Macroeconomia 1
Clase 12
Ciclos Economicos Reales

Prof. McCandless
UCEMA

October 22, 2009

1 Real business cycles

Real Business Cycles

- Business cycles were described earlier
- When we take data and remove the HP filter
- What remains we call a buisness cycle
  - recall that the cycle depends, in part, on the lamda we use in the filter
- Cycles are regular, cyclical movements and co-movements in variables
  - output, consumption, investment, prices, wages, interest rates
  - consumption tends to be smoother than output
  - investment tends to have a larger variance than output

Random shocks

- We cannot include everything important in our model
- Some of what is important for economic outcomes are not under our control
- Some of it we can't explain
- We put all the stuff that our model doesn't explain in "random variables"
  - the values of random variables are determined by "nature'
  - we take them as given

- we don't the what the future values will be
- but we do know the distribution of the possible future values
- we include these future values in our models as expectations

- Path of the economy can be determined (partically) by expectations

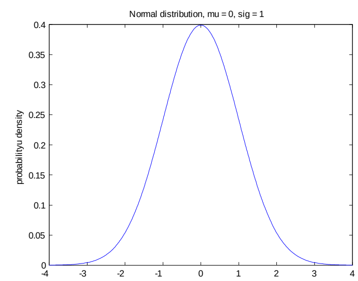
Probabilities

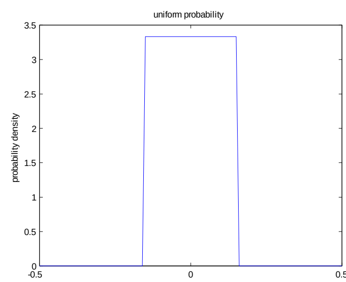
- Random variables can take on a set of values
- This set could be
  - discrete (sella y cara)
  - continuous (between 0 and 300mm of rainfall in a day)
- We assign probabilities to the chances that some value will occur
- These probabilities can be objective or subjective
  - Depends on the information available
- Objective probabilities come from historical experience
  - How often did sella come up tossing a coin?
  - How often did it rain between 1 and 3 mm in 24 hours?
- Subjective probabilities depend less clearly on historical experience
  - my subjective probabilities can be different from yours
  - my information set is different

Probabilities

- Characteristics of probability distributions
  - If every possibility is included, some of probabilities is one (1)
  - probabilities are zero or positive
  - for continuous variables, we have a probability density function
- Examples of continuious probabilities

[5cm]
5cm
Normal
5cm
Uniform
Real Real business cycle models





- Based on Solow type model
- include expectations
- include optimization of consumption, labor decisions, investment decisions, firms
- shocks occur to
  - technology
  - money
  - preferences
  - information
- people have limited information about the world and its future
- Expectations can reflect that
- We will simplify much of these aspects in our model
  - course in Masters program does the models more completely

A simple real business cycle model

- Consumers live two periods

- Have labor (1 unit) that they supply to the market when young
- receive market wage for their labor
- They do not work when old
- They can buy capital when young and hold till old
- When old, they earn rents on their capital and then sell capital that remains to new young
- Each period there is a shock to technology
  - changes in weather
  - political shocks that are not modelled
  - changes in world demand for goods

.The consumer's decision

- Consumers have the utility function

$$u_t^h = \ln c_t^h(t) + \beta \ln c_t^h(t+1)$$

- $c_t^h(s)$ = consumption in period s of person h born in period t
  - person only lives two periods
  - only interesting consumptions are in period $s = t$ and $t + 1$
- budget constraint when young is

$$w_t = c_t^h(t) + k^h(t+1)$$

- budget constraint when old is

$$c_t^h(t+1) = r_{t+1} k^h(t+1) + (1-\delta) k^h(t+1)$$

.Writing out consumer's problem

- Rewrite first budget constraints as

$$c_t^h(t) = w_t - k^h(t+1)$$

  and keep second budget constraint as

$$c_t^h(t+1) = r_{t+1} k^h(t+1) + (1-\delta) k^h(t+1)$$

- the wages and rentals are given
- Choice variable for the consumer is $k^h(t+1)$

