A Model of Inflation and Growth

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July 4, 2008

1 Introduction

Substantial empirical work has found a mixed relationship between inflation and economic growth. In general, the results have been that for countries with relatively low inflation rates, the long term correlation between inflation and growth appears to be positive. For countries with higher inflation rates, the correlation is negative. These results have been found by McCandless and Weber [11] and Barro [2], for example. Boyd and Champ [3] is a quite comprehensive review of the literature on money and inflation and contains studies using more recent data that support the above standard results.

A model for inflation and growth needs to be able to explain the data so it needs to be able to increase growth under some condition and to reduce it under others. While the data implies that very low inflations (and deflations) imply slower growth, positive inflations up to some level increase growth and higher levels of inflation produce slower growth. The model here is able to replicate this result by assuming that high and low growth rates have different causes. In low or negative inflation environments, the inflation comes from money issue or withdrawal by the central bank to or from the financial system: that is, through monetary policy. High inflations are normally associated with money issue being used by the fiscal authorities to meet government expenses or to make direct transfers to households in the form of poverty assistance or social security payments.

The context is that of an endogenous growth model with cash in advance for consumption purchases and a system of financial intermediaries that take cash deposits from the households and make cash loans to the firms to finance working capital that the firms use to pay their labor bill. In this economy, money can enter via transfers to or withdrawals from the financial intermediaries, which is what I call monetary policy, or by transfers directly to the households, which is what I think of as a kind of seigniorage policy. For an economy where only monetary policy is being used, interest rates on lending to the firms have a one to one correspondence with money growth rates, so that every money growth rate rule has an equivalent interest rate rule. Cash in advance models normally have problems with getting velocity to increase with higher inflation rates. This can be done, at least over a range of inflation rates, by imposing a shopping cost for spending period t's wage income in period t, see McCandless [10] for details. When these shopping costs are used in the model, monetary policy with ever higher money issue no longer results in ever higher growth rates. Shopping costs have been left out of the current model to keep it simple.

Previous models of endogenous growth and inflation, such as Roubini and Sala-i-Martin [13] and Jones and Manuelli [7] have studied the tax side of inflation, where more inflation reduces output growth. Neither of these papers contain well articulated financial systems and neither is able to explain how low levels of inflation might increase the growth rate of an economy.

2 The model

The model is a cash in advance economy with four components: households, firms, a competitive sector of financial intermediaries (banks), and a central bank. The households are quite standard except that in each period they deposits some of the money they carried over from the previous period in the financial intermediaries. The firms have a version of an AK production function where the economy's total capital enters as an externality in each firm's production function. In addition, the firms need to pay wages before they sell their goods, so they need to borrow from the financial intermediaries to finance their wage bill. The financial intermediaries borrow cash from the households and lend to the firms. The fact that the financial intermediaries are competitive is important and implies that the economy will behave differently from models such as Carlstrom and Fuerst [4], for example.

The central bank causes the money supply to grow by a constant rate each period. The exercises I do here is to see how different rates of money issue affect the rate of growth of output. This money issue can enter the economy one of two ways, as a direct transfer to the households, as in Cooley and Hansen [5], or as a direct transfer to the financial intermediaries, as in McCandless [10] or [9], Chapter 12. This model has properties similar to that of Cooley and Quadrini [6] but is simpler, so that the effects of money issue are not clouded by, as in their case, job search.

2.1 Households

Households maximize the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + A \ln \left(1 - h_t^i\right)\right],$$

subject to the cash in advance constraint,

$$c_t^i = \frac{m_{t-1}^i}{P_t} + \chi(g_t - 1)\frac{M_{t-1}}{P_t} - \frac{n_t^i}{P_t},$$

and the flow budget constraint (with the cash in advance constraint removed),

$$\frac{m_t^i}{P_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1-\delta) k_t^i + r_t^n \frac{n_t^i}{P_t}.$$

In these equations, c_t^i is time t consumption of household i, h_t^i are the hours it worked, m_{t-1}^i the money it carried over from period $t-1$, k_{t+1}^i is its holdings of capital at the end of period t, and n_t^i are its period t nominal deposits in the financial intermediaries. The economy wide variables are prices, P_t, wages w_t, rental on capital, r_t, interest rate on deposits, r_t^n, the growth rate of money, g_t, the money stock from the last period, M_{t-1}, and the fraction of the new money that goes as lump sum transfers to households, $\chi = \{0, 1\}$.

