Calculos para CES

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$$
\begin{aligned}
L &= \sum_{t=0}^{\infty}\beta^{t}\left[\frac{c_t^{1-\eta}}{1-\eta}+B\frac{(1-l_t)^{1-\eta}}{1-\eta}\right. \\
&\quad +\lambda_t^1\left(m_{t+1}+s_{t+1}-(1+R)\,s_t-P_tl_t\right) \\
&\quad \left.+\lambda_t^2\left(m_t-P_tc_t\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial c_t} &= \frac{1}{c_t^{\eta}}-\lambda_t^2P_t=0 \\
\frac{\partial L}{\partial l_t} &= -\frac{B}{(1-l_t)^{\eta}}-\lambda_t^1P_t=0 \\
\frac{\partial L}{\partial m_{t+1}} &= \lambda_t^1+\beta\lambda_{t+1}^2=0 \\
\frac{\partial L}{\partial s_{t+1}} &= \lambda_t^1-\beta\,(1+R)\,\lambda_{t+1}^1=0
\end{aligned}
$$

first order conditions are

$$
\begin{aligned}
\frac{B}{(1-l_t)^{\eta}} &= \beta\frac{P_t}{P_{t+1}c_{t+1}^{\eta}} \\
\frac{1}{\beta\,(1+R)} &= \frac{P_t\,(1-l_t)^{\eta}}{P_{t+1}\,(1-l_{t+1})^{\eta}}
\end{aligned}
$$

$$m_{t+1}+s_{t+1}=m_t+(1+R)\,s_t+P_tl_t-P_tc_t$$

$$P_tc_t=m_t$$

$$l_t=c_t+\overline{G}$$

money growth rule is

$$\frac{m_{t+1}-m_t}{P_t}=\frac{g_tm_t}{P_t}=\overline{G}$$

In a stationary state (recall that $1+\overline{g}=1+\pi$)

$$
\begin{aligned}
\left[(1+\overline{g})\,\frac{B}{\beta}\right]^{\frac{1}{\eta}}\overline{C} &= 1-\overline{l} \\
\frac{1+\overline{g}}{\beta}-1 &= R
\end{aligned}
$$

$$\overline{C} = \overline{M/P}$$

$$\overline{g}\overline{M/P} = \bar{l} - \overline{C}$$

Given $\overline{g}$, one and use these four equations to find the four variables $\overline{C}, \overline{M/P}, \bar{l}, R$. Once these are found, for this $\overline{g}$, the real income for the government is $\overline{G} = \bar{l} - \overline{C}$.

$$R = \frac{1+\overline{g}}{\beta} - 1$$

$$\begin{aligned}
R &= \frac{1+\overline{g}}{\beta} - 1 \\
\overline{M/P} &= \frac{1}{\left(\left[(1+\overline{g})\,\frac{B}{\beta}\right]^{\frac{1}{\eta}} + (1+\overline{g})\right)} \\
\overline{C} &= \overline{M/P} \\
\bar{l} &= (1+\overline{g})\,\overline{M/P} \\
\overline{G} &= \overline{g}\overline{M/P}
\end{aligned}$$