(0.083)	(0.064)	(0.125)	0.251	0.000	0.83	0.894
$\alpha_f + \alpha_b = 1$							
CBOGAP	0.576	0.424	-0.003				
S1	(0.085)	(0.085)	(0.005)	0.224	0.000	0.87	0.837
S2	(0.068)	(0.068)	(0.006)	0.252	0.000	0.83	0.973
NFB-LS	0.569	0.431	0.076				
S1	(0.067)	(0.067)	(0.043)*	0.223	0.000	0.87	0.804
S2	(0.065)	(0.065)	(0.076)	0.251	0.000	0.83	0.980

Notes: the data (except for the CBOGAP) is from Gali and Gertler (1999) and the sample spans 1960Q1-1997Q4. The GG IV set includes four lags of each of inflation, the output gap, labor income share, term spread of long-short interest rates, commodity price inflation, and wage inflation (non-farm business sector). S1 and S2 refer to Newey-West heteroskedasticity-and-autocorrelation consistent standard errors (12 lags bandwidth kernel) associated with equations (3) and (2) in the text respectively. CBOGAP denotes the output gap based on the real potential GDP estimated by US Congressional Budget Office; NFB-LS refers to labor share of non-farm business sector;  $p$ -over denotes  $p$ -value of Hansen's  $J$ -test;  $p$ -auto denotes  $p$ -value of IV serial correlation test of Godfrey (1994), under the null hypothesis of no serial correlation; \* denotes statistical significance at the 10% level; S.E. is standard error of regression.

the restriction of convex combination on the forward- and backward-looking components (i.e.,  $\alpha_f + \alpha_b = 1$ ), indicating the dominant role of forward-looking behavior. Moreover, coefficient estimates on the output gap (i.e. the CBOGAP) are negative while estimates on the measures of labor income share are positive. Essentially, it is the latter observation that leads GG to conclude that it is the labor income share, rather than the output gap that should be a valid driving force for inflation in the NKPC.

However, although the coefficient estimates in Table 1 seem to support GG's argument, the inference based on NKPC (2) can differ significantly from that based on (3). This impression is confirmed by comparing standard errors associated with the labor share under the two different constructions. For instance, under GG's construction (3), the standard error associated with the coefficient estimate (0.136) on the labor share is 0.077, which implies that the estimate is significant at 10%

level. Nevertheless, this coefficient estimate on the labor share is insignificant at conventional levels when the standard error (0.125) based on the NKPC model (2) is considered. The same scenario occurs when the restriction of  $\alpha_f + \alpha_b = 1$  is imposed. Interestingly, however, the difference between S1 and S2 associated with the output gap appears less distinct, although it turns out to be immaterial since the GDP gap is statistically insignificant under either case.

In essence, our finding here indicates that even if one were ready to accept the claim in GG that the output gap is not a valid driving variable for inflation, the celebrated results of their labor income share being a valid inflation pressure in the NKPC may be accepted only with caution.

The observed differences illustrated above are affected by the introduction of the rational prediction error  $\epsilon_{t+1}$ . The potential influence of this disturbance can also be evaluated by comparing the standard error of the regressions (S.E.) reported in the middle column of Table 1. By construction,

$$\left\{ \begin{array}{l} S.E. = \sqrt{\tilde{\sigma}_\eta^2 + \tilde{\sigma}_e^2 - 2\tilde{\sigma}_{\eta,e}} \quad \text{under (3)} \\ S.E. = \sqrt{\tilde{\sigma}_\eta^2} \quad \text{under (2)} \end{array} \right. \tag{7}$$

where  $\tilde{\sigma}_\eta^2$ ,  $\tilde{\sigma}_e^2$ , and  $\tilde{\sigma}_{\eta,e}$  denote variances of  $\eta$ ,  $e$ , and covariance between the two error terms respectively. Under the assumptions in GG, one would expect  $\tilde{\sigma}_{\eta,e}$  to be negligible and hence the S.E. associated with S1 should be inflated by the variance of the forecasting error. However, the results in Table 1 indicate that the opposite scenario can also occur. This finding implies that in practice the correlation between the forecasting error and the specification disturbance in the NKPC tends to be influentially large.

