Economics 102 | Matthias Doepke
Macroeconomic Theory | Fall 2000

Problem Set 3

Robinson Crusoe

In this problem you are going to analyze the simplest model world one can imagine: The island of Robinson Crusoe. The aim is to find out what is happening on this island in terms of economics; that is, we want to know what is being produced and consumed by Robinson.

To get started with the analysis, we need a list of things Rob cares about and a description exactly how he feels about them. We need an exact description because we want to predict exactly what is going to happen on the island, and Rob is the one who makes things happen there.

Let us assume that the only things Rob cares about are coconuts and his time. We will denote coconuts by c, and since Rob can consume only positive amounts, he is restricted by the constraint $c \geq 0$. Also, Rob has one unit of time at his command. This time can be spend either as labor l or as leisure $1 - l$, and naturally Rob is restricted by $0 \leq l \leq 1$. Having completed the list, we now have to specify how Rob feels about different pairs of (c, l). The standard construct for this is a utility function. The utility function $u(c, l)$ tells us how much utility (how many "utils") Robinson gets out of consuming (c, l). If some pair (c, l) gives more utils than some other pair $(\hat{c}, \hat{l})$, Rob prefers (c, l) over $(\hat{c}, \hat{l})$.

Finally, we have to specify the technology, that is, how many coconuts Rob can get for any amount of work. This information is given by the production function $f(l)$. Since Rob does all the production himself, there are no formal markets in this economy. Therefore preferences and technology are all we need to describe the economy. Let us now see what is going in this model world.

We will consider two different islands, distinguished by different "types" of Robinson. The first one (Rob I) is described by the utility function

$$u(c, l) = c - l, \tag{1}$$

whereas Rob II has preferences

$$u(c, l) = \ln(c) + \ln(1 - l). \tag{2}$$

Question 1:
For a given utility function, if we fix the value of u and solve for c as a function of l, we get what is called an indifference curve.

For both realizations of Rob, plot indifference curves corresponding to $u = 1$ and $u = 2$ in the (c, l) plane.

Question 2:
Now let us assume that the production function on both islands is

$$f(l) = Al^{\alpha}, \tag{3}$$

where $A = 1$ and $\alpha = .5$. This means that the maximum number of coconuts Rob can consume is $c = Al^{\alpha}$. Given this technology, which (c, l) pair will be chosen by Rob I? Which will be chosen by Rob II? Illustrate your results in a diagram showing the production function and indifference curves.

Question 3:
What are the general solutions for arbitrary values of A and α? For what values of these parameters do the solutions make economic sense?

Crusoe and Storage

The following questions extend the Robinson Crusoe model that you analyzed so far to an intertemporal setting. Crusoe lives for two days now, and his utility function is

$$u(c, l) = \ln(c_1) + \ln(1 - l_1) + \beta[\ln(c_2) + \ln(1 - l_2)]. \tag{4}$$

For simplicity, assume $\beta = 1$. In the first period, Robinson can employ the technology

$$f_1(l_1) = 2l_1 \tag{5}$$

for transforming labor into coconuts. Since coconuts do not grow quickly and monkeys eat some of them, it is harder to produce coconuts in the second period. The production function is

$$f_2(l_2) = l_2. \tag{6}$$

However, Crusoe has the option of storing coconuts. If he stores s coconuts in period one, he gets Rs coconuts in period two, where R is a number between zero and one. $R = 1$ means that no coconuts get lost in storage. If some nuts get spoiled or stolen by the monkeys over night, we have $R < 1$.

Question 4:
Write down Robinson's resource constraints for day 1 and 2.

Question 5:
Find Robinson's consumption and labor input in both periods as a function of R. How many coconuts does Robinson store?

Question 6:
What is the effect of an increase in R on storage, consumption, and labor? Interpret your results in terms of income and substitution effects.