

Clase 11

Ciclos Economicos Reales

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1 Real business cycles

Real Business Cycles

- Business cycles were described earlier
- When we take data and remove the HP filter
- What remains we call a business cycle
 - recall that the cycle depends, in part, on the lambda we use in the filter
- Cycles are regular, cyclical movements and co-movements in variables
 - output, consumption, investment, prices, wages, interest rates
 - consumption tends to be smoother than output
 - investment tends to have a larger variance than output

Random shocks

- We cannot include everything important in our model
- Some of what is important for economic outcomes are not under our control
- Some of it we can't explain
- We put all the stuff that our model doesn't explain in "random variables"
 - the values of random variables are determined by "nature"
 - we take them as given
 - we don't know what the future values will be

- but we do know the distribution of the possible future values
- we include these future values in our models as expectations
- Path of the economy can be determined (partially) by expectations

Probabilities

- Random variables can take on a set of values
- This set could be
 - discrete (sella y cara)
 - continuous (between 0 and 300mm of rainfall in a day)
- We assign probabilities to the chances that some value will occur
- These probabilities can be objective or subjective
 - Depends on the information available
- Objective probabilities come from historical experience
 - How often did sella come up tossing a coin?
 - How often did it rain between 1 and 3 mm in 24 hours?
- Subjective probabilities depend less clearly on historical experience
 - my subjective probabilities can be different from yours
 - my information set is different

Probabilities

- Characteristics of probability distributions
 - If every possibility is included, some of probabilities is one (1)
 - probabilities are zero or positive
 - for continuous variables, we have a probability density function
- Examples of continuous probabilities

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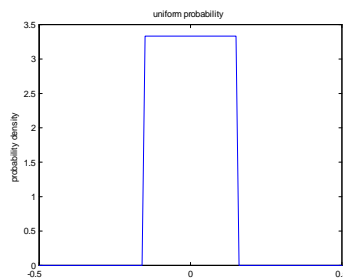
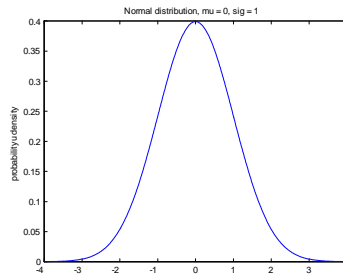
Normal

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Uniform

Real Real business cycle models

- Based on Solow type model



- include expectations
- include optimization of consumption, labor decisions, investment decisions, firms
- shocks occur to
 - technology
 - money
 - preferences
 - information
- people have limited information about the world and its future
- Expectations can reflect that
- We will simplify much of these aspects in our model
 - course in Masters program does the models more completely

A simple real business cycle model

- Consumers live two periods
- Have labor (1 unit) that they supply to the market when young

- receive market wage for their labor
- They do not work when old
- They can buy capital when young and hold till old
- When old, they earn rents on their capital and then sell capital that remains to new young
- Each period there is a shock to technology
 - changes in weather
 - political shocks that are not modelled
 - changes in world demand for goods

The consumer's decision

- Consumers have the utility function

$$u_t^h = \ln c_t^h(t) + \beta \ln c_t^h(t+1)$$

- $c_t^h(s)$ = consumption in period s of person h born in period t
 - person only lives two periods
 - only interesting consumptions are in period $s = t$ and $t + 1$
- budget constraint when young is

$$w_t = c_t^h(t) + k^h(t+1)$$

- budget constraint when old is

$$c_t^h(t+1) = r_{t+1}k^h(t+1) + (1 - \delta)k^h(t+1)$$

Writing out consumer's problem

- Rewrite first budget constraints as

$$c_t^h(t) = w_t - k^h(t+1)$$

and keep second budget constraint as

$$c_t^h(t+1) = r_{t+1}k^h(t+1) + (1 - \delta)k^h(t+1)$$

- the wages and rentals are given
- Choice variable for the consumer is $k^h(t+1)$
 - choosing $k^h(t+1)$ determines $c_t^h(t)$ and $c_t^h(t+1)$

