Economics 102 | Matthias Doepke
Macroeconomic Theory | Fall 2000

Problem Set 4

This problem set will be graded. Questions 1 to 4 can be done in groups of up to four people. You need to use a spreadsheet program like Excel, and you need to pick a country. You need to hand in only one answer for the group. The other questions are to be done individually.

Growth Accounting

In the neoclassical growth model, economic growth is explained by increases in the factors of production, and by technological progress. In the following questions, you will determine the relative contributions of these factors to growth in the country of your choice.

To see how this can be done, consider the neoclassical production function

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha}.$$

Y_t is GDP, A_t a productivity parameter, L_t the number of workers, K_t the aggregate capital stock, and α determines the relative importance of capital and labor in production. We are going to be concerned with explaining growth in GDP per worker. Otherwise, we would have to get into issues like population growth, labor force participation rates and so on, which is complicated and of no particular interest to us at this point. To transform the production function into a per worker production function, we divide by the number of workers L_t on both sides:

$$\frac{Y_t}{L_t} = \frac{A_t L_t^{\alpha} K_t^{1-\alpha}}{L_t} = \frac{A_t L_t^{\alpha} K_t^{1-\alpha}}{L_t^{\alpha} L_t^{1-\alpha}} = A_t \left(\frac{K_t}{L_t}\right)^{1-\alpha}.$$

If we use lower case letters to denote per worker values ($y_t = Y_t/L_t$, $k_t = K_t/L_t$), we can write this as:

$$y_t = A_t k_t^{1-\alpha} \tag{1}$$

Taking (natural) logarithms, this becomes:

$$\ln y_t = \ln A_t + (1-\alpha)\ln k_t$$

Since growth rates can be computed as log-differences, by subtracting this from the equation for the following year we get a formula for growth in GDP per worker:

$$\ln y_{t+1} - \ln y_t = \ln A_{t+1} - \ln A_t + (1-\alpha)(\ln k_{t+1} - \ln k_t)$$

The GDP growth rate is therefore equal to growth in productivity plus $1-\alpha$ times growth in capital per worker. The following questions ask you to compute the size of these factors for the country of your choice. You can get real GDP per worker and capital per worker from the Penn World Tables that you used already for the problem set 1 (there is a link from the course home page). Use as many years as are available in the data set.

Question 1:
Before you can do any computations, you need to know what $1-\alpha$ is. How could you determine $1-\alpha$, and why?

For your computations, assume that $1-\alpha = .4$. This is (roughly) the correct value for the United States, and we assume that all countries use the same production function. In fact, in most countries measures for $1-\alpha$ are close to .4.

Question 2:
Use (1) to compute the productivity factors A_t for your country. Provide a plot of GDP per worker, capital per worker, and productivity for your country.

Question 3:
By using log-differences, compute the GDP growth rate, productivity growth, and growth in capital per worker for each year in your sample. Also compute the average growth rate for these three variables.

Question 4:
Which percentage of average growth per worker is explained by growth in capital, and which percentage by productivity growth? For the period from 1965 to 1992, the average growth rate of output per worker was 2.7% in the United States, and productivity growth averaged 2.3%. How do these numbers compare to your country? Does the neoclassical growth model (i.e., the Solow model) offer an explanation of the performance of your country relative to the United States? If not, what do you think explains the differences?

The Solow Growth Model

Question 5:
Consider an economy which has the production technology:

$$Y_t = A_t L_t^{\alpha} K_t^{1-\alpha}.$$

Assume that there is a competitive firm that hires workers and capital for wage W_t and interest R_t. Formulate the profit maximization problem of the firm and find equilibrium wage and interest. Verify that labor share ($W_t L_t / Y_t$) and capital share ($R_t K_t / Y_t$) are constant in this economy.

Question 6:
Consider an economy in which the production function for output per worker is:

$$y_t = k_t^{1-\alpha},$$

where k_t is capital per worker. Capital depreciates at rate δ, fraction s of output per worker is invested in new capital, and fraction (1-s) is consumed. The law of motion for capital per worker (assuming a constant number of workers) is then:

$$k_{t+1} = s k_t^{1-\alpha} + (1-\delta) k_t,$$

and consumption is given by:

$$c_t = (1-s) k_t^{1-\alpha}.$$

Find the steady-state level of capital, output, and consumption per worker in this economy (as a function of s, α, and δ).

Question 7:
Assume that you would get to choose the savings rate s. What is the highest level of output that you could achieve in steady state? Which savings rate would implement this steady state? Which is the highest level of steady state consumption that can be achieved? Which savings rate implements this steady state?

Population Dynamics

Consider an economy where generation t consumers have preferences

$$u(c_t, n_t) = \ln(c_t) + \ln(n_t)$$

over consumption c_t and the number of children n_t. It takes 2 units of the consumption good to feed a child, therefore the budget constraint is $c_t + 2n_t = w_t$, where w_t is the wage in period t. The production function is

$$Y_t = 16\sqrt{L_t},$$

where L_t is the population size (measured in millions). Labor gets paid its marginal product. The population grows according to the number of children per person, $L_{t+1} = n_t L_t$.

Question 8:
Find the law of motion for population in this economy. That is, maximize the utility of the consumer subject to the budget constraint to find the optimal number of children n_t as a function of w_t, then find the marginal product of labor to determine the wage w_t as a function of L_t, and plug your solutions into the law of motion for population. What is the steady-state level of population? Remember that the steady-state level of population $\bar{L}$ has to satisfy the equation $\bar{L} = n\bar{L}$. What is the wage rate in the steady state?

Question 9:
Assume that the economy is at the steady-level of population when an infectious disease suddenly kills 50% of the population. What is the growth rate of population in the first period after the epidemic?

Question 10:
Now assume that the production function is:

$$Y_t = 16L_t.$$

What is the law of motion for population with this production function? Is there still a steady-state level of population? Explain.