– choosing $k^h(t+1)$ determines $c_t^h(t)$ and $c_t^h(t+1)$

- utility can be written as

$$u_t^h = \ln\left(w_t - k^h(t+1)\right) + \beta \ln\left(r_{t+1} k^h(t+1) + (1-\delta) k^h(t+1)\right)$$

Solving consumer's problem

- Take derivative of utility function

$$u_t^h = \ln\left(w_t - k^h(t+1)\right) + \beta \ln\left(r_{t+1} k^h(t+1) + (1-\delta) k^h(t+1)\right)$$

  with respect to capital

- First order condition is

$$\begin{aligned} \frac{\partial u_t^h}{\partial k^h(t+1)} &= 0 = -\frac{1}{w_t - k^h(t+1)} \\ &\quad + \frac{\beta}{[r_{t+1} + (1-\delta)] k^h(t+1)} (r_{t+1} + (1-\delta)) \end{aligned}$$

- simplifying get

$$\frac{1}{w_t - k^h(t+1)} = \frac{\beta}{k^h(t+1)}$$

- or

$$k^h(t+1) = \beta\left(w_t - k^h(t+1)\right)$$

Solving consumer's problem

- Result is

$$k^h(t+1) = \frac{\beta}{1+\beta} w_t$$

- Consumers save a fraction $\beta/(1+\beta)$ of their income when young
- In this simple economy: expected future rentals do not enter the decision
- This is a result of the log utility function
  - doesn't hold for other types of utility functions
  - but makes our life easy
  - income and substution effects exactly cancel out with log utility functions
- a reasonable value for β is .98 or so (the book uses $\beta = 1$)

. Determining wages

- Assume the aggregate production function in period t is

$$Y_t = A_t K(t)^\theta L(t)^{1-\theta}$$

- Wages equal the marginal product of labor, so

$$w_t = \frac{\partial w_t}{\partial L(t)} = (1-\theta) A_t K(t)^\theta L(t)^{-\theta}$$

- To keep things simple, assume that there is only one person born in each period
- An equilibrium condition, which we can apply now, is that $L(t) = 1$
- That gives that wages are equal to

$$w_t = (1-\theta) A_t K(t)^\theta$$

.Capital determination

- Since there is only one person born in each period
- Another equilibrium condition is that

$$K(t+1) = k^h(t+1)$$

- Putting this and the definition of wages into the first order condition for the young person,

$$k^h(t+1) = \frac{\beta}{1+\beta} w_t$$

we get

$$K(t+1) = \frac{\beta}{1+\beta}(1-\theta) A_t K(t)^\theta$$

- This is the first order difference equation for the capital stock in this economy

. Consumption and investment

- The old sell to the young the $(1-\delta)K(t)$ of their capital that did not wear out
- Investment (the new capital) of the economy in period t is

$$I_t = K(t+1) - (1-\delta) K(t)$$

- Aggregate consumption is equal to the consumption of the young and the old

$$C_t = c_t^h(t) + c_{t-1}^h(t)$$

- The aggregate resourse constraint in period t is

$$(1-\delta)\,K\,(t)+Y_t=K\,(t+1)+C_t$$

- Aggregate consumption can be found from

$$C_t=Y_t+(1-\delta)\,K\,(t)-K\,(t+1)$$

How can we do this model without much math?

- The utility function is in logs
  - this makes the income and substitution effects cancel out
  - that means that the amount that a young person want to save is a constant fraction of income
  - like in the Solow model
  - Savings rate is only a function of β

$$s=\frac{\beta}{1+\beta}\,(1-\theta)$$

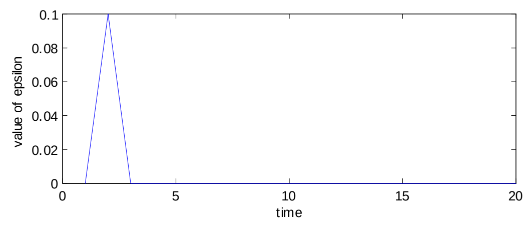
- Income to the young is only their wages
- We assume that the labor supply per person is constant (and always = 1, like in the Solow model)
- A difference from the Solow model: savings of the young $\neq$ investment
- Here savings of the young = next period's capital stock
- Investment is savings of the young minus $(1-\delta)K_t$
- The young buy the remaining capital of the old = $(1-\delta)K_t$