- utility can be written as

$$u_t^h = \ln(w_t - k^h(t+1)) + \beta \ln(r_{t+1}k^h(t+1) + (1-\delta)k^h(t+1))$$

Solving consumer's problem

- Take derivative of utility function

$$u_t^h = \ln(w_t - k^h(t+1)) + \beta \ln(r_{t+1}k^h(t+1) + (1-\delta)k^h(t+1))$$

with respect to capital

- First order condition is

$$\begin{aligned} \frac{\partial u_t^h}{\partial k^h(t+1)} &= 0 = -\frac{1}{w_t - k^h(t+1)} \\ &\quad + \frac{\beta}{[r_{t+1} + (1-\delta)]k^h(t+1)}(r_{t+1} + (1-\delta)) \end{aligned}$$

- simplifying get

$$\frac{1}{w_t - k^h(t+1)} = \frac{\beta}{k^h(t+1)}$$

- or

$$k^h(t+1) = \beta(w_t - k^h(t+1))$$

Solving consumer's problem

- Result is

$$k^h(t+1) = \frac{\beta}{1+\beta}w_t$$

- Consumers save a fraction $\beta/(1+\beta)$ of their income when young
- In this simple economy: expected future rentals do not enter the decision
- This is a result of the log utility function
 - doesn't hold for other types of utility functions
 - but makes our life easy
 - income and substitution effects exactly cancel out with log utility functions

- a reasonable value for β is .98 or so (the book uses $\beta = 1$)

. Determining wages

- Assume the aggregate production function in period t is

$$Y_t = A_t K(t)^\theta L(t)^{1-\theta}$$

- Wages equal the marginal product of labor, so

$$w_t = \frac{\partial w_t}{\partial L(t)} = (1 - \theta) A_t K(t)^\theta L(t)^{-\theta}$$

- To keep things simple, assume that there is only one person born in each period
- An equilibrium condition, which we can apply now, is that $L(t) = 1$
- That gives that wages are equal to

$$w_t = (1 - \theta) A_t K(t)^\theta$$

.Capital determination

- Since there is only one person born in each period
- Another equilibrium condition is that

$$K(t+1) = k^h(t+1)$$

- Putting this and the definition of wages into the first order condition for the young person,

$$k^h(t+1) = \frac{\beta}{1 + \beta} w_t$$

we get

$$K(t+1) = \frac{\beta}{1 + \beta} (1 - \theta) A_t K(t)^\theta$$

- This is the first order difference equation for the capital stock in this economy

.Consumption and investment

- The old sell to the young the $(1 - \delta) K(t)$ of their capital that did not wear out
- Investment (the new capital) of the economy in period t is

$$I_t = K(t+1) - (1 - \delta) K(t)$$

- Aggregate consumption is equal to the consumption of the young and the old

$$C_t = c_t^h(t) + c_{t-1}^h(t)$$

- The aggregate resource constraint in period t is

$$(1 - \delta) K(t) + Y_t = K(t+1) + C_t$$

- Aggregate consumption can be found from

$$C_t = Y_t + (1 - \delta) K(t) - K(t + 1)$$

How can we do this model without much math?

- The utility function is in logs
 - this makes the income and substitution effects cancel out
 - that means that the amount that a young person want to save is a constant fraction of income
 - like in the Solow model
 - Savings rate is only a function of β

$$s = \frac{\beta}{1 + \beta} (1 - \theta)$$

- Income to the young is only their wages
- We assume that the labor supply per person is constant (and always = 1, like in the Solow model)
- A difference from the Solow model: savings of the young \neq investment
- Here savings of the young = next period's capital stock
- Investment is savings of the young minus $(1 - \delta)K_t$
- The young buy the remaining capital of the old = $(1 - \delta)K_t$

