Solutions to the problems in Introduction to Dynamic Macroeconomic Theory

July 10, 2008

EXERCISES

1.1 $C(t) = \alpha\gamma N + (1-\alpha)\gamma N = \gamma N$ for all $t \geqslant 1$ therefore, it is feasible.

1.2 $C(t) = N(t)\alpha\gamma + N(t-1)n(1-\alpha)\gamma = N(t)\alpha\gamma + N(t)(1-\alpha)\gamma = \gamma N(t)$ for all $t \geqslant 1$ therefore, it is feasible.

1.3 $C(t) = 2 = Y(t)$ for all $t \geqslant 1$ therefore, it is feasible.

1.4 $C(t) = 1 = Y(t)$ for all $t \geqslant 1$ therefore, it is feasible.

1.5 If $C(t) < Y(t)$ for some $t \geqslant 1$, there is an alternative feasible allocation with more total consumption of some good and no less of any other good $C(t) = Y(t)$ for all $t \geqslant 1$.

1.6 A $[0.5\gamma, 0.5\gamma]$ allocation is Pareto superior to the $[0.6\gamma, 0.4\gamma]$ allocation, but is noncomparable to a $[0.4\gamma, 0.6\gamma]$ allocation.

1.7 If an allocation is Pareto optimal, then $C(t) = Y(t)$ for all $t \geqslant 1$.

1.8 The allocation determined by $\alpha = 0.5$ is Pareto superior, $c_0^h(1) = 0.5\gamma$; $u(0.5\gamma, 0.5\gamma) = 1.41\gamma^{1/2}$; $c_0^h(1) = 0.25\gamma$; $u(0.75\gamma, 0.25\gamma) = 1.36\gamma^{1/2}$ All members of all generations are better off.

1.9 The allocation determined by $\alpha = 0.1$ is Pareto superior, $c_0^h(1) = 3.6\gamma$; $u(0.1\gamma, 3.6\gamma) = 2.21\gamma^{1/2}$; $c_0^h(1) = 2\gamma$; $u(0.5\gamma, 2\gamma) = 2.12\gamma^{1/2}$.

1.10 The allocation $c_0^h(1) = 1/2$ for $h = 1, 2$; $c_t^h(t) = c_t^h(t+1) = 1/2$ for $h = 1, 2$ for all $t \geqslant 1$ is Pareto superior.

1.11 See page 26.

1.12 $u_t^h = c_t^h(t)c_t^h(t+1)$ $t \geqslant 1$, $h = 1, 2$; $Y(t) = 16$; $c_t^h = [6, 2]$ $h = 1, 2$ This allocation is efficient and the MRS are equal. But, the efficient allocation $c_t^h = [2, 6]$ is Pareto superior.

1.13 If $\alpha < 1/4$ all members of all generations $t \geqslant 1$ are worse off. If $\alpha > 1/4$ the old at period 1 are worse off.

1.14 $0 < \alpha \leqslant 0.5$.

1.15 $0 < \alpha \leqslant \dfrac{1}{1+n}$.

2.1 $r(t) = \dfrac{\frac{\partial u_t^h}{\partial c_t^h(t)}}{\frac{\partial u_t^h}{\partial c_t^h(t+1)}} = \dfrac{[c_t^h(t+1)]^{1/2}}{\beta[c_t^h(t)]^{1/2}}$

$= \dfrac{\{r(t)[\omega_t^h(t) - c_t^h(t)] + \omega_t^h(t+1)\}^{1/2}}{\beta[c_t^h(t)]^{1/2}}$

then, $c_t^h(t) = \dfrac{\omega_t^h(t)}{1+\beta^2 r(t)} + \dfrac{\omega_t^h(t+1)}{r(t)[1+\beta^2 r(t)]}$.

2.2 $s_t^h(r(t)) = \dfrac{\beta^2 r(t)\omega_t^h(t)}{1+\beta^2 r(t)} - \dfrac{\omega_t^h(t+1)}{r(t)[1+\beta^2 r(t)]}$.

2.3 a. $S_t(r(t)) = 100 - \dfrac{50}{r(t)}$

b. $S_t(r(t)) = 75 - \dfrac{50}{r(t)}$.

2.4 a. $r(t) = 1/2$ for all $t \geqslant 1$; $s_t^h(t) = 0$; $c_0^h(1) = 1$; $c_t^h = [2, 1]$.

b. $r(t) = 2$ for all $t \geqslant 1$; $s_t^h(t) = 0$; $c_0^h(1) = 2$; $c_t^h = [1, 2]$.

c. $r(t) = (1/2)^{1/2}$ for all $t \geqslant 1$; $s_t^h(t) = 0$; $c_0^h(1) = 1$; $c_t^h = [2, 1]$.

d. $r(t) = 2/3$ for all $t \geqslant 1$; $s_t^{even}(t) = 1/4$; $s_t^{odd}(t) = -1/4$; $c_0^{even}(1) = 1$; $c_t^{even} = [7/4, 7/6]$; $c_0^{odd}(1) = 1$; $c_t^{odd} = [5/4, 5/6]$.

e. $r(t) = 5/8$ for all $t \geqslant 1$; $s_t^h(t) = 1/5$; $c_0^h(1) = 1$; $c_t^h = [9/5, 9/8]$ $\quad h = 1, 2, ..., 60$; $s_t^h(t) = -3/10$; $c_0^h(1) = 1$; $c_t^h = [13/10, 13/16]$ $\quad h = 61, 62, ..., 100$.

f. $r(t) = 1 \quad t = 1, 3, ...$; $s_t^h(t) = 0$; $c_0^h(1) = 1$; $c_t^h = [1, 1]$; $r(t) = 1/2$ $t = 2, 4, ...$; $s_t^h(t) = 0$; $c_0^h(1) = 1$; $c_t^h = [2, 1]$.