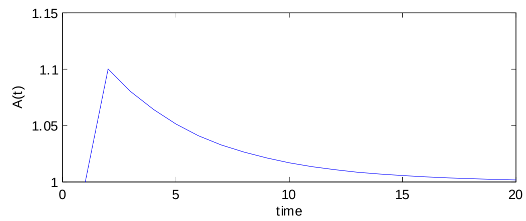
Impulse response

- How does the economy behave if
  - it starts in a stationary state
  - it gets a one time shock to technology
- The stochastic process for technology

$$A_{t+1}=1-\gamma+\gamma A_t+\varepsilon_t$$

- ε_t is the shock (the inpulse)
- An impulse response function shows the path of the variables of the economy after a one time shock to ε_t





Impulse response

- Notice that if $\varepsilon_t = 0$, always, then $A_t = 1$, in a stationary state

$$A = 1 - \gamma + \gamma A$$

- gives

$$A - \gamma A = 1 - \gamma$$

  or

$$A = 1$$

Impulse response

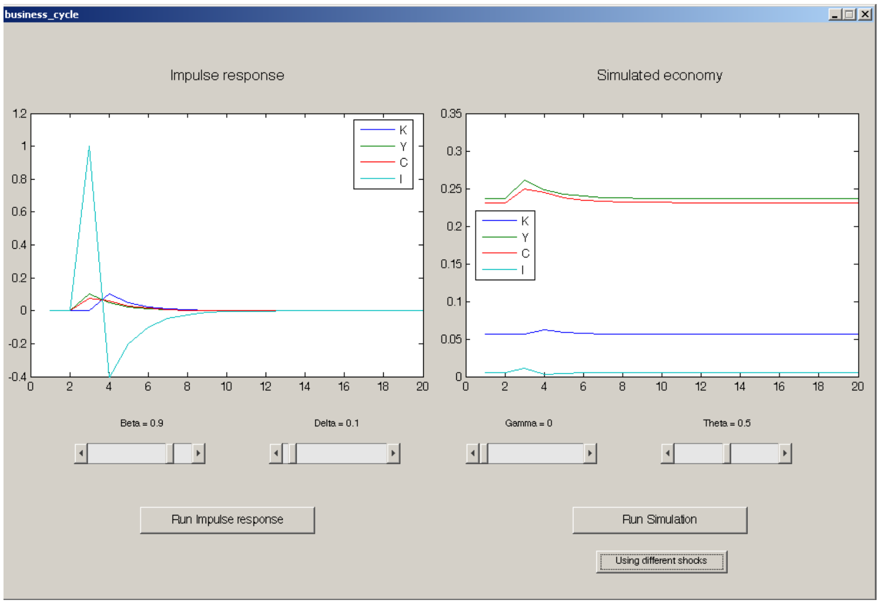
- Start with $A_0 = 1$, $\varepsilon_t = 0$ for all t except $t = 2$, here we set the shock $\varepsilon_2 = .1$
- ε_t follows the path
- if $\gamma = .8$, the A_t follows the path

Impulse response

- Start with $K(0) = \overline{K}$ the stationary state value
- Given the A_t path shown above, use

$$K(t+1) = \frac{\beta}{1+\beta}(1-\theta) A_t K(t)^{\theta}$$

  to calculate the sequence for $K(t)$



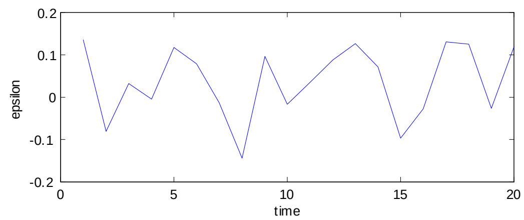
- We usually show responses as the percentage change around the stationary state
- The response for capital is

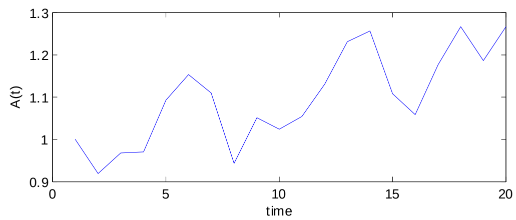
$$response\ K = \frac{K(t+1) - K(t)}{\overline{K}}$$

