On liability dollarization: a simple model with financially closed and open economies *

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Abstract

This paper presents a simple analysis of debt-denomination choice in small open economies, either financially isolated, or integrated in world credit markets. In the model, borrowers are producers of non-traded goods, the relative prices are subject to shocks. The nominal exchange rate is also random, subject to an exogenous (policy) shock. Debt obligations can be denominated in units of traded goods (dollarized contracts) or in local currency. Default can occur, and it is costly to the contract parties. In a financially closed economy, the model implies that the choice of denomination depends on the probability of default with each contract, and on the value of the default cost. If real and nominal exchange rates are positively correlated, dollarized contracts tend to be preferred to (non-contingent) nominal contracts when real shocks are small and there is "excess volatility" in the nominal shock. When foreign risk-neutral investors are added to the model, in addition to the default properties, the choice of denomination also depends on the possibility of hedging relative-price

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risks offered by contracts to residents. The model does not reflect "home country bias": here, in equilibrium, domestic lenders invest their funds in the foreign bond, while local borrowers finance projects in the international market.

1 Introduction

The practices concerning the denomination of financial contracts vary widely from case to case. In some instances, nominal contracting is the norm, even for assets with long maturities. In other circumstances, agents routinely utilize indexed units in a large set of transactions. Some economies are characterized by a large-scale use of foreign currency denominations. Clearly, the denomination of assets may have strong effects on macroeconomic performance. In particular, shocks leading to slight disturbances to a contractual system with liabilities denominated in a certain unit can induce a major breakdown with another unit of denomination. "Dollarized" financial systems are vulnerable to large movements in the real exchange rate. There is ample evidence of this effect in recent crisis episodes. However, dollarization is essentially a market outcome, that is, it is an endogenous phenomenon. Thus, it is important to understand what leads agents to choose such arrangement.

The analysis in this paper concentrates on the decisions of private agents, taking as given the determination of "outside" shocks and policy surprises[1]. We consider a theoretical model where borrowers face three possible alternative contractual forms: nominal (denominated in domestic monetary units), "dollarized " (with payments defined in terms of a traded good with fixed international price) and indexed (where nominal obligations vary with the aggregate price level, possibly measured through a noisy signal). We consider two versions of this model, one without international capital flows and another introducing international (risk-neutral) investors. We find that, in the financially closed economy, preference over contracts depends crucially

[1] We recognize that the denomination of contracts influences the costs and benefits of alternative courses of monetary policies. Dollarization may create "fear of floating" (Calvo and Reinhart (2002)). Conversely, the choice of contractual units can be expected to depend on the anticipated behavior of monetary authorities. The interaction between the features of monetary policies and the types of contracts which are prevalent in the economy is certainly an interesting and relevant matter, but we will not deal with it here.

on their properties concerning the likelihood of default, which can be determined in terms of the distribution of shocks. In this first version of the model results show that when comparing nominal and "dollarized " contracts, the ranking depends on whether monetary policies offset the effects of "real shocks" and on the magnitude of the "excess volatility " of prices that those policies may induce. In the economy with perfect access to international financial markets this comparison is less clear, since international investors introduce now different risk-sharing properties to the contracts traded in the economy.

The main elements of the first version of the model are the following. The economy produces two goods. The output of traded goods is determined exogenously. Producers of non-traded goods must borrow resources in order to finance their production projects from lenders who own the traded good endowments. There are random shocks on those endowments (and, indirectly, on relative prices), and also on the "nominal exchange rate". These are interpreted as induced by monetary policy. Borrowers consume both goods, with Cobb-Douglas utilities, so that indirect utilities depend linearly on the value of income deflated by a consumer-price-index. There may be default on debts; in that case the parties of the transaction incur "settlement" costs. In one setting, lenders are assumed to have the same preferences as borrowers. This represents the case of pure domestic lending.

In the second version, foreign lenders are introduced. These international investors care only for consumption of traded goods, evaluating the return of their loans in terms of those traded goods. Borrowers still maximize their income in terms of the consumption basket. As mentioned above, in addition to the default probabilities, the risk-sharing properties of the contracts also matter in this economy. We can then establish a typology of conditions which determine the contractual unit that agents would prefer.

The notion of "original sin" has received much attention in the literature (see, for example, Haussmann and Panizza (2003)). The argument refers to the inability of many governments and private agents to borrow in international markets by issuing debt denominated in their own currency. The phenomenon seems naturally linked to the incentives that governments may have to reduce the real value of their liabilities (or those of domestic residents) against non-residents (see Chamón (2002) and Tirole (2002)). We are concerned with a related but different issue, namely, the dollarization of domestic financial transactions (which in the colorful wording of the field has been called "domestic" original sin). Our purpose is to study how this phenomenon may emerge as determined by the fundamental conditions of nominal and relative price variability.

Our paper is related to the recent literature which has analyzed the relationship between monetary policy and liability dollarization, in works such as those of Ize and Levy - Yeyati (2003), Ize and Parrado (2002), Jeanne (2003), Broda and Levy-Yeyati (2003), Ize and Powell (2004) and Chang and Velasco (2004). These papers (which are briefly discussed in section 4) have, in a way, more complicated frameworks than the one we use here. Our analysis tries to generate "ground-level" results based on a barebones model, which abstracts away from financial intermediation and information asymmetry, disregards the possibility of externalities, assumes that domestic agents preferences are risk neutral in terms of a CPI basket (to be defined later), and where monetary policies are treated as exogenous.

The rest of the paper is organized as follows. Section 2 presents the economy without access to international capital markets, including the characterization of equilibrium contracts. Section 3 extends the model introducing international risk-neutral lenders. Section 4 presents a detailed discussion on the related literature. Section 5 concludes.

2 The case of a financially closed economy

The economy considered first is open to international trade but, by assumption, it cannot exchange assets with the rest of the world: all financial transactions take place between residents. Even so, it is of course possible that the domestic agents choose to denominate contracts in an international unit of account. In this section we discuss this possibility. First, we present the setup of the model. Then, we analyze the default properties of "dollarized " and nominal contracts under different configurations of shocks, stressing the cases where monetary policies make the nominal exchange rate high when the real exchange is high. Finally, we consider the optimal choice of contract denomination, which turns out to be uniquely associated with the probabilities of default.

The relevant time frame contains two periods, denoted as $t = 0, 1$. There are two goods, one traded with the rest of the world, T, the other one a non-tradeable, N. The relative price of N, denoted p_N, is interpreted as the inverse of the real exchange rate. The economy is financially closed, in the sense that its agents do not borrow or lend in international markets. The trade balance must be zero: since we will not consider changes in the terms of trade, tradeable goods can be considered homogeneous, and everything happens as if, in equilibrium, the production of goods T was used locally.

There exists two groups of agents. The first group is a continuum of *lenders*, who receive an endowment of T goods equal to $\bar{k} > 0$ at date 0, and a random date 1 endowment of T goods equal to $\widetilde{y}_{1T}$, which is expressed as $z\bar{k}$.The variable z thus measures the abundance or scarcity of traded goods in the future, and can have two possible values: low (L) and high (H); the probability of a bad draw is denoted by π. Lenders are assumed to have the following preferences:

$$U^L = \bar{k} - k + E_0^L \left[\left(c_T^L \right)^{\alpha} \left(c_N^L \right)^{1-\alpha} \right]$$

where $k \in \left[0, \bar{k}\right]$ is the amount of T goods lent to entrepreneurs, c_j^L the amount of j goods ($j = T$,N) consumed by lenders at date 1 and E_0^L is the expectation operator according to the lender's date 0 beliefs. Here $\alpha \in (0, 1)$ is the parameter that measures the share the share of total expenditures allocated to tradeables.

There exists a continuum of identical domestic producers of non-traded goods, who are the *borrowers*. At date 0 these borrowers finance projects that require a fixed amount of k units of tradeables to generate a (deterministic) quantity of non-tradeables, Ak. If the value of output is strictly less than the total debt payments required by the contract there is default. In that case, the output of the firm is appropriated by the lenders, and there is a fixed cost of ψ traded goods. This cost, intended to represent the sundry losses incurred by the parties of a defaulted contract, is modelled as a transfer to a third party (say, the judge who enforces the contract), who is also a consumer in the economy, and has the same preferences over future goods as the other agents. Those assumptions simplify the problem by making aggregate demand and the real exchange rate independent of the existence of default, and independent of the size of the parameter ψ.

Entrepreneurs (and judges) are assumed to have preferences represented by an ex-ante expected utility function

$$E_0 \left[\left(c_T^B \right)^{\alpha} \left(c_N^B \right)^{1-\alpha} \right]$$

where c_j^B is the amount of j goods ($j = T$,N) consumed by borrowers at date 1 and E_0 is the expectation operator according to the agent's date 0 beliefs.

Remark 1 *Given the symmetry of date-1 preferences, the results of this section would apply also to the case where lending finances the production of traded goods, with suitable re-interpretations of the variables, such as replacing the price of the output of the project relative to that of tradeables, p_N, by 1, and the share of the produced good in consumption spending, α, by $1-\alpha$.*

The nominal price at date 1 of the T good (the nominal exchange rate) is called $\widetilde{S}$, and assumed to be an exogenous random variable, presumably under the control of the monetary authority. This variable is supposed to be exogenous with respect to the decisions of agents regarding the standard of denomination: this ignores the potential feedbacks of the contractual pattern on the incentives of monetary policies

The absolute price of N goods in period 1 is equal to $\widetilde{S}p_N$. We assume that:

Assumption S is a strictly positive random variable with support in the set $\mathcal{S} = \{S_L, S_H\}$, where $0 < S_L < S_H$. The random variable z has support in the set $Z = \{z_L, z_H\}$ with $0 < z_L < z_H$. Let $\Omega \equiv \mathcal{S} \times Z$. The joint distribution is denoted $q(z, S)$, where $0 \leqq q(z, S) \leqq 1$ and $\sum_{z,S} q(z, S) = 1$. Furthermore, assume that $q(z_i, S_i) = 0$, with $i = L, H$. Let $\pi \equiv q(z_L, S_H)$.

Since it is an empirical regularity that nominal and real exchange rates appear to be positively correlated, we concentrate on this case, although the results can easily be generalized to cover different patterns of correlation between real and nominal shocks.