2.5 2.4a. $c_t^h = [1, 2]$ is Pareto superior. All members of all generations $t \geqslant 1$ have the same utility level, but the old at period 1 are better off.

2.4c. $c_t^h = [1, 2]$ is Pareto superior.

2.4d. $c_t^{even} = [7/6, 7/4]$; $c_t^{odd} = [5/6, 5/4]$ is Pareto superior.

2.4e. $c_t^h = [9/8, 9/5]$ $\quad h = 1, 2, ..., 60$; $c_t^h = [13/16, 13/10]$ $\quad h = 61, 62, ..., 100$ is Pareto superior.

2.4f. $c_t^h = \begin{cases} [0.8, 1.25] & t = 1, 3, ... \\ [1.75, 1.2] & t = 2, 4, ... \end{cases}$ is Pareto superior.

3.1 a. $r(t) = 2$ for all $t \geqslant 1$; $s_t^h(t) = 0$; $c_0^h(1) = 2$; $c_t^h = [1, 2]$ is Pareto superior to that obtained in the absence of the scheme. All members of all generations $t \geqslant 1$ have the same utility level, but the old at period 1 are better off .

b. $r(t) = 4$ for all $t \geqslant 1$; $s_t^{even}(t) = 1/4$; $s_t^{odd}(t) = -1/4$; $c_0^{even}(1) = 2$; $c_t^{even} = [3/4, 3]$; $c_0^{odd}(1) = 2$; $c_t^{odd} = [1/4, 1]$ is noncomparable to that obtained in the absence of the scheme.

c. The maximum that can be given to every person when old is 2.

$r(t) = 3$ for all $t \geqslant 1$; $s_t^h(t) = 0$; $c_0^h(1) = 3$; $c_t^h = [1, 3]$ is Parteo superior to that obtained in the absence of the scheme.

d. Each person when young is taxed 1 unit and that is transfered to the old.

$r(t) = 3$ for all $1 < t < 10$; $s_t^h(t) = 0$; $c_0^h(1) = 3$; $c_t^h = [1, 3]$

$r(t) = 2$ for all $t > 10$; $s_t^h(t) = 0$; $c_t^h = [1, 2]$

It is Parteo superior to that obtained in the absence of the scheme.

3.2 $c_t^h(t) + \dfrac{c_t^h(t+1)}{(1-z_\gamma)r(t)} = g\left[\omega_t^h(t) + \dfrac{\omega_t^h(t+1)}{(1-z_\gamma)r(t)}\right]$ Equivalent : Changes in income and consumption taxes that left g constant did not change the equilibrium pattern of consumption.

3.3 a. $r(1) = 8/3$; $s_1^h(1) = 0$; $c_0^h(1) = 2.25$; $c_1^h = [3/4, 2]$.

b. $r(1) = 8/3$; $s_1^h(1) = 1/4$; $c_0^h(1) = 2.25$; $c_1^h = [3/4, 2]$.
c. $r(1) = 4$; $s_1^h(1) = 1/4$; $c_0^h(1) = 2.25$; $c_1^h = [3/4, 3]$.

3.4 $S_1(r(1)) = 100 - \dfrac{50}{r(1)} = 25$, $r(1) = 2/3$, $s_1^h(1) = 1/4$, $c_1^h = [7/4, 7/6]$; $S_2(r(2)) = 100 - \dfrac{50}{r(2)} = 16.67$, $r(2) = 0.6$, $s_2^h(2) = 0.167$, $c_2^h = [1.83, 1.1]$ is Pareto superior. It is not Pareto optimal, because all generations are worse off than if the government borrowed 50 units and rolled that borrowing over.

3.5 2.4b. It is not a feasible rolling over policy.

2.4d. $S_t(r(t)) = 75 - \dfrac{50}{r(t)} = 25$; $r(t) = 1$ for all $t \geqslant 1$; $s_t^{even}(t) = 1/2$; $s_t^{odd}(t) = 0$; $c_t^{even} = [1.5, 1.5]$; $c_t^{odd} = [1, 1]$.

3.6 $r(t) = 1.5$; $c_1^h = [4/3, 2]$.

3.7 $t_1^h(1) = 0.28125$; $t_1^h(2) = 0.3$.

4.1 $\delta = 1.2$; $b^h(0) = 0.17$; $\delta = 0.8$; $b^h(0) = 0$.