All of the above is based on the assumption that  $\eta_t$  in (2) is white noise so that lagged values of inflation are legitimate instruments. However, the results of the serial correlation tests ( $p$ -auto) for  $\eta_t$  in Table 1 suggest that in all regressions, the null hypothesis of no serial correlation (up to 4 lags) can be rejected at 1% significance level. The presence of serial correlation in the stylized NKPC model casts doubt on the validity of lagged values of inflation (and possibly real variables) as instruments. In principle, we would expect Hansen's (1982)  $J$ -test (overidentifying restrictions test) to provide an auxiliary diagnosis. Unfortunately, it is widely known that the  $J$ -test can have noticeably low power in detecting invalid moment conditions, in particular when the estimated residuals manifest serial correlation. Consequently, the  $J$ -test could be deceptive and this may well be the case here:  $p$ -values of  $J$ -test ( $p$ -over) are surprisingly large.

Overall, the presence of serial correlation in the estimated residuals implies that the assumption of white noise  $\eta_t$  in the stylized NKPC model (2) is unlikely to hold. The empirical finding of serial correlation may be unsurprising in that the theoretical derivation of the NKPC does not predict the absence of serial correlation in  $\eta_t$ . More crucially, the stylized NKPC specified by GG may be empirically insufficient to capture U.S. inflation dynamics. On theoretical grounds, if we interpret one period as being fairly short (say one quarter in quarterly models), it is likely that agents who follow “the rule of thumb” of backward-looking behavior may look at past inflation over more than one period. Therefore, we develop and test an extended specification of the NKPC in the following section.

#### IV. An extended model

The previous section presented evidence that the error term in the stylized NKPC developed by GG is serially correlated. Gali et al. (2001) also acknowledge that the independent and identically distributed assumption on  $\eta_t$  may be too strong. The serial correlation may of course, come from different sources and so give rise to various implications. One important implication of the serial correlation uncovered is that the stylized specification is not sufficient to capture all of the U.S inflation dynamics. Therefore, extending the lagged inflation structure could be a rewarding innovation. More importantly, as we show below, this extension can be derived from the micro economic foundations of inflation similar to that used in GG.

To be specific, we assume an economic environment similar to Calvo’s (1983) model, in which each firm is able to revise its prices in any given period with a fixed probability  $(1 - \theta)$  and, hence, there is a probability  $\theta$  that it must keep its price unchanged. As in GG, we also assume both “forward-” and “backward-looking” firms co-exist in the economy with a proportion of  $\omega$  and  $(1 - \omega)$  respectively. Further, we extend the rule of the recent pricing behavior of the backward-looking firms to incorporate a weighted process of past inflation, instead of a stylized one lag of inflation inertia.

Based on the regular assumptions in Calvo’s model and log-linear approximations, it is possible to obtain the (log) aggregate price level as

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^*, \quad (8)$$

where  $p_t^*$  is the new price set in period  $t$ . Following GG, let  $p_t^f$  be the price set by forward-looking firms and  $p_t^b$  be the price set by backward-looking firms at time  $t$ .

Then the new price (relative to the aggregate price) can be expressed as a convex combination of  $p_t^F$  and  $p_t^B$ , viz.

$$p_t^* - p_t = \omega(p_t^F - p_t) + (1 - \omega)(p_t^B - p_t). \tag{9}$$

Next, following Woodford (2003), the pricing behavior of the forward-looking firms can be written as

$$p_t^F - p_t = \theta\beta \sum_{s=0}^{\infty} (\theta\beta)^s E_t \pi_{t+s+1} + (1 - \theta\beta) \sum_{s=0}^{\infty} (\theta\beta)^s E_t \varsigma y_{t+s}, \tag{10}$$

where  $\beta$  denotes a subjective discount factor,  $\varsigma$  is introduced by the procedure of log-linearization (see Woodford for a discussion of the economic implications of  $\varsigma$ ), and  $y_t$  denotes the real marginal cost or real output gap. Iterating (10) gives

$$p_t^F - p_t = \theta\beta E_t \pi_{t+1} + (1 - \theta\beta)\varsigma y_t + \theta\beta E_t (p_{t+1}^F - p_{t+1}). \tag{11}$$

In GG, the backward-looking firms are assumed to follow the rule of thumb of backward-looking behavior in their price setting, viz.

$$p_t^B - p_t = p_{t-1}^* - p_t + \pi_{t-1}. \tag{12}$$

As emphasized in the literature, this is an elegant innovation in that the backward-looking firms can now set their prices to the average price determined in the most recent of price adjustments with a correction for inflation.