2.1 The equilibrium relative price of non-tradeables.

It is easy to show that the equilibrium price p_N^* does not depend on the particular contract that lenders and borrowers sign ex - ante. Let $\tau(S, z, p_N)$ be the transfer of resources (denominated in tradeables) from borrowers to lenders given S, z and p_N. Clearly the Cobb-Douglas assumption about preferences for L, B and J imply that the aggregate demand for tradeables in period 1 is equal to

$$c_T^B(p_N, S) + c_T^L(p_N, S) + c_T^J(p_N, S) = \left(\frac{\alpha}{2S}\right)\left(I_1^B(S, p_N, z) + I_1^L(S, p_N, z) + I_1^J(S, p_N, z)\right)$$

where $I_1^h(S, p_N, z)$ is the period 1 ex-post income of $h = L, B, J$.

$$\begin{aligned} I_1^B(S, p_N, z, k) &\equiv \begin{cases} ASp_N\bar{k} - \tau(S, p_N, z) & \text{if no default} \\ ASp_N\bar{k} - \tau(S, p_N, z) - S(1-\chi)\psi & \text{if default} \end{cases} \\ I_1^L(S, p_N, z) &\equiv \begin{cases} Sz\bar{k} + \tau(S, p_N, z) & \text{if no default} \\ Sz\bar{k} + \tau(S, p_N, z) - S\chi\psi & \text{if default} \end{cases} \\ I_1^J(S, p_N, z) &\equiv \begin{cases} 0 & \text{if no default} \\ \psi & \text{if default} \end{cases} \end{aligned}$$

This is because whatever is paid by debtors needs to go to lenders or the judge. Therefore $I_1^B(S, p_N, z) + I_1^L(S, p_N, z) + I_1^J(S, p_N, z) = ASp_N\bar{k} + Sz\bar{k}$. Therefore in equilibrium we must have $\left(\frac{\alpha}{S}\right)\left(ASp_N\bar{k} + Sz\bar{k}\right) = z\bar{k}$, the aggregate per capita supply of tradeables in period 1. From here it is clear that the value of p_N satisfies:

$$p_N^*(z) = \left(\frac{z}{A}\right)\left(\frac{1-\alpha}{\alpha}\right)$$

Hence, the relative price of non tradeables depends only on the relative supply of both goods, which varies univocally with the shock on the endowment of tradeables (given that the output of N is fixed and non-stochastic, granting that the projects are financed).

For future use, it will be useful to define the price of the consumption basket in terms of traded goods as $\widehat{P} = \frac{p_N^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} = \Phi^{-1}p_N^{1-\alpha}$ (where $\Phi \equiv \alpha^\alpha(1-\alpha)^{1-\alpha}$), while the consumer price index is $P = S\widehat{P}$. The price level P is the minimum expenditure for a consumer to obtain one unit of u. This resembles the standard composite index used in the literature.[2]

Although alternative contracts have quite different features, it will be shown below that they all share the following simple property:

Claim For any type of contract, if $I_1^{LC}(p_N, S)$ is the nominal return in $t = 1$ actually obtained in a state (p_N, S) by a lender who lends a unit of goods in $t = 0$, then $E\left(\frac{I_1^{LC}}{P}\right) = 1$. That is, lenders will require a unitary expected return in terms of the consumption basket.

[2] See, for example, Obstfeld and Rogoff (1996), chapter 10.

2.2 Contract choice and default

This section computes the expected utilities for borrowers and lenders, and study the factors that affect the choice of the contract denomination. We start by assuming two possible contracts, one that represents dollarized debt contracts and the other being a contract denominated in local units of account (nominal contract).

In the first contract contractual payments are denominated in the tradeable good, which we interpret as a "dollar"contract, since the world-prices of the tradable good is assumed to be equal to one dollar, i.e., we do not consider shocks on world prices. Clearly, if shocks on those prices were possible, they should also be contemplated in the choice of contracts of the agents. Taking this effect into account would entail a reasonably simple extension of this analysis, that we will not pursue here.

The contract specifies that for every unit of T good borrowed by the entrepreneur in period 0 she must return R_T units of T goods (if the debt is honored) in period 1. Default does not occur if $R_T \leqq p_N(z)\ A$, otherwise it happens. If default takes place, the third party (the judge) seizes the whole amount of the non-tradeable good produced by the entrepreneur at date 1 and transfers it to the lender. The social cost of default is equal to ψ units of T goods and completely borne by lenders (this does not affect results). We will denote by $\phi \equiv \psi/k$.

Let $\widehat{\Delta}$ be the set of states where the dollar contract induces no default, i.e., the pair of S and z such that $R_T \leqq p_N(z)\ A$. Let $\widehat{D}$ be defined as the complement of $\widehat{\Delta}$, i.e., set of states where there is default with the T contract. With this notation at hand, we can characterize the ex-post income of borrowers and lenders:

$$I_B(S,z) = \begin{cases} [p_N(z)\, A - R_T]\, S\bar{k} & \forall (S,z) \in \widehat{\Delta} \\ 0 \quad \forall (S,z) \in \widehat{D} \end{cases}$$

$$I_L(S,z,k) = \begin{cases} Sz\bar{k} + SR_T k & \forall (S,z) \in \widehat{\Delta} \\ Sz\bar{k} + Sp_N(z)\, Ak - S\psi & \forall (S,z) \in \widehat{D} \end{cases}$$

The lender maximizes her expected utility choosing $k \in [0, \bar{k}]$ subject to utility being at least equal to the expected utility of not lending, $\bar{\bar{k}} + \bar{k} \sum_{(z,S)} q(S,z) \left(\frac{z}{\widehat{P}(z)}\right)$. Recalling the definitions of the nominal price index $P(S,z)$ and the "dollar " price index $\widehat{P}(z)$, Cobb-Douglas utilities imply the first order condition for the lender:

$$1+\sum_{(S,z)\in\widehat{D}} q(z,S)\frac{\psi}{\bar{k}\widehat{P}(z)}=\sum_{(S,z)\in\widehat{\Delta}} q(S,z)\frac{R_T}{\widehat{P}(z)}+\sum_{(S,z)\in\widehat{D}} q(S,z)A\frac{p_N(z)}{\widehat{P}(z)}$$

This substantiates the claim that the expected return, per unit of lending, in terms of the consumption basket will equal 1. Replacing this equality in the ex-ante indirect utility function of the borrower, denoted as U_T^B , then:

$$U_T^B=(1-\alpha)A^{1-\alpha}\bar{k}\sum_{(S,z)} q(S,z)z^{\alpha}-\bar{k}-\psi(1-\alpha)A^{1-\alpha}\sum_{(S,z)\in\widehat{D}} q(z,S)z^{\alpha}$$

The lender obtains the expected utility equal to her reservation utility, that is, the utility of not lending.

The second contract specifies that, for every unit of T good received at date 0, the borrower will pay R_{nom} nominal units in period 1. If the borrower does not honor her debt, the third party forces the borrower to transfer its entire output of non tradeables to the lenders, after paying the costs of default. We define Δ as the set of states where entrepreneurs do not default on debt, (that is, where $R_{nom}\bar{k}\leqq Sp_NA\bar{k}$). Let D be the complement of Δ, i.e., the set of states such that borrowers default. We can actually write down the entrepreneur's ex-post income at date 1:

$$I_1^B(S,z)=\begin{cases}\bar{k}\left[ASp_N(z)-R_{nom}\right], & \text{if } R_{nom}\leqq ASp_N\bar{k}\\ 0, & \text{otherwise}\end{cases}$$

When lenders give k units of T to borrowers in period 0, Therefore their ex-post income in period 1 is:

$$I_1^L(S,z,k)=\begin{cases}Sz\bar{k}+R_{nom}k, & \text{if } R_{nom}\leqq ASp_N(z)\\ Sz\bar{k}+Sp_N(z)Ak-S\psi, & \text{otherwise}\end{cases}$$

The lender will choose $k\geq 0$ so as to maximize over k in $\left[0,\bar{k}\right]$ the expected utility subject to the constraint that this expected utility is at least as large as her utility of not lending. The solution to this problem is characterized by the following equality:

$$\sum_{(S,z)\in\Delta} q(z,S)\left(\frac{R_{nom}}{P(S,z)}\right) + \sum_{(S,z)\in D} Aq(z,S)\frac{p_N(z)}{\widehat{P}(z)} = 1 + \sum_{(S,z)\in D} q(z,S)\frac{\phi}{\widehat{P}(z)}$$

If R_{nom} satisfies this equality then the constraint also holds with equality, and so k must be equal to $\bar{k}$. Thus, we will consider any equilibrium with $k = \bar{k}$. The lender also considers $p_N(z)$ as exogenous, so the individual k does not affect p_N for the individual lender. Moreover, in this case the expected value of the CPI-deflated return (net of expected and deflated default cost) to the lenders must also be equal to one.

Lenders again obtain the reservation utility. For borrowers, after replacing in R_{nom} by the expression coming from the first order condition of the lender, her utility, denoted as U^B_{nom}, is equal to:

$$U^B_{nom} = (1-\alpha)A^{1-\alpha}\bar{k}\sum_{(S,z)} q(S,z)z^{\alpha} - \bar{k} - \alpha A^{1-\alpha}\psi \sum_{(S,z)\in D}\frac{q(S,z)}{z^{1-\alpha}}$$

Therefore, under non - tradeable denominated debt contracts the level of the equilibrium indirect utility for entrepreneurs depends on which states the entrepreneur finds it optimal to default on her debt.

Given that lenders obtain the same expected utility whether the contract is in dollars or nominal, borrower's expected utilities are the key to determine which of the two contracts arise in equilibrium. The following result provides the explicit formula to study what determines this choice:

Lemma 2 *The conditions that make one type of contract preferable to another depend only on their probabilities of default and the deflated values of the cost of default.*

Proof. Computing the difference between the borrower's indirect utility with nominal contracts and that with contracts denominated in T goods (for lenders the difference is always 0) yields

$$U^B - U^B_T = -\psi\left(\sum_{(S,z)\in D}\frac{q(S,z)}{\widehat{P}(z)} - \sum_{z\in\widehat{D}}\frac{q(S,z)}{\widehat{P}(z)}\right)$$

■

Remark 3 *If the cost of default, ψ, were specified as a constant in terms of the consumption basket (instead of being determined as a quantity of traded goods), then the preference over contracts would result simply from comparing their probabilities of default.*

It is obvious that $sgn\left(U^B - U_T^B\right)$ must be the opposite of $sgn\left[\sum_{(S,z)\in D}\frac{q(S,z)}{\widehat{P}(z)} - \sum_{z\in\widehat{D}}\frac{q(S,z)}{\widehat{P}(z)}\right]$. Given that the functional form $\widehat{P}(z)$ is the same in D and $\widehat{D}$, and given that $q(S,z) \in (0,1)$ and $p_N > 0$ in equilibrium then the sign of this last term is determined by whether $\#D$ is strictly greater or strictly less than $\#\widehat{D}$. This is an important difference with respect to the case to be studied below, where lenders care only about the consumption of traded goods. Here, the symmetry of preferences allows to characterize the dominance much more easily, by just looking at the states in which default occurs.

Having established how the interest rate is determined by the above mentioned expected return condition, and that default states are the main factor for the choice of contract denomination, we proceed by studying the cases of default for each type of contract.