The first order conditions that come from the household's maximization problem are (after aggregating over the unit mass of identical households)

$$\begin{aligned}
\frac{A}{w_t\,(1-H_t)} &= \frac{1}{r_t^n C_t}, \\
\frac{w_{t+1}\,(1-H_{t+1})}{w_t\,(1-H_t)} &= \beta\left(r_{t+1} + (1-\delta)\right), \\
\frac{A}{w_t\,(1-H_t)} &= \beta \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}.
\end{aligned}$$

In addition, the aggregate versions of the budget constraint and cash in advance conditions are

$$\frac{M_t}{P_t} + K_{t+1} = w_t H_t + r_t K_t + (1-\delta) K_t + r_t^n \frac{N_t}{P_t}$$

and

$$C_t = \frac{M_{t-1}}{P_t} + \chi (g_t - 1) \frac{M_{t-1}}{P_t} - \frac{N_t}{P_t}.$$

2.2 Firms

There are a continuum of identical firms where each firm j has the production function

$$y_t^j = \lambda_t \left(k_t^j\right)^{\theta} \left(h_t^j\right)^{1-\theta} K_t^{1-\theta},$$

where y_t^j is the output of the one good produced by firm j in period t and λ_t is the country wide technology level, which we will keep constant in this paper, so $\lambda_t = \lambda$. There is an externality effect on each firm's output that comes from the aggregate capital stock, much as in Romer [12] or Lucas [8] or Barro [1].

Firms maximize their profits

$$\pi_t^j = y_t^j - r_t^f w_t h_t^j - r_t k_t^j$$

where r_t^f is the gross interest rate that the firms pay to the financial intermediaries to borrow funds to cover their wage bill. I assume that the wage bill must be paid before the goods are sold, so firms must borrow to finance this cost. First order conditions that come from maximizing profits, are

$$r_t^f w_t = (1-\theta)\,\lambda \left(k_t^j\right)^{\theta} \left(h_t^j\right)^{-\theta} K_t^{1-\theta}$$

and

$$r_t = \theta\lambda \left(k_t^j\right)^{\theta-1} \left(h_t^j\right)^{1-\theta} K_t^{1-\theta}.$$

Aggregating over a unit mass of identical firms, the factor market conditions are

$$r_t^f w_t = (1-\theta)\,\lambda H_t^{-\theta} K_t$$

and

$$r_t = \theta\lambda H_t^{1-\theta}.$$

The aggregate production function is

$$Y_t = \lambda H_t^{1-\theta} K_t$$

2.3 Financial intermediaries

The banks or financial intermediaries take deposits in cash from the households at the beginning of a period and lend it to the firms as working capital for the wage bill. Since the firms are paid for their goods at the end of the period, they also pay off their loans then. Banks are competitive. Banks can also receive direct transfers of money from the central bank. The aggregate budget constraint of the banks is

$$w_t H_t = (1-\chi)\,(g_t - 1)\,\frac{M_{t-1}}{P_t} - \frac{N_t}{P_t}$$

where $\chi = 0$ is the case where all money issue goes to the financial intermediaries. All of the income of the banks gets paid out to the depositors so the zero profit condition for the financial intermediaries is

$$r_t^n \frac{N_t}{P_t} = r_t^f w_t H_t.$$

This final condition is different from that frequently used in the limited participation model of Carlstrom and Fuerst [4], for example. In their models, $r_t^n = r_t^f$ and income to the financial system that results from the transfers from the central bank goes to the households as lump sum dividends. Making the financial intermediaries competitive is very important in terms of the results of monetary policy.

Finally, the central bank's growth rule for money is

$$M_t = \overline{g} M_{t-1}$$

and we study only cases where $\overline{g}$ is a constant.

3 Balanced growth rate paths

We limit the study here to time paths of the economy where growth is constant and balanced. There is no uncertainty in the economy, money growth rates are constant and technology is a constant. The object is to compare the growth rates of output that result from different growth rates of the money stock and for the two policies where money is injected either directly to households or to the financial intermediaries.

It is fairly easy to show that there exists an equilibrium for this model where the growth rate of output, γ^y, is constant and where the growth rates for consumption, wages, the real money stock, the real bank deposits, and capital are

$$\gamma^c = \gamma^w = \gamma^{\frac{M}{P}} = \gamma^{\frac{N}{P}} = \gamma^k = \gamma^y,$$

and where r, r^n, r^f, π, and H are constants. In that balanced growth path, one has the model

$$\begin{aligned}
\gamma^c &= \beta\left(r+1-\delta\right), \\
\gamma^c &= \frac{w_t\left(1-H\right)\beta}{\pi A C_t}, \\
\gamma^c &= \frac{\beta r^n}{\pi}, \\
r &= \theta\lambda H^{1-\theta}, \\
C_t &= \left[1+\chi(\overline{g}-1)\right]\frac{M_t}{\overline{g}P_t} - \frac{N_t}{P_t}, \\
\frac{M_t}{P_t} + K_{t+1} &= w_t H + rK_t + (1-\delta)K_t + r^n\frac{N_t}{P_t}, \\
r^f w_t &= \left(1-\theta\right)\lambda H^{-\theta}K_t, \\
w_t H &= \left(1-\chi\right)\left(\overline{g}-1\right)\frac{M_t}{\overline{g}P_t} - \frac{N_t}{P_t}, \\
r^n\frac{N_t}{P_t} &= r^f w_t H, \\
\gamma^c &= \frac{\overline{g}}{\pi} \\
\gamma^c &= \gamma^k \equiv \frac{K_{t+1}}{K_t}
\end{aligned}$$