However, inflation inertia in (12) is confined to one single lag, which may neglect the importance of other historical inflation in predicting current inflation. In particular, if we interpret one period as being short, the backward-looking agents are likely to take more than one period to respond fully to changes in actual inflation. Therefore, it would appear reasonable to replace  $\pi_{t-1}$  in (12) with a weighted average of inflation over several periods in the past. As such, we extend (12) in the following way:

$$p_t^B - p_t = (p_{t-1}^* - p_t) + \pi_{t-1} + \rho^*(L)\Delta\pi_{t-1}, \tag{13}$$

where  $\rho^*(L) = \rho_1^* + \rho_2^*L + \rho_3^*L^2 + \dots + \rho_q^*L^{q-1}$  is a polynomial in lag operator. For quarterly models, lag order of one-year (i.e.  $q = 4$ ) appears to be plausible, although a more (or less) extended period could be chosen following some empirical rule established over time.

Combining (8)-(13), it is shown in the Appendix that the extended specification (econometric model) of the New Keynesian Phillips Curve takes the form.

$$\pi_t = \alpha_f E_t \pi_{t+1} + \alpha_b \pi_{t-1} + \alpha_p(L) \Delta \pi_{t-1} + \alpha_y y_t + \eta_t, \quad (14)$$

where

$$\left\{ \begin{array}{l} \alpha_f = \theta \beta \psi^{-1} \\ \alpha_b = [(1 - \omega) - \theta \beta (1 - \omega)(1 - \theta) \rho_1^*] \psi^{-1} \\ \alpha_p(L) = (1 - \omega)(1 - \theta) [\rho^*(L) + \theta \beta \rho'(L)] \psi^{-1} \\ \alpha_y = \omega(1 - \theta \beta)(1 - \theta) \xi \psi^{-1} \\ \psi = \theta + (1 - \omega)(1 - \theta) + \theta \beta (1 - \omega) + \theta \beta (1 - \omega)(1 - \theta) \rho_1^* \\ \rho^*(L) = \rho_1^* + \rho_2^* L + \rho_3^* L^2 + \rho_4^* L^3 \\ \rho'(L) = \rho_2^* + \rho_3^* L + \rho_4^* L^2 \end{array} \right. \quad (15)$$

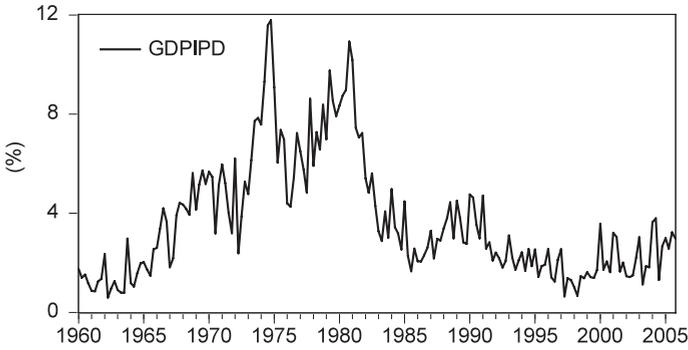
## V. Empirical results

This section presents our GMM estimation results for the extended NKPC model (14). The data used in our empirical studies are obtained from the Federal Reserve Bank of St. Louis spanning 1960Q1-2005Q4. Inflation is measured by the quarterly growth rate of GDP implicit deflator price index (annualized and seasonally adjusted) and denoted by GDPIPD. To investigate the role of the output gap and labor income share in the extended model, we concentrate on the CBOGAP and the NFB-LS (i.e. labor income share) as defined in section III. In the robustness analysis, however, we also consider the real output gap derived from the Hodrick-Prescott filter and a fitted quadratic function of time. The data for inflation and the real variables are plotted in Figure 1 and 2 respectively.

Because the extended model is free of serial correlation in the empirical estimations, lagged inflations in the model are now valid instruments. In addition, given the fact that the NKPC is one of the key equations in monetary policy analysis (see, for example, Clarida et al. 1999), we adopt as a baseline IV set in the regression four lags of each of the real variable, the unemployment rate, and short- and long-term interest rates, based on the following considerations.