2.3 Default conditions for dollar and nominal contracts

We start with T-good contracts. Recall that in this case there is no default when $R_T \leq Ap_N(z)$. The following result provides a complete characterization of the conditions for default in each possible state:

Proposition 4 *If dollar contracts were traded in equilibrium, the following statements hold:*

T1 There is no default (i.e., $\widehat{D} = \emptyset$) if and only if $\pi A^{1-\alpha} z_L^{\alpha} \geq 1$

T2 There is default in the state of high real exchange rate (i.e., $\widehat{D} = \{(S_H, z_L)\}$) iff

$$Ap_N(z_L) < \left\{\frac{1+\pi\left[\frac{\phi - Ap_N(z_L)}{\widehat{P}(z_l)}\right]}{(1-\pi)}\right\}\widehat{P}(z_H) \leq Ap_N(z_H) \qquad (1)$$

T3 The value of z_H is assumed to be large enough so that there is no default in the state of low real exchange rates ($\widehat{D}$ can never be equal to $\{(S_L, z_H)\}$.

The proof of this proposition is in the appendix. Since lenders care about the purchasing power of their income in terms of the consumption basket, the dollar interest rate will depend on the expected purchasing power of traded goods. No-default implies that traded goods should be abundant enough even in the "bad state "so as to make the relative price of non-tradeables sufficient for the borrowers to service their debt. That is: the real exchange rate should not be "too high " in that state. More formally, if we define *discount factors* $\widehat{\pi}_L \equiv \frac{\pi}{\widehat{P}(z_L)}$ and $\widehat{\pi}_H \equiv \frac{1-\pi}{\widehat{P}(z_H)}$, and the corresponding expectations with respect to these discount factors, then no default with T good-contracts obtains when $Ap_N(z_L)$ multiplied by the expected discount factor is at least one.

Note that if $A\, p_N(z_L)\, E\left[\frac{1}{\widehat{P}}\right] < 1$ then the inequality (1) in the proposition clearly holds since it implies $A\ p_N(z_L)\ \left[\frac{(1-\pi)}{\widehat{P}(z_H)} + \frac{\pi}{\widehat{P}(z_l)}\right] < 1 + \phi\frac{\pi}{\widehat{P}(z_l)}$, or A $p_N(z_L)\ E\left[\frac{1}{\widehat{P}}\right] < 1 + \phi\ \frac{\pi}{\widehat{P}(z_L)}$. The second inequality is the same as

$$1 + \phi\frac{\pi}{\widehat{P}(z_L)} < AE\left[\frac{p_N}{\widehat{P}}\right]$$

where $E\left[\frac{p_N}{\widehat{P}}\right]$ indicates the *expected* value of p_N relative to the discount factor. It is worth noting that $\frac{p_N}{\widehat{P}}$, the real price of non-tradeables in terms of the CPI index, is proportional to p_N^{α}. The condition implies that, in order for the borrower to be solvent in state H, the expected purchasing power of the output of non-tradeables financed by the loan must exceed the unitary return required by lenders, plus the expected cost of default (in terms of the consumption basket), which occurs in the state L. Of course, if this condition is not met, that is, if the relative price of non-tradeables is not sufficiently high to make the project cover its opportunity cost, there will be no lending.

A sufficient condition for all inequalities to hold is z_H sufficiently high and z_L low, together with a not so-large value of ϕ. Clearly, a high degree of real volatility may generate default with T-good contracting.

Regarding default conditions on nominal contracts, a similar result is attainable[3].

[3] If one assumed uncorrelated nominal and real shocks conditions for no-default ave interesting implications. It can be shown in this case that the nominal interest rate would be equal to $\left[E\left(\frac{1}{S\widehat{P}}\right)\right]^{-1} = \left[E\left(\frac{1}{S}\right)\right]^{-1}\left[E\left(\frac{1}{\widehat{P}}\right)\right]^{-1}$ by independence. So no-default means that, for the values, p_{NL}, S_L, it must be the case that $Ap_{NL}S_L \geq R_{nom}$, or $Ap_{NL} \geq$

Proposition 5 *When nominal contracts are traded in equilibrium, the following statements hold:*

N1 If $z_L \geq \frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}}+\frac{1-\pi}{z_H^{1-\alpha}}\frac{S_H}{S_L}\right)}$ *or either if* $\left(\frac{z_H^{1-\alpha}}{1-\pi}\right)\left(\frac{1}{(1-\alpha)A^{1-\alpha}}-\pi z_L^{\alpha}\right) \leq \frac{z_L}{z_H}\frac{S_H}{S_L} \leq \frac{\left(\frac{\pi}{z_L^{1-\alpha}}\right)}{\frac{1}{(1-\alpha)A^{1-\alpha}}-(1-\pi)z_H^{\alpha}}$*, then there is no default* ($D = \emptyset$)*.*

N2 When real shocks have a sufficiently greater variance relative to the nominal shocks such that $A\, p_N\left(z_L\right)\, S_H\, E\left[\frac{1}{P}\right] < 1 < 1+\phi\widehat{\pi}_L \leq E_{\widehat{\pi}}\left[Ap_N\right]$ *holds, then there is default at state where the real and nominal exchange rates are high (i.e.,* $D = (S_H, z_L)$*).*

N3 When nominal shocks have a sufficiently greater variability than nominal shocks, such that $A\ p_N\left(z_H\right)\ E_{\widehat{\pi}}\left(\frac{S_L}{S}\right) < 1 < 1+\phi\widehat{\pi}_H \leq E_{\widehat{\pi}}\left(Ap_N\right)$ *holds, then there is default at state where the real and nominal exchange rates are low (i.e.,* $D = (S_L, z_H)$*).*

Remark 6 (about N1) *It is not difficult to show that the condition on* z_L *in* ***N1*** *is weaker in the case of nominal contracts than that of "dollarized" contracts. Again, similar arguments imply that the condition on* z_H *is stronger in the case of nominal contracts relative to that of "dollarized" ones.*

The intuition of this result is simple. First, given the negative correlation between the nominal and real exchange rates, the nominal depreciation reduces the "dollar value " of the nominal debt in the state where the relative price of non-tradeables is low. This may avoid a default which would occur under T contracts, provided that the magnitude of the reduction in that dollar value is large enough and, at the same time, the amplitude of the nominal exchange variation between both states is not so high as to cause default in state z_H.

Remark 7 (about N2) *The first inequality in* ***N2*** *is stronger than the first inequality which specifies conditions for default with dollar contracts when* $z = z_L$*. Hence, if there is default with nominal contracts in the state* (S_H, z_L)

$\frac{\left[E\left(\frac{1}{P}\right)\right]^{-1}}{S_L E\left(\frac{1}{S}\right)} > \left[E\left(\frac{1}{P}\right)\right]^{-1}$. Therefore, the condition for no default is stronger for nominal contracts than is the case with traded-goods contracts, given the excess volatility in prices introduced by monetary policies.

then there must be default in the same state with T-good contracts. In this case, the real exchange rate in the z_L state is high so as to induce default on "dollar " contracts, while the movements in the nominal exchange rate reduce the dollar value of nominal debts in that state, but not sufficiently so as to avoid default.

Remark 8 (about N3) *This case can be compatible with, either, default or no default with T-good contracts.*

N3 is the case where the nominal shock has a large variability, since the nominal price of non-tradeables must be higher in the state where p_N is low. That is, it is necessary for the nominal shock to more than offset the movements in the real exchange rate. The situation can be qualitatively described as follows. In the event where the real exchange rate is high, the nominal depreciation raises the aggregate price level. In that instance, the dollar value of nominal debts is so much reduced that the non-tradeable producers can repay their debts. However, the nominal interest rate is so high that default occurs in the case of low nominal-low real exchange rate.

This possibility may be relevant to rationalize the reluctance of agents to make nominal contracts in economies with fixed exchange rates where it is feared that in the case of exit from the peg, nominal prices may undergo a large jump. This reluctance would be stronger if there is uncertainty about the size of the devaluation in the high-real exchange rate case, because then there could be states of default on nominal debts also in that case.

2.4 Ranking of contracts

Given that default states define the borrower's preference for one contract or the other, we analyze the ranking in each case. Lemma 2 clearly shows that, whenever one contract leads to more default states than a second contract, then the latter will be preferred.

Thus, when there is no default in either case, or when there is default both with the T contract and the nominal contract in the state (S_H, z_L) , the borrower is indifferent between both types of contracts.

Remark 9 *If the nominal exchange rate is higher in the state of high real exchange rate, as supposed here, it has been shown that the conditions for no default in state z_L are weaker for nominal contracts. Therefore, the set* *of states corresponding to Case D1, N2 (no default with T-contracts, default with nominal contracts with the z_L shock) is an empty set.*

If there is default with the dollar contract when the exchange rate is high, and no default with nominal contracts, clearly $U^B - U_T^B = -\ \psi\ \frac{\pi}{\widehat{P}(z_L)} < 0$. Therefore here the nominal contract dominates the $T - good$ contract. In this case, the nominal shocks (or, figuratively, monetary policies) act in such a way that, while default would occur in state z_L, the project is solvent with the nominal contract because the "dollar " value of obligations is reduced sufficiently (but not so much as to disturb the repayment of debts in state z_H by raising the nominal interest rate too high). This offsetting property of the nominal shock facilitates the servicing of nominal debts on projects that finance the production of non-tradeables[4].

Suppose now that the volatility of the nominal exchange rate is so large that, while there is no default under the dollarized contract, the debtor of nominal obligations would default in the low real exchange rate state. This occurs because the nominal interest would be so large (given the chance of very high prices in the z_L state) that the real rate would be "excessive " in the state of low nominal prices, even if this is a "good " state in terms of the relative price of output. The situation corresponds to $Ap_{NL}\ E\left(\frac{1}{\widehat{P}}\right) \geq 1$ and also to $\frac{Ap_{NH}}{\widehat{P}_H} < \frac{\left[E\left(\frac{1}{P}\right)\right]}{P_{HL}}$, where $P_{HL} = S_L p_{NH}$ is the nominal price level in the state z_H. Here, with very variable nominal prices, and sufficiently high prices of non-tradeables in the state of high real exchange rate, T-good contracts are clearly preferred.

Suppose that there is default under nominal contracts with low exchange rates and default with dollar contracts with high exchange rates. We first need to show that no mutually contradictory inequalities arise here. The default conditions for nominal contracts are $Ap_N\left(z_H\right) S_L E\left[\frac{1}{P}\right] < 1 < 1+\phi\widehat{\pi}_H$

[4]In a system where loans may also finance the production of traded goods (and, assuming, for the sake if this illustrative argument that the distribution of relative prices, $\widehat{P}$, remains unchanged) given that the nominal exchange rate is lower when the relative price of tradeables is low conditions for no default on nominal credits relative to T- credits would be strengthened. This would reduce (or, in some instances, eliminate) the range of values of the ratio S_H/S_L for which there would be no default on any nominal contract, and where, therefore, these contracts would (weakly) dominate the T contracts. In addition, it may be the case that in a setting with financing to both sectors, the result could be the "coexistence " of different types of contracts, to be used in different types of loans, although this last conjecture remains to be shown more formally.