- Given the time path for K, we calculate the time paths for $Y(t)$, $C(t)$, $I(t)$
- We use these time paths to calculate their responses as percentage change around their stationary states

Impulse response
Simulations

- In simulations, we normally begin with $K(0) = \overline{K}$, in a stationary state
- We find a path of ε_t generated by a random (psuedo-random) number generator in the computer
- The random numbers come from a chosen distribution
  - distribution can be uniform
  - distribution can be normal





- ∗ one needs to be careful with normal distributions
- ∗ they can produce (not very often) very large negative numbers as the shock
- ∗ in that case, technology can become negative (too strange to think about)
- ∗ need to keep shocks from becoming too small

Simulations

- Example of an ε_t path with uniform shocks

Simulations

- The time path for technology is

$$A_{t+1} = 1 - \gamma + \gamma A_t + \varepsilon_t$$

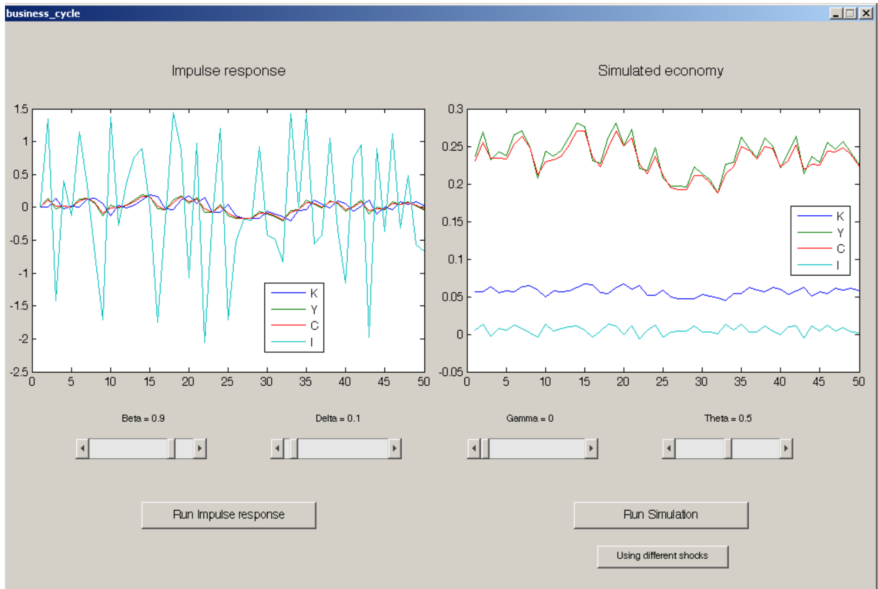
- Beginning with $A_0 = 1$
- In the example we use $\gamma = .8$
- Get time path for technology of

Simulations

- Beginning with $K(0) = \overline{K}$, use

$$K(t+1) = \frac{\beta}{1+\beta}(1-\theta) A_t K(t)^{\theta}$$

to find the time path for the capital stock



- Use (recall that $L(t) = 1$)