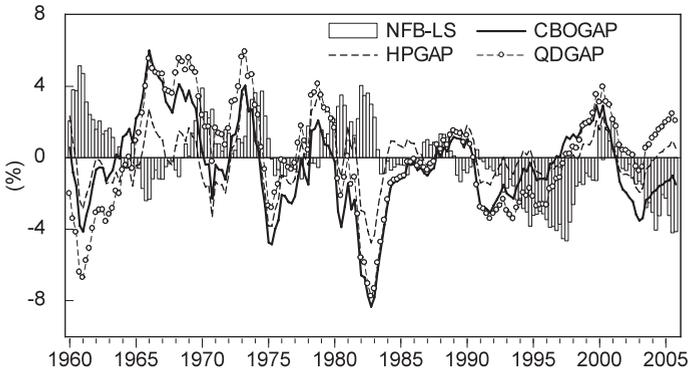
First, because the real variable ( $y$ ) in the regression is assumed to drive inflation, lagged values of  $y$  and hence the unemployment rate (by Okun's Law), are also

Figure1. The U.S. inflation series: 1960Q1-2005Q4



Notes: GDPIPD denotes quarterly growth rate of the GDP implicit price deflator (annualized and seasonally adjusted).

Figure 2. Labor income share and alternative output gap measures



Notes: NFB-LS refers to labor income share of non-farm business sector; CBOGAP denotes the output gap based on the real potential GDP estimated by US Congressional Budget Office; QDGAP denotes quadratically detrended log real GDP; HPGAP denotes Hodrick-Prescott filtered log real GDP (with penalty parameter=1600).

expected to be valid instruments for the real variable, in the current period and for future inflation. Second, the correlation between interest rates and inflation can be understood in terms of the standard AS/AD system. The important relationship between long-term interest rates and inflation is implied in the recent work by McGough et al. (2005). Therefore, our baseline IV set appears to be a sufficient candidate to capture the dynamics for the extended NKPC.

Based on this construction, we summarize in Table 2 the key coefficient estimates of the extended model in conjunction with the statistics of interest over the sample. In addition to the same underlying statistics as presented in Table 1, we also report p-values of the joint significance test on the extra lagged inflation from order two to four ( $\tilde{\alpha}_p(L)$ ).

Several interesting findings emerge from Table 2. First, the coefficient estimates of  $\tilde{\alpha}_y$  are intuitive and show that CBOGAP obtains a positive sign and is statistically significant at the 10% level (without convex restriction) and at 5% level (with convex restriction) respectively. The magnitude of the estimates on the CBOGAP are 0.11, indicating that a 1% increase in the output gap will lead to 0.11% rise in inflation, *ceteris paribus*. This is well in accord with the established economic theory as indicated, for example, in Roberts (1995).

However, coefficient estimates on the labor income share appear quite different from that on the CBOGAP. Table 2 shows that although the estimates on the labor share are also positive, they are insignificant in all cases, which suggest that the labor share is not a significant driving force for inflation in the extended NKPC, a finding in broad agreement with Rudd and Whelan (2006) but in contrast to GG.

Perhaps a more important finding in Table 2 is that serial correlation is generally absent in the extended model. The absence of serial correlation indicates that the extended NKPC model (14) is more plausible than the stylized specification (2) in

**Table 2. Estimation results of the extended NKPC**

	$\tilde{\alpha}_f$	$\tilde{\alpha}_b$	$\tilde{\alpha}_y$	$\tilde{\alpha}_p(L)$	$\rho$ -auto	$\bar{R}^2$	$\rho$ -over
CBOGAP	0.270 (0.232)	0.731 (0.214)	0.110* (0.058)	0.006	0.865	0.81	0.635
NFB-LS	0.640 (0.151)	0.356 (0.136)	0.030 (0.047)	0.057	0.078	0.82	0.579
$\alpha_f + \alpha_b = 1$							
CBOGAP	0.269 (0.207)	0.731 (0.207)	0.110** (0.056)	0.006	0.864	0.81	0.706
NFB-LS	0.645 (0.136)	0.355 (0.136)	0.028 (0.044)	0.040	0.082	0.82	0.658

Notes: the sample spans 1960Q1-2005Q4. Inflation expectations are measured by fitted values of the realized future inflation on the baseline IV set (see text for fuller details). Heteroskedasticity-robust standard errors are reported in parentheses. The statistics reported under  $\tilde{\alpha}_p(L)$  refer to p-values of joint significance test on lagged inflation of order 2 to 4. \* and \*\* denote statistical significance at the 10% and 5% levels respectively. Other notations follow those in Table 1.

explaining U.S. inflation dynamics. Another important finding is that the p-values of the joint significance tests on the extra lagged inflation (from order two to four) are smaller than 5% in most regressions (0.057 in the second regression), indicating a statistically significant role of the extra lagged inflation in the NKPC.