$\leqq E_{\widehat{\pi}}[Ap_N]$. The values of (z_L, z_H) consistent with these inequalities (and $z_L < z_H$) are rather special, and not always exist. However, when ψ is sufficiently small then it is possible to find values of z_L and z_H satisfying all relevant inequalities. If these conditions are met, then the difference $U^B - U_T^B$ is just equal to $-\psi[\widehat{\pi}_H - \widehat{\pi}_L]$. This is strictly negative if and only if

$$\frac{z_H}{z_L} > \left(\frac{1-\pi}{\pi}\right)^{\frac{1}{1-\alpha}}$$

so in this case (under the conditions of case 2) the T - good contract dominates the nominal contract. Note that since $\frac{z_H}{z_L} > 1$ this implies that $\pi < \frac{1}{2}$. Accordingly, in order for the "dollarized " contract to be preferred, the chances of default under that contract should be lower than those under the nominal contract. However, this asymmetry derives from the special assumption that the cost of default is defined in terms of the T-good. That means that default is particularly expensive in terms of the consumption basket if there is default with that contract, since it occurs when tradeables have a high relative price.

2.5 On mixed-denominated and indexed contracts

Given the assumption that agents can only choose "pure " contracts wholly denominated in one unit of account, one may wonder how the results would change when contracts may be written partially in both units of account and, in particular, whether there exists a contract with a combination of denominations such that this contract is default-free. That depends on parameters. In fact, it is easy to see that if z_L is sufficiently low no contract made as a combination of nominal and $T-$goods can prevent default in state (S_H, z_L), because in this state the T-good value of the output produced by borrowers would not be enough to cover the "dollar " portion of the debt.

However, in other cases, a "convex combination" of tradeable good and nominal contracts can certainly eliminate default. Actually, it is clear that allowing for mixed-denomination loans can never generate inferior outcomes that the pre-imposed corner solution. However, the fact remains that most observed contracts seem to be quite "simple", and liable to induce default in some contingencies. An analysis of why financial markets are "incomplete" in this dimension goes well beyond the scope of this discussion.

Regarding contracts where payments are linked to some measure of domestic prices, practical applications of indexing to the CPI, or similar price indices, must take into account the presence of a reporting lag. Since the *current* price index is not available at the time a payment is due, the adjustment clause must be based on the inflation rate of a past period. For instance, a contract made in month t, with maturity in $t+k$, would adjust by the price ratio P_{t+k-1}/P_{t-1}, instead of P_{t+k}/P_t . Thus, the discrepancy between P_t/P_{t-1} and P_{t+k}/P_{t+k-1} would create a difference between the adjustment index and the true price change. The real value of the payment made in $t+k$ would then be random. The magnitude of that difference, which increases with the degree of price volatility, can be quite sizeable in conditions of high and variable inflation, which may help to explain the diffusion of "dollar" contracts instead of indexed debt, in instances of extreme price instability.

On the other hand, in the context of the model discussed here, it may be argued that the price p_N of the individual producer is imperfectly correlated with the non-traded-goods component of the price index and that, therefore, indexing to P would have an additional random component compared with a hypothetical adjustment by an index proportional to $Sp_N^{1-\alpha}$. Also, and more simply, there could be a measurement error in the price index. Agents may expect that the potential discrepancy between measured and actual changes in the aggregate price level relevant as "consumption deflator " would increase with the time horizon over which the change is observed. In any case, it seems pertinent to contemplate the case of "noisy " indexation.

The possible sources of noise have a different nature, and relate in a different way to other shocks:

- In the case of the reporting lag, the error in the price level for a contract made in t and maturing in $t+k$ would be given by: $(P_{t+k}/P_{t+k-1})/(P_t/P_{t-1})$. The shock on the contemporaneous price level would affect the real value of payments in the same way for both the indexed as well as the non-indexed debt. The non-indexed nominal contract would be unaffected by the shock on $P_t/P_{t-1,}$ but the real value of payments would clearly be influenced by the shocks on the aggregate price level intervening during the interval $\{t, t+k-1\}$, which would not be the case for indexed contracts. The properties concerning default with indexation would then depend on the magnitude of the one-period shocks on absolute prices, as with nominal contracts. The difference between both contracts would arise from the degree of mean-reversal of prices: negative correlations in the movements in the inflation rate across periods would tend to reduce prediction errors in the real returns in nominal contracts relative to those with indexation[5]. For the purpose of this exercise, the existence of the reporting lag would imply that the preference of "dollarized " over indexed contracts would be influenced by similar considerations such as those that make nominal contracts more or less attractive relative to dollar contracts.

- If there are *idiosyncratic* shocks on the price p_N of the output of the debtor, but the aggregate price index is measured without error, the determination of interest rates would be the same as in the case without such shocks. The default conditions would be affected by the idiosyncratic "noise", but much in the same way as would apply to nominal or dollar contracts.

- In the case of measurement errors, in the first approximation it may be assumed that they are independent of the shocks on the nominal and real exchange rates (although, in fact, those errors are likely to increase when absolute and relative prices are more variable).

In any case, it can be seen that, in the context of this model, the "unobservability" of the true price at the time of specifying the payments would be the crucial consideration to prefer contracts other than the domestic-price-indexed. In effect, suppose that all agents (borrowers and lenders) can observe the CPI realized at the beginning of period 1. Consider a contract that specifies an interest rate expressed in units of the price index $P(S, z)$; we first assume that this price is publicly observable without cost or error. The contract specifies that, if no default occurs, the value of pesos to be paid is equal to $P(S, z)\, r$, where r is determined endogenously in equilibrium[6]. Then, the following result can be obtained:

[5] As an accounting illustration of the problem, consider a nominal contract with real return approximated by $R + p_t - p_{t+k}$, where p is the log of the price level. Writing $p_{t+1} - p_t = \varepsilon_{t+1}$,that real return would be $R - \sum_{t+1,t+k} \varepsilon_{t+i}$. In the case of the indexed contract, the real rate would be given by: $R_I + p_{t+k-1} - p_{t-1} - (p_{t+k} - p_t) = R_I + \varepsilon_t - \varepsilon_{t+k}$. Thus, the relative magnitude of the variances of the real returns turns out to depend on the comparison between: $var(\varepsilon_t - \varepsilon_{t+k})$ and $var(\sum_{t+1,t+k} \varepsilon_{t+i})$, which is a function of the degree of autocorrelation of the shock.

[6] Note that r must be less than A for at least one value of z, otherwise the entrepreneur prefers not to borrow any amount of T good in period 0.

Proposition 10 *The condition for no default under the indexed contract is given by:*

$$\frac{Ap_{NL}}{\widehat{P}_L} \geq 1$$

or:

$$1 \leq \Phi A^{1-\alpha} \left[z_L \left(\frac{1-\alpha}{\alpha} \right) \right]^{\alpha}$$

Then, r is equal to one.

Clearly, with this indexed contract, if there is default in only one state, it must be when $z = z_L$. It is clear that the condition for no default under the indexed contract is weaker than that which applies for a T-contract. Therefore, the "dollar " contract can never dominate a perfectly indexed contract. More formally, the ex-post utility for the borrower with indexed contracts is given by $\bar{k} \left[\frac{Ap_N(z)}{\widehat{P}(z)} - 1 \right]$. So the ex-ante utility is $V_{ind}^B = \bar{k}$ $\left[\widehat{\pi}_L Ap_N(z_L) + \widehat{\pi}_H Ap_N(z_H) \right] - \bar{k}$, which is equal to the utility reached by the borrower with the other two contracts when no default occurs under both of them. This has the implication that the perfectly indexed contract can never be dominated by the T - good debt contract.

What happens if the borrower has to choose among the three contracts? We still can prove[7] that the choice depends on the states of default under each contract, as well as the relative probabilities of those states. Thus, when no default occurs with indexed contracts, these are preferred to non-indexed contracts (either nominal or dollarized) except when there is no default in these other cases. Therefore, the preference for either nominal or T contracts would be based on the impossibility to define and implement contracts with "perfect " indexation.

Note that if we compare the condition for no default with an indexed contract (in the last proposition) with that for the nominal contract, which can be expressed as:

$$\frac{Ap_{NL}}{\widehat{P}_L} \geq \frac{1}{\pi + (1-\pi)\frac{P_{HL}}{P_{lh}}}$$

[7] The formal proof is available upon request.

(where P_{ij} denotes the aggregate price level in the state (S_i, z_j)) we see that this latter inequality is weaker than that applying to indexed contracts if $P_{HL} > P_{LH}$, that is, if the price level is higher when the real exchange rate is high. There may be cases, then, where the nominal contract is preferred to the indexed contract. This requires the preceding condition, in addition to a sufficiently high real exchange rate in the (S_H, z_L) case (but not too high so as to cause also default with the nominal contract). With some parameter values, there can be default for the indexed contract in state (S_H, z_L) , and for the nominal contract in the (S_L, z_H) state.

3 The case of a financially open economy

The previous analysis assumed that lenders are domestic agents (who consume both traded and local non-traded goods) and that the interest rate is determined internally. The discussion served to highlight that "dollarization "may arise in some instances, even without the participation of foreign residents in credit transactions. In any case, the setup of the model can be modified to represent also the case of an economy where the domestic financial market is linked to the international market.

Thus, we consider here the polar situation where there is perfect capital mobility. The assumption will be that there is a default-free external interest rate R^* specified in units of good T , or "dollars " (once again, we are abstracting away possible movements in international prices) which defines the opportunity cost of funds for local agents. The assumption is that risk neutral foreign traders stand ready to borrow/lend elastically if the expected rate of return on domestic contracts, measured in terms of tradeable goods, equals R^*. This determines the domestic interest rate. All other features of the model remain the same. In particular, if default occurs, the creditor must pay the default cost to the (internal) "judges " in order to appropriate the residual value of the project.

If there are actual transactions in assets with foreign agents, the demand for non-traded goods will naturally depend on the distribution of wealth between the parties in the transactions for the different possible contingencies. Therefore, the price p_N would not be simply a function of the real shock z, and independent of the details of the contracts. This effect may be incorporated into the analysis, but it would complicate the calculations without adding much substantive content to the results. Consequently, in the following we will proceed with the analysis of the properties of contracts as functions of the price p_N, without deriving this price in general equilibrium from the real shock z and the distribution of wealth between residents and not residents determined by the outcomes of the contracts themselves.

Moreover, in this financially open economy, domestic agents would have the option of buying international assets. Therefore, the identity of the lenders who finance the local production of non-tradeables is endogenously determined along with the unit of denomination of contracts. We will assume that domestic prospective lenders have access to a no-default international bonds with a sure return R^* in terms of goods T. Consequent with the criterion of simplifying the setup of the model to the limit, we will make the extreme assumptions that there are no costs, or restrictions, for international lending (in a way that wholly abstracts away the potential for home-country effects) and, further, that monetary policies are independent of the nationality of lenders in the domestic market. Here, there is no “original sin ”, in the sense that external agents may be willing to supply finance in domestic currency, if the case arises[8].