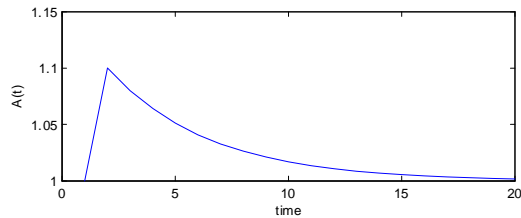
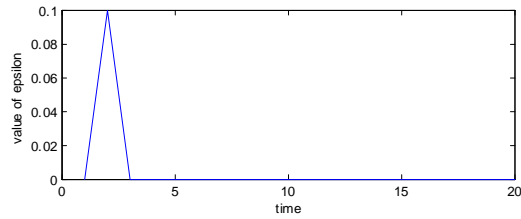
Impulse response

- How does the economy behave if
 - it starts in a stationary state
 - it gets a one time shock to technology
- The stochastic process for technology

$$A_{t+1} = 1 - \gamma + \gamma A_t + \varepsilon_t$$

- ε_t is the shock (the impulse)
- An impulse response function shows the path of the variables of the economy after a one time shock to ε_t

Impulse response



- Notice that if $\varepsilon_t = 0$, always, then $A_t = 1$, in a stationary state

$$A = 1 - \gamma + \gamma A$$

- gives

$$A - \gamma A = 1 - \gamma$$

or

$$A = 1$$

Impulse response

- Start with $A_0 = 1$, $\varepsilon_t = 0$ for all t except $t = 2$, here we set the shock $\varepsilon_2 = .1$
- ε_t follows the path
- if $\gamma = .8$, the A_t follows the path

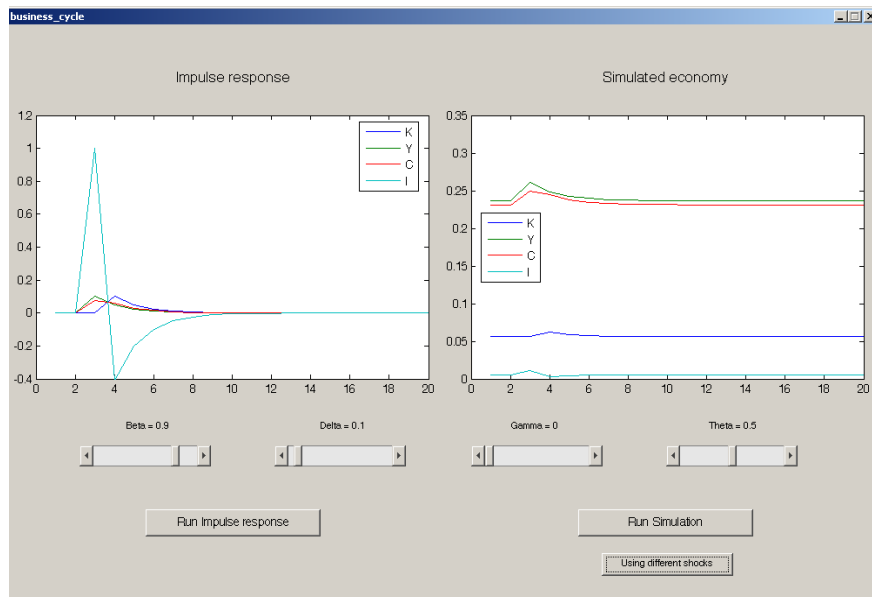
Impulse response

- Start with $K(0) = \bar{K}$ the stationary state value
- Given the A_t path shown above, use

$$K(t+1) = \frac{\beta}{1+\beta} (1-\theta) A_t K(t)^\theta$$

to calculate the sequence for $K(t)$

- We usually show responses as the percentage change around the stationary state



- The response for capital is

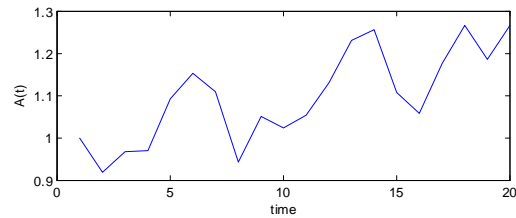
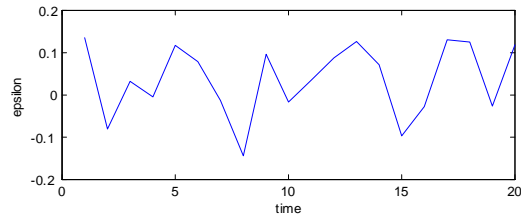
$$response\ K = \frac{K(t+1) - K(t)}{\bar{K}}$$