and, given the policy variables $\overline{g}$ and χ and an initial K_0, solves for the variables $\left\{\gamma^c, r, w_0, H, \pi, C_0, r^n, \frac{M_0}{P_0}, \frac{N_0}{P_0}, K_1, r^f\right\}$. From these, the time path for the economy can be calculated directly.

The model is too complicated to solve analytically so it is solved numerically for an example economy. In this economy, $A = 1.72$, $\beta = .96$, $\delta = .024$, $\lambda = .5$ and $\theta = .4$.[1] The economy begins with $K_0 = 10$ and is run for 20 periods. Figure 1 shows the time paths for capital in five cases: one for when the money

[1] A and δ come from Hansen. $\theta = .4$ gives .4 as the fraction of output that goes to capital.

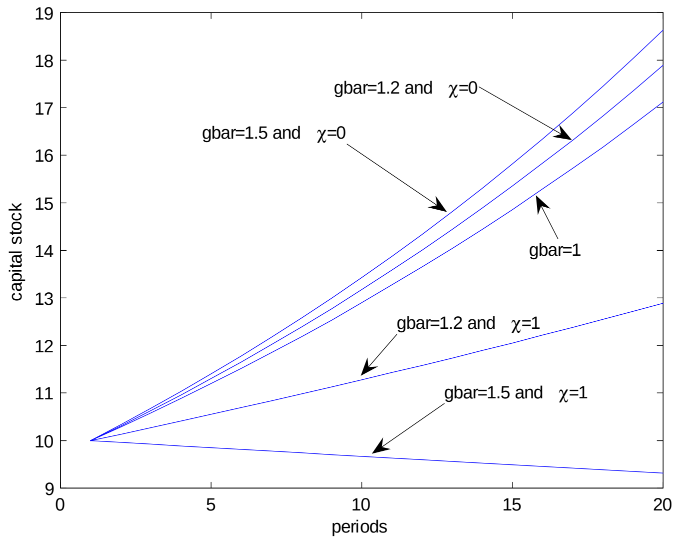

Figure 1: Time paths of capital stock

stock is held constant, $\overline{g} = 1$, two for when the money stock grows at the gross rate $\overline{g} = 1.2$ and where $\chi = 0$ or $\chi = 1$ and two for when the money stock grows at the gross rate $\overline{g} = 1.5$ and where $\chi = 0$ or $\chi = 1$. Recall that $\chi = 0$ is the case where money issue by the central bank goes to the financial intermediaries and $\chi = 1$ is where it goes directly to the households.

Table 1 shows the values for the constant growth rates and for r, r^n, r^f, π, and H in the three example economies shown in Figure 1. The table confirms that the highest growth rates are achieved with money transfers to the financial intermediaries and the higher the money growth rate, the higher the growth rate of the economy. With the same rate of money growth but with transfers to the households, the results are just the opposite: lower growth rates occur with higher inflation rates, even reaching cases where the net economic growth is negative. The economies with money transfers to the financial system have higher labor supplies, lower interest rates on borrowing for financing working capital, higher rentals on capital and lower inflation rates than the economies with transfers to the households and with the same rates of growth of the money supply. In the economy with transfers to the households, the capital stock (and other growing variables) grows slower, rentals on capital are lower than the no money growth economy, labor supply is smaller, and these economies have the higher inflation rate for the same rate of money issue.

Lower values of β cause the impacts of inflation to be greater and easier to see in the graphs, for that reasons I chose $\beta = .96$. Given the other parameters, with $\lambda = .5$ the resultant growth rates are near the averages in the data.

	γ^k	r	r^n	r^f	π	H
$\overline{g} = 1$	1.0287	0.0956	1.0417	1.0417	0.9721	0.2920
$\overline{g} = 1.2$ $\chi = 0$	1.0311	0.0981	1.2500	0.8207	1.1638	0.3049
$\overline{g} = 1.5$ $\chi = 0$	1.0333	0.1004	1.5625	0.6238	1.4517	0.3169
$\overline{g} = 1.2$ $\chi = 1$	1.0134	0.0797	1.2500	1.2500	1.1841	0.2156
$\overline{g} = 1.5$ $\chi = 1$	0.9963	0.0618	1.5625	1.5625	1.5056	0.1413

Table 1: Results for three monetary policies

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