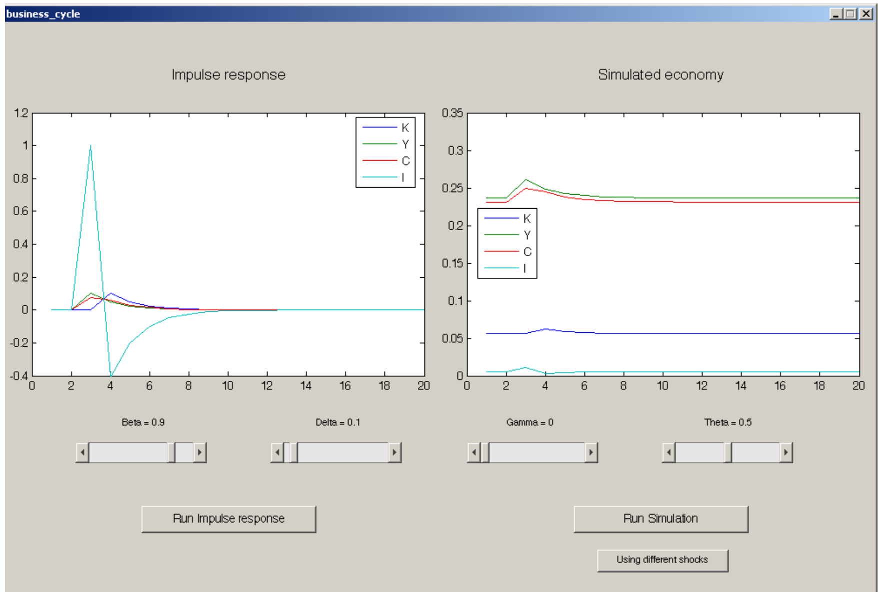
$$I_t = K(t+1) - (1-\delta)K(t)$$

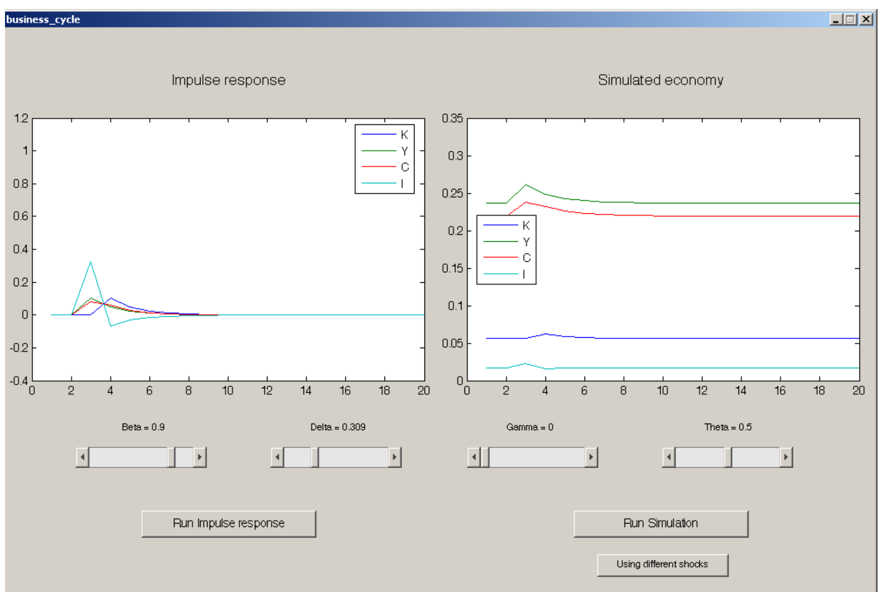
$$Y_t = A_t K(t)^{\theta}$$

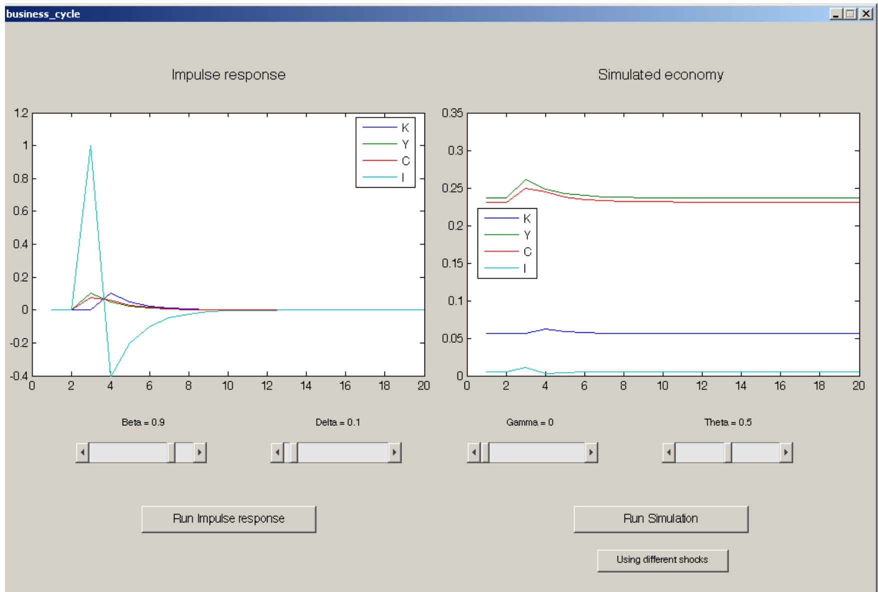
$$C_t = Y_t + (1-\delta)K(t) - K(t+1)$$

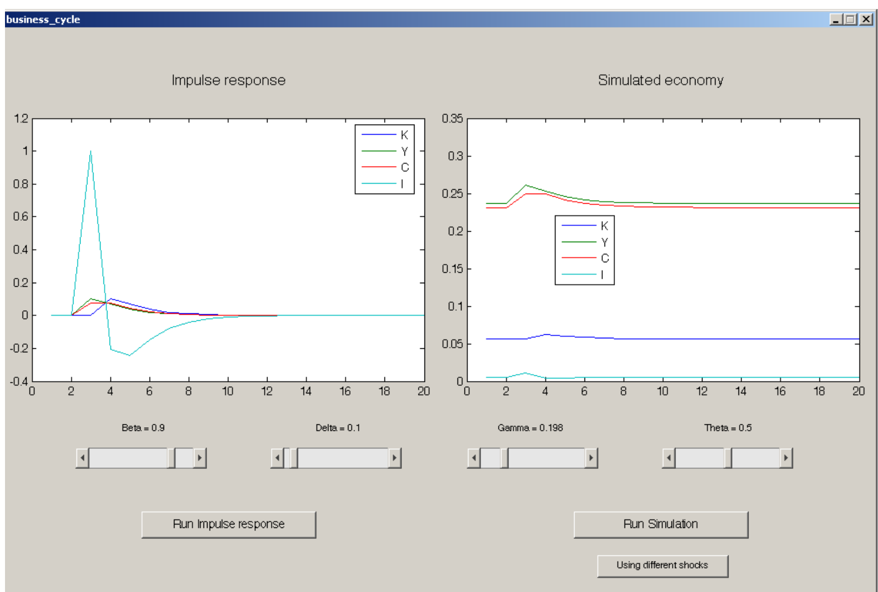
to find the time paths for the other variables

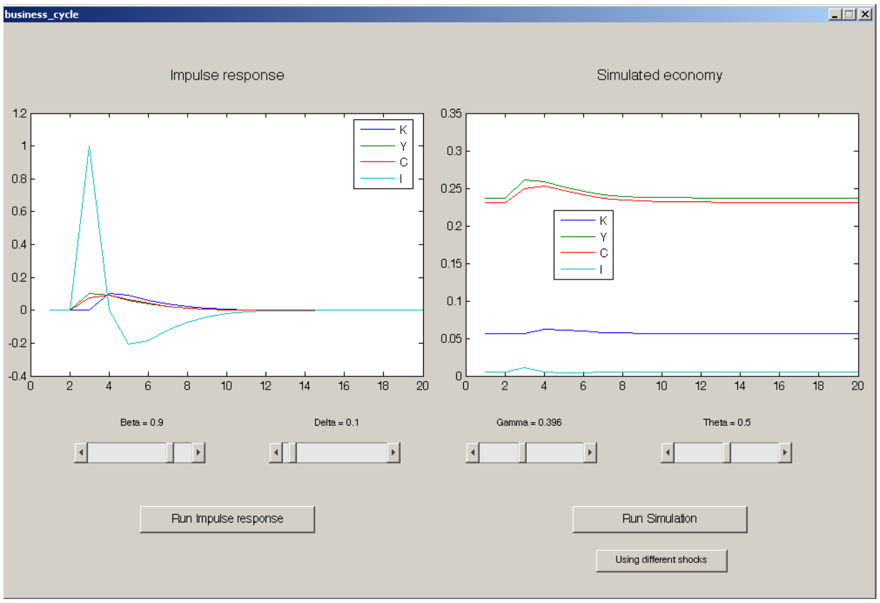
Example of a simulation
Effect of changing delta (delta = .1)
Effect of changing delta (delta = .309)
Effect of changing gamma (gamma = 0)
Effect of changing gamma (gamma = .198)
Effect of changing gamma (gamma = .396)
Simulation (gamma = 0)
Simulation (gamma = .198)
Simulation (gamma = .396)
Simulation (gamma = .99)