- Given the time path for K , we calculate the time paths for $Y(t)$, $C(t)$, $I(t)$
- We use these time paths to calculate their responses as percentage change around their stationary states

Impulse response

Simulations

- In simulations, we normally begin with $K(0) = \bar{K}$, in a stationary state
- We find a path of ε_t generated by a random (psuedo-random) number generator in the computer
- The random numbers come from a chosen distribution
 - distribution can be uniform
 - distribution can be normal
 - * one needs to be careful with normal distributions
 - * they can produce (not very often) very large negative numbers as the shock



- * in that case, technology can become negative (too strange to think about)
- * need to keep shocks from becoming too small

Simulations

- Example of an ε_t path with uniform shocks

Simulations

- The time path for technology is

$$A_{t+1} = 1 - \gamma + \gamma A_t + \varepsilon_t$$

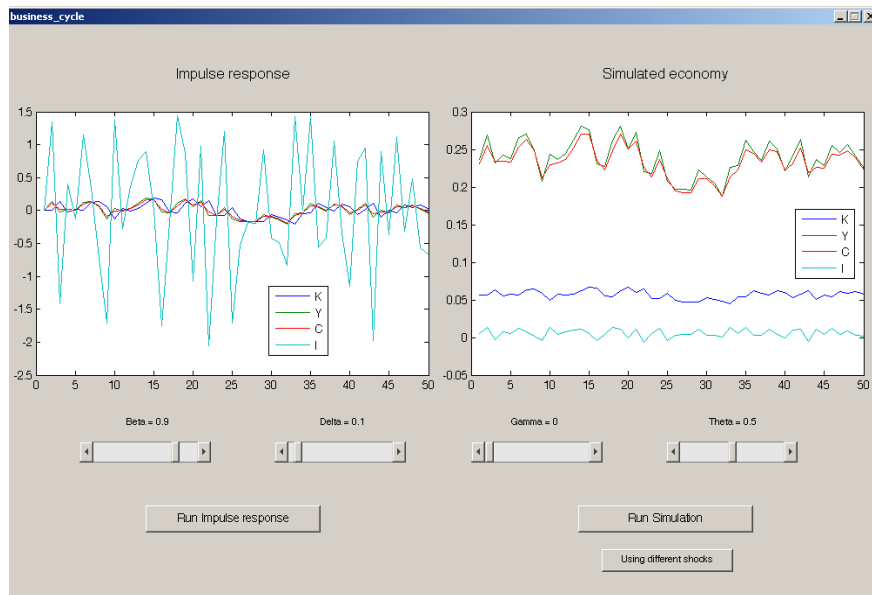
- Beginning with $A_0 = 1$
- In the example we use $\gamma = .8$
- Get time path for technology of

Simulations

- Beginning with $K(0) = \bar{K}$, use

$$K(t+1) = \frac{\beta}{1+\beta} (1-\theta) A_t K(t)^\theta$$

to find the time path for the capital stock



- Use (recall that $L(t) = 1$)

$$I_t = K(t+1) - (1 - \delta)K(t)$$

$$Y_t = A_t K(t)^\theta$$

$$C_t = Y_t + (1 - \delta)K(t) - K(t+1)$$

to find the time paths for the other variables

Example of a simulation

Effect of changing delta (delta = .1)

Effect of changing delta (delta = .309)

Effect of changing gamma (gamma = 0)

Effect of changing gamma (gamma = .198)

Effect of changing gamma (gamma = .396)

Simulation (gamma = 0)

Simulation (gamma = .198)

Simulation (gamma = .396)

Simulation (gamma = .99)