As shown below, all this implies that, in a variety of cases, the outcome of the model would be that domestic agents, both lenders and borrowers, interact with international markets and not among themselves. Given the assumptions, it is clear that the terms of contracts of whatever denomination will be determined by the condition that the expected return to lenders (net of default costs), measured in terms of traded goods, is equal to R^*. But borrowers will still care about their real income in terms of the consumption basket. This introduces another element into the analysis, since the preference over contracts will depend not only on their properties concerning default, but also on the features related to the allocation of the relative-price risk. It will be seen that, within the class of contracts that are compatible with the arbitrage conditions induced by the openness to international markets, the domestic assets offer a lower utility to domestic lenders (who wish to maximize the expected return in terms of the CPI basket) than the international bond that gives a sure return R^* in units of goods T.

Clearly, the extreme form of international intermediation associated with this result is not observed in practice. But, of course, accounting for the

[8] A possible extension of the model may be to introduce a restriction to foreign lending in domestic units which, in the cases it would be operative, may result either in dollarization with foreign lenders, or in the payment of a premium to domestic lenders so that they accept to lend locallly.

tendency to lend in domestic markets would require contemplating other effects, and not only those of the shocks induced by shifts in real endowments and in the value of the nominal unit, which are the focus of this discussion..

3.1 Interest-rate characterization for each contract type

In this subsection we characterize the conditions under which international investors are willing to lend to local borrowers for each contract type. We start by taking the T-good (dollar) contract. Let $\widehat{\Delta}'$ denote the set of no-default states under dollar contracts, while $\widehat{D}'$ is the set of default states. Let R_T' be the (promised) gross interest rate on this dollar contract. Given the price p_N of non-traded goods at each state, the contractual rate of return in dollar terms, R_T' will be determined by the condition:

$$R_T' \Pr(\widehat{\Delta}') + E_{\widehat{D}}' \left[Ap_N - \phi \right] = R^*$$

For financing to be at all feasible, it the mean value in terms of tradeables of the output of the project per unit of investment must exceed R^*. We will assume that this holds.

Consider the nominal contract. Let Δ' be the set of states where there is no default under the nominal contract, and D' the set of states where default occurs. If R_{nom}' is the contractual nominal return, the arbitrage condition implies:

$$R_{nom}' E_{\Delta'} \left[\frac{1}{S} \right] + E_{D'} \left[A[p_N - \phi \right] = R^*$$

These two equalities will be used to determine the interest rates under each default condition.

3.2 Default conditions and the lenders's choices

In this section we consider the typology of cases of default on "dollarized " and nominal contracts on which the expected return to lenders in terms of tradeable goods is R^*. We also analyze whether local lenders would chose to buy domestic or international bonds, given the terms of the contracts and the simplifying assumptions on the absence of costs and restrictions on international capital movements. We denote p_{Ni} as the relative price of the non-tradeable good when $z = z_i$, with $i = L, H$.

3.2.1 Default conditions

We start with the dollar contract. In this case there is no default if and only if $Ap_N \geq R'_T$. Thus, the following proposition can be shown:

Proposition 11 *Suppose that a dollar contract is traded in equilibrium (with international investors). Then*

T'1 *If $R^* \leq Ap_{NL}$, then there is no default ($\widehat{D}' = \emptyset$)*

T'2 *If $Ap_{NL} < R^* < R^* + \pi\phi \leq AE[p_N]$, then there is default in the state (S_H, z_L)*

Obviously, for a dollar contract to induce no default in any state, the "dollar" value of output in the "bad state" must be sufficient to service the debt at the no default interest rate. The condition in **T'1** refers to the levels of the values of output. For a given mean, it will be more easily satisfied the smaller is the variability of the real exchange rate. Regarding **T'2**, the first inequality $Ap_{NL} < R^*$ must hold since otherwise the parties would choose a contract with $R = R^*$ and there will be no default in any state[9].

Now, suppose that the contract traded is the nominal one. The default characterization in this case is summarized in the following result:

Proposition 12 *Suppose that a nominal contract is traded in equilibrium. Then:*

N'1 *If $\frac{R^*-(1-\pi)Ap_{NH}}{\pi Ap_{NH}} \leq \frac{S_L}{S_H} \leq \frac{(1-\pi)Ap_{NL}}{R^*-\pi Ap_{NL}}$ then there is no default under nominal contracts*

N'2 *If $Ap_{NL}S_H E\left(\frac{1}{S}\right) < R^* < R^* + \pi\phi \leq AE[p_N]$ then there is default in the state (S_H, z_L)*

N'3 *If $Ap_{NH}S_L E\left(\frac{1}{S}\right) < R^* < R^* + (1-\pi)\phi \leq AE[p_N]$ then there is default in the state (S_L, z_H)*

[9] Clearly, if $\psi > 0$, the condition in **T'2** need not be satisfied even though the expected value of non-tradeable production per unit of investment is larger than R^*. In that case, there may be no credit denominated in tradeables.

We see that, for a nominal contract to induce no default, the two conditions in **N'1** imply the (natural) requirement that the expected value of output in terms of traded goods exceed R^*, but it is not necessary that $Ap_{NL} > R^*$. Given that the nominal exchange rate would be higher in the state with low real prices of non-tradeables, the condition on p_{NL} is weaker than in the case of "dollarized" contracts. For similar reasons, the condition on p_{NH} is stronger. Such inequalities bound the ratio between the nominal exchange rates in both states (that is, the variability of the nominal variable) as a function of the real prices of non-tradeables. Nominal contracts can "avoid" default when "dollar" denominated contracts are subject to default if changes in the nominal rate counteract the effects of "real" shocks on the prices of non-tradeables and, at the same time, the variability of the nominal exchange rate is not "too large".

A corollary of **N'1** is that, provided that the values of non-tradeable output satisfy the expected value condition $E\left[Ap_N\right] > R^*$, there is no default with nominal contracts if $S_L p_{NH} = S_H p_{NL}$, that is, if monetary policy "stabilizes" the nominal price of non-traded goods. That condition reproduces the situation that would be generated with contracts with payments indexed to the prices of non-traded goods (given that there is no uncertainty on the volume of output).

N'2 implies that, in order to have default in the state p_{NL} with nominal contracts, there must also be default with traded-goods denominated contracts: the rise in the nominal exchange rate does not compensate for the fall in the "dollar" value of output. In this case, the nominal contract is equivalent to a "dollar" contract, since the traded-good value of payments in the p_{NH} state must be equal for both contracts, so that it covers the expected value of "dollar losses" resulting from the liquidation of the project in the state of high real exchange rate.

Finally, **N'3**, nominal contracts induces default in state (S_L, z_H) when the nominal exchange rate is "so high" in the state (S_H, z_L) that this drives the nominal interest rate to a point where the nominal value of output in the state p_{NH} is insufficient to service the debt. Thus, $\frac{S_H}{S_L}$ has to be high enough so as to compensate for the fact that $Ap_{NH} > R^*$. This corresponds to a situation with a large "nominal variability", which would tend to make nominal contracts less attractive.

Interestingly, this latter case may hold even if $Ap_{NL} < R^*$, provided that ϕ is sufficiently small. Nominal impulses can then "reverse" the states of default, and make the firm solvent in state (S_H, z_L) (which may be a state of default with T-contracts) and subject to default in state (S_L, z_H) (the "good state" with T-contracts). The logic is that the nominal exchange rate is expected to be very high in the state of high real exchange rate. Then, the nominal contract will include a large "inflation premium " to contemplate this event. This premium would be offset by high nominal prices if the state of high exchange rates materializes. In the other state, although the real price of non-tradeables would be high, low nominal prices would make the producers insolvent.

Finally, if the conditions for "solvency in at least one state" are not met, there will not be nominal contracting. When real exchange rate and high nominal exchange rate are positively correlated, since the conditions for the firm to be insolvent in the p_{NL} state with nominal contracts imply those for insolvency in that state for T-contracts, it follows that, if there is no possibility of credit in nominal terms, "dollar" credits are also not feasible.

3.2.2 The choices of domestic lenders

Here, we ask whether in each case of default, a domestic lender, with risk-neutral preferences in terms of the CPI basket, would be better off accepting the local contract or buying an international bond with a sure return R^*. Given the expression for the interest rate, the lender's ex ante utility derived from the local dollar-denominated bond would be:

$$V_{dollar}^{L,local} \equiv \left[\frac{R^* - E_{\widehat{D}'}(Ap_N - \phi)}{\Pr(\widehat{\Delta}')} \right] E_{\widehat{\Delta}'} \left(\frac{1}{\widehat{P}} \right) + E_{\widehat{D}'} \left[\frac{Ap_N - \phi}{\widehat{P}} \right]$$

and that derived from the nominal contract is:

$$V_{nom}^{L,local} \equiv \left[\frac{R^* - E_{D'}(Ap_N - \phi)}{E_{\Delta'}(\frac{1}{S})} \right] E_{\Delta'} \left[\frac{1}{S\widehat{P}} \right] + E_{D'} \left[\frac{Ap_N - \phi}{\widehat{P}} \right]$$

The lender's expected utility from investing outside the country is:

$$V_{foreign}^{L,local} \equiv R^* E \left[\frac{1}{\widehat{P}} \right]$$

The choice is obtained by comparing either $V_{dollar}^{L,local}$ or $V_{nom}^{L,local}$ with $V_{foreign}^{L,local}$.

For dollar contracts, the following result is obtained:

Proposition 13 *If the economy is open to international capital flows, then local agents do not strictly prefer to lend to local borrowers in dollar-denominated contracts (either if this implies no default or if there is default with high exchange rates).*

Clearly, there is indifference for local investors between lending to borrowers than lending abroad when the dollar contract implies no default. In the case of default with high exchange rates, we see that, for the same expected rate of return in terms of tradeable goods, the individual values the fact that the international bond offers a repayment in those goods when their price is high domestically, while the domestic contract concentrates its excess return when tradeables are cheap.

For nominal contracts, we obtain a slightly different result:

Proposition 14 *If the economy is open to international capital flows, then local agents would not lend to local borrowers with nominal contracts if either there is no default with those contracts or if there is default in state* (S_H, z_L). *If the nominal contract induces default at the state* (S_L, z_H) *then local investors prefer to lend to local borrowers under those contracts instead of buying international bonds if and only if*

$$\max\left\{(Ap_{NH} - \phi), Ap_{NH} S_L E\left(\frac{1}{S}\right)\right\} < R^* < R^* + (1-\pi)\phi \leq E[Ap_N]$$

The conditions imply that either there cannot be lending to local borrowers in tradeable-goods denominated contracts, or there is no default on those contracts.

Given that both contracts have the same mean return, the question is which one of them pays a higher dollar return in the state with high real exchange rate. The condition that local agents prefer to lend domestically is equivalent to say that in its no-default state, the nominal contract has a higher dollar return than the riskless (in dollar terms) international bond. This is equivalent to the statement that, in the state of default, the dollar return to creditors is lower than the return on the international bond.

While the proposition seems intuitive, it need not be satisfied: it can be that the dollar payment to lenders in the case of default is higher than the payment with no default, if the "liquidation" dollar value of the project in the state with low real exchange rate exceeds the dollar value of the contractually determined nominal payment in the other state, which would be reduced as a consequence of the high nominal exchange rate.