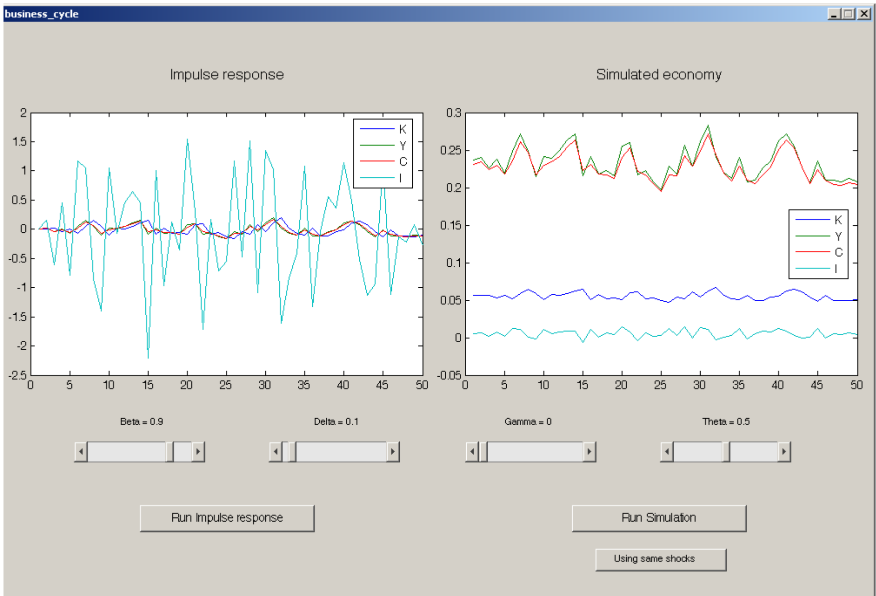

business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0
Theta = 0.5
Run Impulse response
Run Simulation
Using different shocks

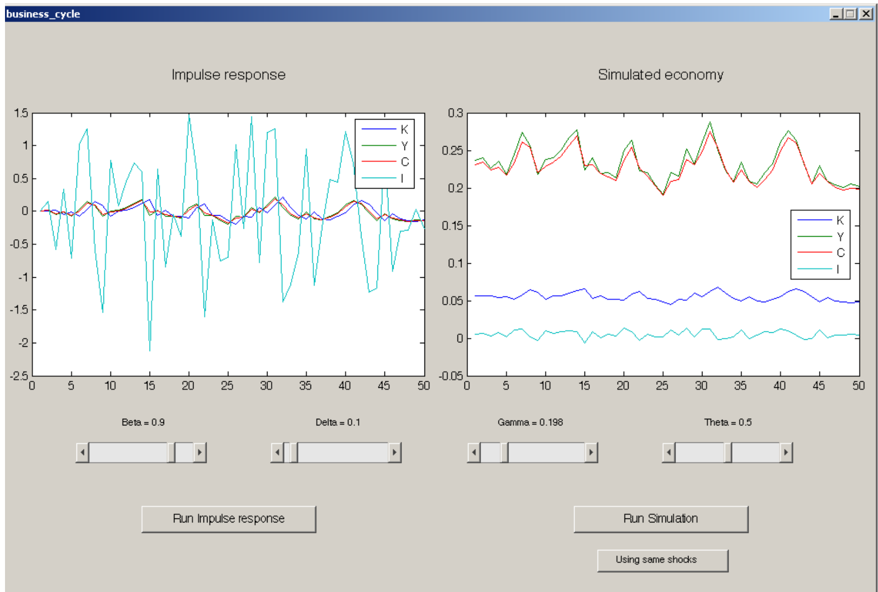

business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.309
Gamma = 0
Theta = 0.5
Run Impulse response
Run Simulation
Using different shocks