To summarize the results reported in Table 2, we find that the extended NKPC model is free of serial correlation and the conventional output gap is still a valid driving force for inflation in the extended model. The labor income share, however, is statistically insignificant in the NKPC once serial correlation is accounted for. Overall, the estimation results provide clear support for the extension of the lagged inflation dynamics in the NKPC.

To assess the robustness of these baseline findings, Table 3 reports GMM estimation results for the extended model over the same sample period as that in GG, namely 1960Q1-1997Q4, while Table 4 presents GMM results for the model using the alternative real output gap measures. In both tables, serial correlation is absent and extra values of lagged inflation are statistically significant at conventional levels. In addition, Table 3 shows that the CBOGAP remains significant (at 10% level) with a slight moderation in magnitude over the shorter sample of 1960-1997. Interestingly, however, the results in Table 4 suggest that the coefficient estimates on the real output gap derived using the quadratically detrended method and Hodrick-Prescott filter appear to be insignificant. Nonetheless, both of the two real GDP gap measures maintain their intuitive signs and the magnitude of the estimate on the latter is close to the baseline estimate on the CBOGAP.

**Table 3. GMM estimation results of the extended NKPC over 1960Q1-1997Q4**

	$\tilde{\alpha}_f$	$\tilde{\alpha}_b$	$\tilde{\alpha}_y$	$\tilde{\alpha}_p(L)$	$\rho$ -auto	$\bar{R}^2$	$\rho$ -over
CBOGAP	0.337 (0.227)	0.675 (0.211)	0.107* (0.062)	0.011	0.914	0.80	0.733
NFB-LS	0.685 (0.159)	0.321 (0.141)	0.029 (0.051)	0.055	0.205	0.81	0.564
$\alpha_f + \alpha_b = 1$							
CBOGAP	0.321 (0.206)	0.679 (0.206)	0.107* (0.062)	0.014	0.909	0.81	0.801
NFB-LS	0.675 (0.139)	0.325 (0.139)	0.033 (0.047)	0.036	0.219	0.81	0.628

Notes: refer to notes in Table 2.

**Table 4. GMM estimation results of the extended NKPC using alternative output gap**

	$\tilde{\alpha}_f$	$\tilde{\alpha}_b$	$\tilde{\alpha}_y$	$\tilde{\alpha}_p(L)$	$\rho$ -auto	$\bar{R}^2$	$\rho$ -over
QDGAP	0.489 (0.157)	0.500 (0.144)	0.050 (0.038)	0.015	0.200	0.82	0.444
HPGAP	0.433 (0.215)	0.556 (0.185)	0.137 (0.107)	0.024	0.369	0.82	0.459
$\alpha_f + \alpha_b = 1$							
QDGAP	0.516 (0.153)	0.484 (0.153)	0.045 (0.035)	0.017	0.176	0.82	0.521
HPGAP	0.463 (0.164)	0.537 (0.164)	0.126 (0.087)	0.024	0.322	0.82	0.539

Notes: the sample spans 1960Q1-2005Q4. QDGAP denotes quadratically detrended log real GDP. HPGAP denotes Hodrick-Prescott filtered log real GDP (with penalty parameter=1600). The notations follow those in Table 2.

For completeness, we also evaluate the extended NKPC using the IV choice of GG. The corresponding results (not reported here) suggest that both the output gap measures and the labor income share obtain positive signs, but are statistically insignificant. This may be induced by the excessive number of variables in the IV set of GG, as suggested by Mavroeidis (2005). Of course, the IV set in GG was initially designed for estimating the stylized NKPC and may not necessarily perform well in the extended model. Nonetheless, even based on GG's IV choice, extra lagged inflation is significant in all regressions, providing further support for the extension of the lagged inflation dynamics in our version of the NKPC.