3.3 The choice of borrowers and the ranking of T-good and nominal contracts

The utility maximization of the borrower is equivalent to maximizing the expected value of real income measured in units of the consumption basket. It is clear that the borrower will prefer the contract with the lowest real value of expected payments in terms of the domestic CPI basket.

Suppose that there is no default with the nominal contract (**N'1**). Then one can show that:

Proposition 15 *If there is no default under the nominal contract, then the borrower prefers (ex-ante) the nominal contract to the "dollar" contract, whether the dollar contract implies default in* (S_H, z_L) *or not.*

The proof is in the appendix. This is a significant difference with the case of purely domestic lending with no default: the nominal contract has better properties from the point of view of the borrower than the "dollar ", when there is no default in *both* contacts. The reason is that, while the expected value of payments is the same in terms of T goods, it is lower for nominal contracts in terms of the consumption basket, since it implies lower "dollar" payments when those goods are more expensive[10]. When there is default with the dollar contract, the properties regarding default and "hedging" reinforce one another to favor the nominal contract.

The following result shows a case of indifference:

Proposition 16 *When both dollar and nominal contracts induce default in state* (S_H, z_L) *the borrower is indifferent between them.*

This result is similar to the corresponding default case in the economy without international lenders. Thus, even when international investors lend

[10]The immediate conclusion that the market would gravitate towards nominal contract would apply provided that the distributions of the variables remain unchanged irrespective of the typical features of contracts. In fact, it might be expected that the incentives for monetary policies may change according to the contract mix, especially in the case of foreign lending.

to local borrowers, if each contract implies default (only) with high exchange rates, then borrowers are indifferent in terms of debt denomination.

We already saw that, according to proposition 14, if the conditions in that proposition hold, local investors lend through a nominal contract as long as this induces default in (S_L, z_H). This implies that there cannot be default with dollar contacts. Thus, if local borrowers would choose between a dollar contract offered by foreign lenders and a nominal contract offered by local lenders, then conditions are not compatible with cases **T'2** and **N'3** simultaneously.

Now, consider the case where local borrowers choose between a non-default-inducing dollar contract and a nominal one inducing default at (S_L, z_H) offered by a local lender. Then:

Proposition 17 *Given the conditions in proposition 14, then local borrowers always choose a non-default-inducing T-good contract rather than a nominal one (that implies default with low exchange rates)*

An important corollary is that in (almost) all instances the source of lending would be international:

Corollary 18 *All the cases considered above imply that, in equilibrium, lending to the international market is weakly dominant for local investors, and will be strictly preferred except when the dollar contract implies no default (in which case there is indifference).*

This states that perfect access to international financial markets implies that all the equilibrium borrowing *must* come from abroad. Thus, additional elements would have to be introduced to the model in order to explain the typical observation of a high density of financial transactions between residents of a given economy, that is, the "triangular" pattern of lending abroad and borrowing from abroad obtained in this corollary does not appear as a commonly observed feature.

In any case, we end this section considering the case of default in the state (S_L, z_H) under the nominal contract. Suppose that the borrower compares such nominal contract with a dollar contract that induces no default. The proof of proposition 17 gives that the difference between the dollar contract and the nominal one is:

$$(1-\pi)\left[(Ap_{NH} - R^*)\left(\frac{1}{\widehat{P}_H} - \frac{1}{\widehat{P}_L}\right) + \frac{\phi}{\widehat{P}_L}\right] \equiv \eta$$

Since $Ap_{NH} > R^*$ by assumption, the sign of the expression is ambiguous. The nominal contract concentrates returns to the borrower in the states where the purchasing power of tradeables is high. The effect is seen favorably by the borrower, who cares about the return in terms of the CPI basket. Against this, the cost of default favors the "dollar contract". Thus, this will be chosen by the borrower if ϕ is large enough.

Suppose that the borrower now chooses between the same nominal contract as before (inducing default in the state of low exchange rates) and a dollar contract that yields default in state (S_H, z_L). The corresponding expected returns for the borrower are, for the $T-$debt contract, is $\frac{E[Ap_N]-R^*-\pi\phi}{\widehat{P}_H}$, while for the nominal contract is $\frac{E[Ap_N]-R^*-(1-\pi)\phi}{\widehat{P}_L}$. Thus, the difference in payoffs can be written as:

$$\left(\frac{1}{\widehat{P}_H} - \frac{1}{\widehat{P}_L}\right)(E(Ap_N) - R^*) + \phi\left(\frac{1-\pi}{\widehat{P}_L} - \frac{\pi}{\widehat{P}_H}\right)$$

The first term "favors " the nominal contract, for the reason stated above. That is, the "dollar " payoff for the borrower accrues in the high real exchange rate with the nominal contract, and therefore has a high purchasing power over domestic goods. However, the default costs, determined in tradeable goods, are also paid in a high-price state. The choice of T-contracts would result if the probability π is low enough and the magnitude of the default cost ϕ large enough.

4 A closer look at the literature

The analysis has been carried out under extreme simplifying assumptions, which allow a very stylized description of the choices of agents as a function of fundamental variables, and may orient the search for useful extensions (like those that would qualify the strong, but clearly counterfactual result that in the financially open economy local agents trade assets with non-residents, and not among themselves). The literature in the area tends to use conceptually more complicated frameworks, so that their results would be, in a sense, complementary with those obtained here.

The arguments of this paper are related to those in Jeanne (2003). This author constructs a partial equilibrium model with one good and two units of account. He also considers the interaction between future nominal exchange rate shocks and the physical returns of the projects of the borrowers (in our case, the emphasis is on real exchange rate shifts). In that model, the equilibrium contract minimizes the default probability. This resembles our choice criterion in the case of a financially closed economy, where all that matters are the properties of the contract concerning default.

On their side, Ize and Parrado (2002) present an open economy model, with two tradeable goods (home and foreign) but without a non-tradeable commodity, as in this paper. In their model, the focus is on the determination of the equilibrium fraction of loans made in foreign currency, which is determined by the volatilities of the price index and real shocks; default is excluded from the argument. Our work is more simple in structure, as we do not consider "mixtures" of contracts. This also marks a difference with Ize and Levy-Yeyati (2003), who use a CAPM-type of framework, where investors face a hedging problem. The returns are (implicitly) measured in CPI units, as in our model. However they consider financial intermediaries, which are absent in our framework. Their result on the imperfect *peg* case is similar to our case of default under nominal contracts with low real and nominal exchange rates. In this regard both results refer to a *peso problem* that can be generated from the respective models. However, they rely on the assumption of risk-averse depositors, while in our case it is not risk aversion but default costs that generate the result.

Broda and Levy-Yeyati (2003) and Ize and Powell (2004) represent systems with financial intermediation, which is absent in our setup. The first of these models introduces a market failure: the liquidation procedures for the intermediaries in the event of bankruptcy generate an externality that implies excessive deposit dollarization relative to the optimal level. In the case of Ize and Powell, they model an interaction between banks and a regulator / monetary authority with explicit objectives, and study the links between the actions of the authority and the degree of dollarization of contracts. One common aspect that is shared with our paper is one of the three possible sources of default that arise in the model. This is what they call the "output-induced " credit risk. However, the model considers that borrowers produce tradeable goods. Default occurs when the real exchange rate is overvalued (low values of $\frac{p_T}{p_N}$). In our setup, borrowers, having chosen dollarized debt, default when the real exchange rate is *high*. This is key to understand the differences between the results of both models[11].

[11] Although we do not show this formally, we expect that, in our model, borrowers

Neumeyer (1998) addresses the issue of trading in nominal and real securities using a competitive general equilibrium CAPM framework. This model assumes only one commodity, and no issues of relative price volatility are considered. Real shocks are associated with shocks on future endowments. That model stresses the trade-off between (excess) nominal volatility (which would tend to induce exchange rate pegging) and real volatility (which would favor floating). In our case, the exchange rate policy is taken as given, and we explore how it interacts with real exchange rate volatility to explain the decisions on contract denomination in the credit market.

Although we do not consider the determination of monetary policies, the issue is certainly close to the problem of interest. The matter has been treated by Chang and Velasco (2004) in a quite different setting. They study the dollarization of (external) debt with an endogenous nominal exchange rate policy. They obtain two equilibria, one where dollarization is low and the Central Bank sets a flexible exchange rate regime, and one where dollarization is high and the Central Bank pegs the local currency to the foreign currency. It is worth noting that they assume risk-averse borrowers and no default. It would be interesting to see if and how our model may be extended to deliver similar results. This would call for a richer model where nominal exchange rate *shocks* are not treated as exogenous but represented through a model of Central Bank policy choices.

5 Concluding Remarks

We have studied decisions regarding the nature of debt contracts in a very simple framework. Those simplifications allowed us to concentrate on the effects of the default properties of contracts, and the symmetries or asymmetries in the preferences of agents over different types of goods, in two extreme settings, one where the economy is financially closed, and the other where it is perfectly integrated with international markets.

The exploration in this paper tends to confirm some basic intuitions. Unsteady and erratic monetary policies, which make uncertain the real outcome of nominal contracts and which also make indexed nominal contracts less at-

producing tradeable instead of non-tradeable goods may yield different results regarding debt denomination choice, with possible coexistence of dollarized and nominal debts in equilibrium if both tradeable and non-tradeable producers operate as borrowers in the same credit market.

tractive (due to reporting lags), tend to induce dollarization of liabilities as long as the volatility of the real exchange rate is not too large. However, the vulnerability of those contracts to real exchange rate shocks (which may result in some instances in widespread defaults) is one of the (large) costs associated with the failure to provide a workable "domestic" unit of account for transactions among residents.

We have obtained conditions for the probability distributions of nominal and real shocks for which individuals would choose to denominate either in nominal terms or in "dollar " terms contracts that provide finance to the production of non-traded goods. The results are driven by two forces: the effect of the variabilities and correlation of the nominal and the real exchange on the default properties of the contracts, and the incentives for domestic agents to concentrate claims denominated in a certain unit in states where the value of the unit of account (deflated by the appropriate consumer price index) is comparatively high.

Besides these general considerations, the analysis has suggested that the choice of a contractual unit depends in a non-trivial way on the characteristics of monetary policies (not only their variability, but also their correlation with the real shocks) and those of real impulses that act upon the relative price of traded and non-traded goods. In addition, the preferences of the agents may also matter. In the case where all agents who participate in the financial market care about consumption of the domestic basket (composed by traded and non-traded goods) corresponding to a market with local agents only, the ranking of different contracts depends only on the costs of default that each contract may generate.

By contrast, when the expected returns on assets are determined in terms of "dollars " (as would be the case with a financially open economy), there is an additional consideration regarding the allocation of relative-price risks that affect the "dollar " value of the consumption basket across states. However, we obtain the strong (and counterfactual) result that, under the assumptions of the model, all local savings would be lent abroad, in "dollars".