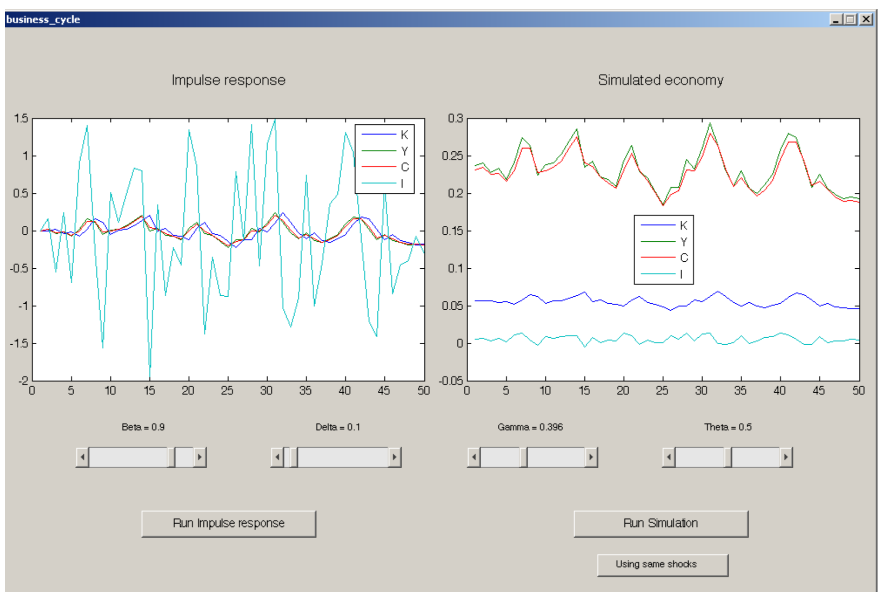

business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0
Theta = 0.5
Run Impulse response
Run Simulation
Using different shocks

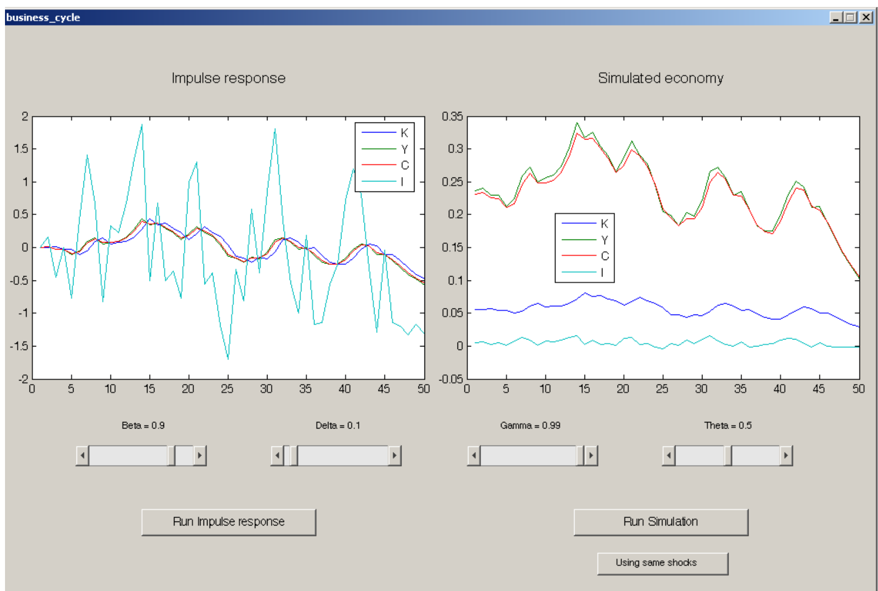

business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0.198
Theta = 0.5
Run Impulse response
Run Simulation
Using different shocks


business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0.396
Theta = 0.5
Run Impulse response
Run Simulation
Using different shocks


business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0
Theta = 0.5
Run Impulse response
Run Simulation
Using same shocks


business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0.198
Theta = 0.5
Run Impulse response
Run Simulation
Using same shocks


business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0.396
Theta = 0.5
Run Impulse response
Run Simulation
Using same shocks


business_cycle
Impulse response
Simulated economy
K
Y
C
I
Beta = 0.9
Delta = 0.1
Gamma = 0.99
Theta = 0.5
Run Impulse response
Run Simulation
Using same shocks