## VI. Discussion

The foregoing section extends the stylized specification of the NKPC with micro economic foundations by assuming that the backward-looking agents care about inflation over the past several quarters rather than one period in their price-setting behavior. The extended model appears to be free of serial correlation and the empirical results support our underlying extension. Once serial correlation is accounted for, we find that the conventional real output gap rather than the labor income share is the more valid driving force for inflation.

A natural question to ask with respect to the extended model is why do backward-looking firms choose to set their prices based on inflation over the past few quarters,

instead of just one quarter as specified in GG? We suggest that at least three reasons can be made to explain this. First, because U.S. inflation during the 1960-2005 period appears fairly persistent (Zhang and Clovis 2009) and inflation over the past few periods, (in addition to the very nearest past), may also contain important information for future inflation. Indeed, the results in Stock and Watson (1999) imply exactly this: that past inflation beyond order one also has important predictive power on future inflation.

In addition, running an autoregressive (AR) process for the U.S. GDP deflator inflation over the underlying sample, for example, gives rise to an optimal lag order of five, based on either Akaike information criterion (AIC) or Bayesian information criterion (BIC), implying the importance of past inflation beyond lag order one in inflation dynamics.

Thirdly, the extension of the stylized NKPC model is also in line with recent research on nominal rigidities of the U.S. economy. In particular, Christiano et al. (2005) find that average duration of the price and wage contracts is more than one quarter (roughly two to three quarters). In turn, it is very likely that the “backward-looking” firms also consider past inflation of a similar time duration in revising their prices.

Therefore, extending the stylized NKPC model, from both theoretical and empirical grounds, appears to be an appropriate development and a valuable contribution to the debate. Of course, the current study is not the only one which has contributed to extending the micro founded short-run inflation dynamics. Several specifications based on the stylized NKPC have been proposed currently, however it is our analysis that explicitly addresses the overlooked issue of serial correlation in the NKPC. For example, Bardsen et al. (2004) extend the baseline NKPC based on variable addition and encompassing tests for European inflation dynamics. Interestingly, these authors find that the fourth lag of inflation in conjunction with lagged GDP gap should be present in the European inflation dynamics. Jondeau and Bihan (2005) also focus on the dynamics of inflation in Europe, as well as for the United States. Employing West’s (1997) approach of filtering moment conditions prior to computing the GMM weighting matrix, they estimate the NKPC with a composite error as in our equation (3). Jondeau and Bihan (2005) argue that a moving average of three lags and leads in the inflation representation in the NKPC fits the data better for both the United States and for Europe. However, this proposal is also likely to induce additional serial correlation if the restriction of moving average process in the regressors is not valid. Recent studies by Cogley and Sbordone (2005), Roberts (2005), and McCallum (2007), among others, are further contributions

in searching this area for modifications on the stylized NKPC. Taken as whole, an encouraging, though still tentative consensus is emerging that shows that enriching inflation dynamics in the stylized NKPC is likely to be more empirically coherent with the leading micro economic theory.

## VII. Conclusions

The GMM estimation in Gali and Gertler (1999) is often criticized on the grounds that the approach may suffer from finite sample problems, weak IV, or some identification problems. The current paper, however, contributes to the literature by addressing the serial correlation problem in the stylized NKPC that had hitherto been neglected and by comparing inferences between different constructions of inflation expectations approximations. Our results show that the stylized specification of the NKPC is significantly serially correlated. In addition, even under the assumption of independence and identical distribution made in Gali and Gertler (1999) regarding the structural and the forecasting errors, the different constructions of inflation expectations can also give rise to significantly different inferences.

These findings lead us to develop and estimate an extended model with richer lagged inflation dynamics. Our empirical results suggest that the extended model provides a good description of U.S. inflation dynamics, with the conventional output gap as a valid real driving variable for inflation. In all regressions of the extended NKPC model, however, the labor income share is statistically insignificant, which indicates that the celebrated finding in Gali and Gertler (1999) does not hold once serial correlation in the NKPC is properly accounted for.

Overall, our analysis underscores the need for extending the stylized NKPC of rational expectations, it also provides some supporting evidence for “output gap-based” NKPC, while at the same time casts doubt on the validity of the celebrated “marginal-cost based” NKPC. Although not the focus of the present paper, the relative importance of inflation components entails further investigation. Zhang, Osborn, and Kim (2008) provide important contributions in this direction and further research would seem most warranted.