Leaving aside questions about the incentives for monetary policies (which this model assumes to be exogenous, and independent of the choice of contracts by private agents and the nationality of the lenders) it turns out that nominal contracts tend to be more preferred to "dollar " contracts in the international-arbitrage case. Also, when considering the inclusion of contracts with payments linked to the aggregate price level (possibly with noise) it does not seem obvious that indexed contracts are unambiguously superior or inferior when compared to the other alternative debt, without considering the specifics of the shocks that may act upon the outcomes.

Natural extensions of the argument would be to incorporate risk aversion over real income, or different motives for borrowing (e.g. by producers of traded goods), which could result in situations with multiple contractual forms, and to contemplate a finer representation of costs and restrictions for cross-country lending. As previously stated, another noticeable simplification of the argument in this paper is in the assumption that the "nominal shock" being independent of the denomination of assets. In this regard, to the extent that "asset dollarization" would discourage monetary variability, it need not be the case that large monetary shocks are actually observed in "dollarized economies". The relevant parameter to define the incentives for private agents to dollarize would be the variability under contracting in "domestic units".

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A Proofs of propositions

Proof of proposition 4. There is never default with a T good denominated contract iff $SR_T \leqq SAp_N(z_L) = S\left(\frac{1-\alpha}{\alpha}\right)z_L$. The equilibrium interest rate must satisfy $E\left[\frac{R_T}{\widehat{P}}\right] = 1$, which implies:

$$R_T = \frac{1}{E\left[\frac{1}{\widehat{P}}\right]} = \frac{1}{\frac{\pi}{\widehat{P}(z_L)} + \frac{1-\pi}{\widehat{P}(z_H)}}$$

For this to be an equilibrium it must be the case that $E\left[\frac{1}{\widehat{P}}\right] Ap_N\left(z_L\right) \geq 1$, which is equivalent to $\pi A^{1-\alpha} z_L^{\alpha} \geq 1$.

When there is default at state (S_H, z_L) it can be shown that R_T is equal to:

$$R_T = \left\{ \frac{1 + \pi\left[\frac{\phi - Ap_N(z_L)}{\widehat{P}(z_l)}\right]}{(1-\pi)} \right\} \widehat{P}\left(z_H\right)$$

Thus, there is no if and only if

$$Ap_N\left(z_L\right) < 1 < \left[\frac{1 + \pi\left[\frac{\phi - Ap_N(z_L)}{\widehat{P}(z_l)}\right]}{(1-\pi)} \right] \widehat{P}\left(z_H\right) \leq Ap_N\left(z_H\right)$$

For this to happen we first need that:

$$\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\psi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\,\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)} \right) \leq z_H^{\alpha}$$

Clearly, for given z_L, there exists values for z_H such that this holds. In fact there exists a unique threshold value $z_H\left(z_L\right)$ such that, for any $z_H > z_H\left(z_L\right)$ the inequality holds strict. On the other hand we also need that z_L be sufficiently low so that the borrower does not repay his debt under the non-default interest rate, that is $\frac{1}{A^{1-\alpha}\alpha\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)} > z_L\left(\frac{1-\alpha}{\alpha}\right)$, or equivalently,

$$\frac{1}{A^{1-\alpha}\left(1-\alpha\right)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)} > z_L$$

It can be shown that $\frac{1}{A^{1-\alpha}(1-\alpha)\left(\frac{\pi}{z_L^{1-\alpha}} + \frac{1-\pi}{z_H^{1-\alpha}}\right)}$ is less than $\left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right)$, so that $z_L < \left(\frac{\frac{1}{A^{1-\alpha}} + \frac{\alpha\chi}{k}\frac{\pi}{z_L^{1-\alpha}} - (1-\alpha)\pi z_L^{\alpha}}{(1-\alpha)(1-\pi)}\right) z_H^{1-\alpha}$, so default occurs effectively at z_L. This ends the proof. ■

Proof of Proposition 5. No default under nominal contract implies the following value for the gross interest rate:

$$R_{nom} = \frac{1}{\left(\frac{1-\pi}{P(S_L,z_H)} + \frac{\pi}{P(S_H,z_L)}\right)} = \frac{S_H S_L}{\left(\frac{\pi}{\widehat{P}(z_L)} S_L + \frac{1-\pi}{\widehat{P}(z_H)} S_H\right)} = \frac{S_H S_L}{\widehat{\pi}_L S_L + \widehat{\pi}_H S_H}$$

For this to be an equilibrium we need that $Ap_N(z_L) \geq \frac{R_{nom}}{S_H}$ and $Ap_N(z_H) \geq \frac{R_{nom}}{S_L}$. Note that since $\frac{\pi}{z_H^{1-\alpha}} > 0$ and $\frac{S_H}{S_L} > 1$ then $\frac{S_H}{S_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}} > \frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}} > \frac{\pi}{z_L^{1-\alpha}}$ for any $S_j's$ and $z_k's$. So

$$\frac{1}{A^{1-\alpha}\alpha\left(\frac{S_H}{S_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}}\right)} < \frac{1}{A^{1-\alpha}\alpha\frac{\pi}{z_L^{1-\alpha}}} = \frac{z_L^{1-\alpha}}{\alpha A^{1-\alpha}\pi}$$

Hence as z_L is large enough $\frac{z_L^{1-\alpha}}{\alpha A^{1-\alpha}\pi}$ must be less than z_L since the first is a strictly concave function. Therefore z_L is greater than $\frac{1}{A^{1-\alpha}\alpha\left(\frac{S_H}{S_L}\frac{1-\pi}{z_H^{1-\alpha}} + \frac{\pi}{z_L^{1-\alpha}}\right)}$. As $z_H > z_L$, so happens with z_H. On the other hand, since $\frac{1-\pi}{z_H^{1-\alpha}} + \frac{S_L}{S_H}\frac{\pi}{z_L^{1-\alpha}} > \frac{1-\pi}{z_H^{1-\alpha}}$ then

$$\frac{1}{A^{1-\alpha}\alpha\left(\frac{1-\pi}{z_H^{1-\alpha}} + \frac{S_L}{S_H}\frac{\pi}{z_L^{1-\alpha}}\right)} < \frac{1}{A^{1-\alpha}\alpha\left(\frac{1-\pi}{z_H^{1-\alpha}}\right)} = \frac{z_H^{1-\alpha}}{A^{1-\alpha}\alpha(1-\pi)}$$

Given that this is a strictly concave function, there exists z_L is large enough so that

$$z_H > z_L > \frac{z_H^{1-\alpha}}{A^{1-\alpha}\alpha(1-\pi)}$$

This shows the first part of **N1**. Suppose $E\left[\frac{1}{P}\right] Ap_N(z_L) < 1$. We basically need that

$$S_H Ap_N(z_L) E\left(\frac{1}{P}\right) \geq 1; \quad S_L Ap_N(z_H) E\left(\frac{1}{P}\right) \geq 1$$

which holds if and only if

$$\frac{1}{\widehat{\pi}_L Ap_N(z_H)} - \frac{\widehat{\pi}_H}{\widehat{\pi}_L} \leq \frac{S_L}{S_H} \leq \frac{1}{\left[\frac{1}{\widehat{\pi}_H Ap_N(z_L)} - \frac{\widehat{\pi}_L}{\widehat{\pi}_H}\right]}$$

which rewritten yields the second condition in **N1**.

On the other hand, default at (S_H, z_L) with nominal contracts imply:

$$R_{nom}^{(S_L,z_H)} = \left(\frac{1 + \frac{\pi}{\widehat{P}(z_L)} \left[\phi - Ap_N (z_L) \right]}{(1-\pi)} \right) P(S_L, z_H) = \left(\frac{1 + \frac{\pi}{\widehat{P}(z_L)} \left[\phi - Ap_N (z_L) \right]}{(1-\pi)} \right) S_L \widehat{P}(z_H)$$

For this to be consistent with an equilibrium we need to get that $A\ S_H\ p_N (z_L) < R_{nom}^{(S_L,z_H)} \leqq A\ S_L\ p_N (z_H)$, equivalent to have $A\ p_N (z_L) < \left(\frac{1+\frac{\pi}{\widehat{P}(z_L)}[\phi - Ap_N(z_L)]}{(1-\pi)} \right)$ $\frac{S_L}{S_H}\ \widehat{P}(z_H)$ and at the same time $\left(\frac{1+\frac{\pi}{\widehat{P}(z_L)}[\phi - Ap_N(z_L)]}{(1-\pi)} \right)\ \widehat{P}(z_H) < Ap_N (z_H)$, or:

$$Ap_N (z_L) S_H E\left[\frac{1}{P}\right] < 1 < 1 + \phi\widehat{\pi}_L < E_{\widehat{\pi}}\left[Ap_N\right]$$

as appears in **N2**.

If there is default at (S_L, z_H) then

$$R_{nom}^{(S_H,z_L)} = \left(\frac{1 + \frac{1-\pi}{\widehat{P}(z_H)} \left[\phi - Ap_N (z_H) \right]}{\pi} \right) S_H \widehat{P}(z_L)$$

This is consistent with an equilibrium when $R_{nom}^{(S_H,z_L)} \leqq S_H A p_N (z_L)$ and $R_{nom}^{(S_H,z_L)} > S_L\ Ap_N (z_H)$. These two imply that:

$$1 + \phi\widehat{\pi}_H \leqq E_{\widehat{\pi}}\left[Ap_N\right]$$

and

$$Ap_N (z_H) < \frac{\left[1 + \phi\widehat{\pi}_H\right] S_H}{\widehat{\pi}_H S_H + \widehat{\pi}_L S_L}$$

which implies the inequalities in **N3**. ■

Proof of Proposition 10. If the total absence of default were true then the first order conditions from the lender imply

$$\sum_{(S,z)} q(z,S) \frac{rP(S,z)}{P(S,z)} = 1$$

or just simply $r = 1$. On the other hand, there is no default under any state if and only if

$$\widehat{P}(z) \leq Ap_N(z)$$

for all z, which is equivalent to

$$1 \leq SA\left[p_N(z_L)\right]^{\alpha}$$

The condition in the proposition is exactly this inequality. This ends the proof. ■

Proof of Proposition 11. The case T'1 is trivial. For T'2 we have that, from the arbitrage condition, the interest rate is:

$$R_T' = \frac{R^* - \pi Ap_{NL} + \pi\phi}{1 - \pi}$$

In addition, the value of output in the "good state" must allow repayment at that interest rate:

$$Ap_{NH} - R^* \geq \pi\left[A\left(p_{NH} - p_{NL}\right) + \phi\right]$$

Reorganizing this:

$$E\left[Ap_N\right] \geq R^* + \pi\phi$$

so the inequalities in the statement of this proposition hold. ■

Proof of Proposition 12. The proof of the case **N'1** is simple. If there is no default, the nominal interest rate R_{nom}' is equal to:

$$R_{nom}' = \frac{R^* S_L S_H}{\pi S_L + (1 - \pi) S_H}$$

Clearly, the no-default conditions are:

$$Ap_{NL} \geq \frac{R_{nom}'}{S_H} \quad \text{and} \quad Ap_{NH} \geq \frac{R_{nom}'}{S_L}$$

which hold when $Ap_{NL} \geq R^*$. If this not true, i.e., if $Ap_{NL} < R^*$, the no-default conditions for nominal contracts can be written as:

$$\frac{R^* - (1 - \pi) Ap_{NH}}{\pi Ap_{NH}} \leq \frac{S_L}{S_H} \leq \frac{(1 - \pi) Ap_{NL}}{R^* - \pi Ap_{NL}}$$

as in the statement of this proposition.

For the case **N'2** the proof goes as follows. The nominal interest rate would be given by:

$$R'_{nom} = \frac{S_L}{1-\pi}\left[R^* - \pi A p_{NL} + \pi\phi\right]$$

There is default in the state (S_H, z_L) if and only if:

$$Ap_{NL} < \frac{R^* S_L}{\pi S_L + (1-\pi) S_H}$$

(a stronger requirement than that which establishes default in the state p_{NL} with traded-goods contracts). Given the expression for the interest rate, the solvency condition in state p_{NH} has the same expression than in the case with traded-goods contracts:

$$Ap_{NH} - R^* \geq \pi\left[A\left(p_{NH} - p_{NL}\right) + \phi\right]$$

as stated by the statement of the proposition.

The proof of **N'3** is similar. The gross interest rate must now satisfy

$$R'_{nom} = \left(\frac{S_H}{\pi}\right)\left(R^* + (1-\pi)\phi - (1-\pi) A p_{NH}\right)$$

The condition for default in state (S_L, z_H) is:

$$Ap_{NH} < \frac{R^* S_H}{\pi S_L + (1-\pi) S_H}$$

In order for the revenue in state p_{NL} to allow repayment of the debt, it must be true that $Ap_{NL} \geq R^* - (1-\pi)\left[A\left(p_{NH} - p_{NL}\right) - \phi\right]$, or:

$$E\left[Ap_N\right] \geq R^* + (1-\pi)\phi$$

as desired. ■

Proof of Proposition 13. In case **T '1**, it is trivial to show that there is no difference between $V^{L,local}_{dollar}$ and $V^{L,local}_{foreign}$, since both imply a gross interest rate of R^*, so that lenders are strictly indifferent regarding where to invest. In case **T '2** it can be shown, after some algebraic manipulation, that the difference between the utilities from local and international lending is:

$$\pi\left[\frac{1}{\widehat{P}_H} - \frac{1}{\widehat{P}_L}\right]\left[R^* - \left(Ap_{NL} - \phi\right)\right]$$

which is clearly negative, since $\widehat{P}_H > \widehat{P}_L$ and there is default in state p_{NL}. ■

Proof of Proposition 14. For the case **N'1,** the difference between $V_{nom}^{L,local}$ and $V_{foreign}^{L,local}$ is $\left[\frac{R^*}{E\left[\frac{1}{S}\right]}\right] E\left[\frac{1}{S\widehat{P}}\right] - R^* E\left[\frac{1}{\widehat{P}}\right]$. But, given the assumption about the positive correlation between nominal and real exchange rates:

$$E\left[\frac{1}{S\widehat{P}}\right] < E\left[\frac{1}{S}\right] E\left[\frac{1}{\widehat{P}}\right]$$

Thus, the agent derives utility from the fact that the international contract offers a comparatively high dollar return when the real price of tradeables is high.

Suppose that the nominal contract implies default in state (S_H, z_L), which is case **N '2.** Then, the difference of utilities $(V_{nom}^{L,local} - V_{foreign}^{L,local})$ is:

$$\left[\frac{R^* - \pi(Ap_{NL} - \phi)}{(1-\pi)\frac{1}{S_L}}\right](1-\pi)\frac{1}{S_L\widehat{P}_H} + \left[\frac{Ap_{NL} - \phi}{\widehat{P}_L}\right]\pi - R^*\left[\frac{1-\pi}{\widehat{P}} + \frac{\pi}{\widehat{P}_L}\right]$$

which is equivalent to the condition for dollar contracts in this state:

$$Ap_{NL} - \phi - R^*$$

Now, in order for the case to apply, it must be:

$$Ap_{NL}S_H\left[\frac{1-\pi}{S_L} + \frac{\pi}{S_H}\right] < R^*$$

Therefore, given that $S_H > S_L$, the international contract is preferred.

Finally, suppose that the nominal contract induces default in the case of the low real exchange rate (case **N'3**). The difference $V_{nom}^{L,local} - V_{foreign}^{L,local}$ is now:

$$\left[\frac{R^* - (1-\pi)(Ap_{NH} - \phi)}{\pi\frac{1}{SH}}\right]\frac{\pi}{S_H\widehat{P}_L} + \left[\frac{Ap_{NH} - \phi}{\widehat{P}_H}\right](1-\pi) - R^*\left[\frac{\pi}{\widehat{P}_L} + \frac{1-\pi}{\widehat{P}_H}\right]$$

This difference reduces to $[R^* - (Ap_{NH} - \phi)]\left[\frac{1}{\widehat{P}_L} - \frac{1}{\widehat{P}_H}\right](1-\pi)$, the sign of which is that of:

$$R^* - (Ap_{NH} - \phi)$$

Thus the inequalities in the statement of the proposition come from the sign of this expression. It can be verified that, if $R^* - (Ap_{NH} - \phi) > 0$ is satisfied and dollar lending is feasible, in the sense that $AE(p_N) > R^*$, then there would be no default on the dollar contracts, since $Ap_{NL} > R^*$. ■

Proof of Proposition 15. When there is no default under any contract, the proof is a simple comparison of the ex-ante borrower's utility from either contract, i.e.,between $V_{nodef}^{B,dollar} = E\left(\frac{Ap_N}{\widehat{P}}\right) - E\left[\frac{R^*}{\widehat{P}}\right]$ and $V_{nodef}^{B,nom} = E\left(\frac{Ap_N}{\widehat{P}}\right)$ $- R^* \frac{E\left[\frac{1}{S\widehat{P}}\right]}{E\left[\frac{1}{S}\right]}$. This reduces to compare $-\frac{1}{\pi\widehat{P}(p_{NL})+(1-\pi)\widehat{P}(p_{NH})}$ and $-\frac{\pi}{\widehat{P}(p_{NL})} - \frac{1-\pi}{\widehat{P}(p_{NH})}$. To prove this first part, suppose that the $T - good$ contract gives more utility. Hence $-\frac{\pi}{\widehat{P}(p_{NL})} - \frac{1-\pi}{\widehat{P}(p_{NH})} \geq -\frac{1}{\pi\widehat{P}(p_{NL})+(1-\pi)\widehat{P}(p_{NH})}$, or

$$\frac{\pi}{\widehat{P}\left(p_{NL}\right)} + \frac{1-\pi}{\widehat{P}\left(p_{NH}\right)} \leq \frac{1}{\pi\widehat{P}\left(p_{NL}\right) + (1-\pi)\,\widehat{P}\left(p_{NH}\right)}$$

but $\frac{1}{\widehat{P}}$ is a strictly convex function of $\widehat{P}$, so we must have that $\frac{\pi}{\widehat{P}(p_{NL})} + \frac{1-\pi}{\widehat{P}(p_{NH})}$ $> \frac{1}{\pi\widehat{P}(p_{NL})+(1-\pi)\widehat{P}(p_{NH})}$, a contradiction. Then: Thus, the difference $V_{nodef}^{B,dollar}$ $- V_{nodef}^{B,nom}$ can be shown to be equal to:

$$-\pi(1-\pi)\frac{(S_H - S_L)}{\pi S_L + (1-\pi)S_H}\left(\frac{1}{\widehat{P}_L} - \frac{1}{\widehat{P}_H}\right) < 0$$

and so $V_{nodef}^{B,dollar} < V_{nodef}^{B,nom}$.

When there is default with the T good contract in state (S_H, z_L) and no default with the nominal one, the expected value of real income for the T - good contract is:

$$\frac{1-\pi}{\widehat{P}_H}\left[Ap_{NH} - \frac{R^* - \pi(Ap_{NL} - \phi)}{1-\pi}\right]$$

while for the nominal contract the ex-ante utility takes the same expression as before. Hence the decision is made according to the sign of:

$$\begin{aligned} & \frac{(1-\pi)\,Ap_{NH} + \pi Ap_{NL}}{\widehat{P}_H} - \frac{R^* + \phi\pi}{\widehat{P}_H} - \\ -E\left[\frac{Ap_N}{\widehat{P}}\right] + \frac{\pi}{\widehat{P}_L}\frac{R^* S_L}{\pi S_L + (1-\pi)\,S_H} + & \frac{(1-\pi)}{\widehat{P}_H}\frac{R^* S_H}{\pi S_L + (1-\pi)\,S_H} \end{aligned}$$

This can be reduced to:

$$\pi\left[\left(\frac{1}{\widehat{P}_H}-\frac{1}{\widehat{P}_L}\right)\left(Ap_{NL}-\frac{R^*S_L}{\pi S_L+(1-\pi)S_H}-\frac{\phi}{\widehat{P}_H}\right)\right]$$

But in this case, since there is no default with the nominal contract in state p_{NL} :

$$Ap_{NL}-\frac{R^*S_L}{\pi S_L+(1-\pi)S_H}>0$$

■

Proof of Proposition 16. In the case **T'2 - N'2**, it must be the case that $Ap_{NL}<\frac{R^*S_L}{\pi S_L+(1-\pi)S_H}$, and therefore, $Ap_{NL}<R^*$. The expected values of income are, for the T-good contracts, as before:

$$\frac{1-\pi}{\widehat{P}_H}\left[Ap_{NH}-\frac{R^*-\pi(Ap_{NL}-\phi)}{1-\pi}\right]$$

and, for the nominal contract:

$$\frac{1-\pi}{\widehat{P}_H}[Ap_{NH}-\frac{1-\pi}{S_H}S_H(R^*-\pi(Ap_{NL}-\phi)]$$

Both expressions are clearly equal. Therefore, the ex-ante utility obtained under any contract is the same. Thus the borrower is indifferent between the nominal and the dollar contract. ■

Proof of Proposition 17.

Expected real incomes are, for the dollarized contract, $E\left[\frac{Ap_N-R^*}{\widehat{P}}\right]$ and, for the nominal contract $\frac{E(Ap_N)-R^*-(1-\pi)\phi}{\widehat{P}_L}$. The difference between expected incomes (T-denominated minus nominal) can be expressed as:

$$(1-\pi)\left[(Ap_{NH}-R^*)\left(\frac{1}{\widehat{P}_H}-\frac{1}{\widehat{P}_L}\right)+\frac{\phi}{\widehat{P}_L}\right]\equiv\eta$$

If the conditions in Proposition 14 hold, then:

$$Ap_{NH}-\phi<R^*$$

and so:

$$\eta>(1-\pi)\frac{Ap_{NH}-R^*}{\widehat{P}_H}>0$$

Therefore, the nominal contract is inferior from the point of view of borrowers. ■