### Appendix: derivation of the extended hybrid New Keynesian Phillips curve

Note that subtracting  $p_t$  from both sides of (8) gives

$$p_t^* - p_t = \frac{\theta}{1-\theta} \pi_t, \quad (\text{A1})$$

and iterating (9) one period ahead produces

$$p_{t+1}^* - p_{t+1} = \omega(p_{t+1}^F - p_{t+1}) + (1-\omega)(p_{t+1}^B - p_{t+1}), \quad (\text{A2})$$

which in turn gives

$$(p_{t+1}^F - p_{t+1}) = \frac{1}{\omega} \left[ \frac{\theta}{1-\theta} \pi_{t+1} - (1-\omega)(p_{t+1}^B - p_{t+1}) \right]. \quad (\text{A3})$$

In order to derive the last item in the square bracket of (A3) in terms of the inflation rate, lag (A1) one period and subtract  $p_t$  from both sides of the resulting equation

$$p_{t-1}^* - p_{t-1} - p_t = \frac{\theta}{1-\theta} \pi_{t-1} - p_t, \quad (\text{A4})$$

and derive

$$p_{t-1}^* - p_t = \frac{\theta}{1-\theta} \pi_{t-1} - \pi_t. \quad (\text{A5})$$

Substituting (A5) into (13) gives

$$p_t^B - p_t = \frac{1}{1-\theta} \pi_{t-1} + \rho^*(L) \Delta \pi_{t-1} - \pi_t. \quad (\text{A6})$$

Combining (A6) and (A3) gives

$$(p_{t+1}^F - p_{t+1}) = \frac{1}{\omega} \left\{ \left[ \frac{\theta}{1-\theta} + (1-\omega) \right] \pi_{t+1} - (1-\omega) \left[ \frac{1}{1-\theta} \pi_t + \rho^*(L) \Delta \pi_t \right] \right\}. \quad (\text{A7})$$

Then, substituting (A7) into (11), we derive

$$p_t^F - p_t = \frac{\theta\beta}{\omega(1-\theta)} E_t \pi_{t+1} - \frac{\theta\beta(1-\omega)}{\omega} \left[ \frac{1}{1-\theta} \pi_t + \rho^*(L) \Delta \pi_t \right] + (1-\theta\beta) \zeta_t. \quad (\text{A8})$$

Now, combining (9), (A1), (A6), and (A8), we obtain

$$\begin{aligned} \frac{\theta}{1-\theta}\pi_t = (1-\omega) & \left[ \frac{1}{1-\theta}\pi_{t-1} + \rho^*(L)\Delta\pi_{t-1} - \pi_t \right] \\ & + \omega \left[ \frac{\theta\beta}{\omega(1-\theta)}E_t\pi_{t+1} - \frac{\theta\beta(1-\omega)}{\omega} \left[ \frac{1}{1-\theta}\pi_t + \rho^*(L)\Delta\pi_t \right] + (1-\theta\beta)\zeta y_t \right]. \end{aligned} \quad (\text{A9})$$

Note that

$$\rho^*(L)\Delta\pi_t = \rho_1^*\pi_t - \rho_1^*\pi_{t-1} + \rho'(L)\Delta\pi_{t-1}, \quad (\text{A10})$$

where  $\rho'(L) = \rho_2^* + \rho_3^*L + \rho_4^*L^2$ . Now, rearranging (A9) gives

$$\begin{aligned} & \left[ \frac{\theta}{1-\theta} + (1-\omega) + \theta\beta(1-\omega) \left( \frac{1}{1-\theta} + \rho_1^* \right) \right] \pi_t \\ & = \frac{\theta\beta}{1-\theta} E_t\pi_{t+1} + \left[ \frac{1-\omega}{1-\theta} - \theta\beta(1-\omega)\rho_1^* \right] \pi_{t-1} \\ & \quad + (1-\omega) [\rho^*(L) + \theta\beta\rho'(L)] \Delta\pi_{t-1} + \omega(1-\theta\beta)\zeta y_t, \end{aligned} \quad (\text{A11})$$

which produces the extended (economic) model of the NKPC, viz.

$$\pi_t = \alpha_f E_t\pi_{t+1} + \alpha_b \pi_{t-1} + \alpha_p(L)\Delta\pi_{t-1} + \alpha_y y_t, \quad (\text{A12})$$

which corresponds to the econometric model (14) in Section IV (by adding a standard model specification error), where parameters values are specified in equation (15).

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