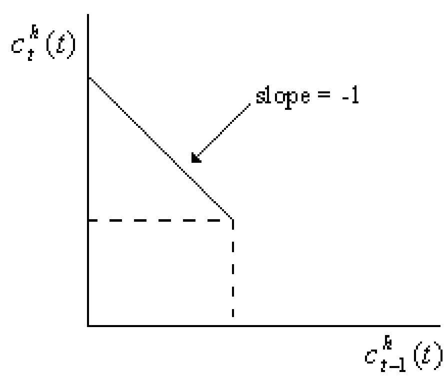
4.2 The preferred consumption point changes. a. $\uparrow c_0^h(1)$; $\downarrow c_1^h(1)$ b. $\downarrow c_0^h(1)$; $\uparrow c_1^h(1)$.

4.3 $u_1^h[c_1^h(1), c_1^h(2), u_0^h(c_0^h(1))]$; $c_0^h(1) = \omega_0^h(1) + b^h(1) - t_0^h(1)$; $c_1^h(1) = \omega_1^h(1) - b^h(1) - t_1^h(1)$; $c_1^h(2) = \omega_1^h(2)$. Each member h of generation 1 chooses a bequest, $b^h(1)$, that maximizes the utility function subject to the budget constraints.

4.4 $b^{odd}(0) = \dfrac{2r(1) - 1}{3r(1)}$; $b^{even}(0) = \dfrac{3r(1) - 1}{3r(1)}$.

4.5 $r(1) = 0.57$.

4.6



$b^h(t) = 0$

5.1 $l^h(t) = \omega_t^h(t) - c_t^h(t) - p_k(t)b_k^h(t)$; $c_t^h(t+1) = \omega_t^h(t+1) + r(t)[\omega_t^h(t) - c_t^h(t) - p_k(t)b_k^h(t)] + p_{k-1}^{h,e}(t+1)b_k^h(t)$; $r(t)c_t^h(t) + c_t^h(t+1) = r(t)\omega_t^h(t) + \omega_t^h(t+1) - b_k^h(t)[r(t)p_k(t) - p_{k-1}^{h,e}(t+1)]$.

5.2 If $r(t)p_k(t) > p_{k-1}^e(t+1)$ $b_k^h(t) = 0$ for all h of generation t. If $r(t)p_k(t) < p_{k-1}^e(t+1)$ $b_k^h(t) = \infty$ for all h of generation t. Neither of these cases can be an equilibrium.

5.3 $r(t) = 1.5$; $p_k(t) = 0.67$; $s_t^h(t) = 0.67$; $c_t^h = [1.33, 2]$.

5.4 $r(t) = 1$; $p_k(t) = 0.5$; $s_t^h(t) = 0.5$; $c_t^h = [1.5, 1.5]$.

5.5 $p_k(t) = \dfrac{-(p^e_{k-1}(t+1))^2 + [(p^e_{k-1}(t+1))^4 + 8(1+p^e_{k-1}(t+1))p^e_{k-1}(t+1)]^{1/2}}{2(1+p^e_{k-1}(t+1))}$.

5.6 $r(1) = 0.83$; $r(2) = 0.75$; $p_2(1) = 1.6$; $p_1(2) = 1.33$; $p_0(3) = 1$; $c^h_0(1) = 1.4$; $c^h_1 = [1.6, 1.33]$; $c^h_2 = [1.66, 1.25]$; $c^h_3 = [1.75, 1]$; $s^h_1(1) = 0.4$; $s^h_2(2) = 0.33$; $s^h_t(t) = 0$ for all $t \geqslant 3$.

5.7 $r(1) = 0.9$; $r(2) = 0.625$; $p_2(1) = 1.78$; $p_1(2) = 1.6$; $c^h_0(1) = 1.44$; $c^h_1 = [1.56, 1.4]$; $c^h_2 = [1.6, 1]$; $c^h_t = [2, 1]$ for all $t \geqslant 3$.; $s^h_1(1) = 0.44$; $s^h_2(2) = 0.4$. If the borrowing and taxing scheme shifts the tax burden to other generations, then the concept of Ricardian equivalence does not hold. Passing on taxes to other generations does change the consumption pattern and the gross interest rate that occurs in the economy.

5.8 $r(1) = 0.67$; $r(2) = 0.6$; $p_2(1) = 2.5$; $p_1(2) = 1.67$; $c^h_0(1) = 1.25$; $c^h_1 = [1.75, 1.17]$; $c^h_2 = [1.83, 1.1]$; $c^h_3 = [1.9, 1]$; $s^h_1(1) = 0.25$; $s^h_2(2) = 0.17$; $s^h_t(t) = 0$ for all $t \geqslant 3$.

5.9 a. Under perfect foresight hypothesis $p^e_{k-1}(t+1) = p_{k-1}(t+1)$

$$r(t+k-1) = \frac{p_0(t+k)}{p_1(t+k-1)} = \frac{1}{p_1(t+k-1)};\; p_1(t+k-1) = \frac{1}{r(t+k-1)}$$
$$r(t+k-2) = \frac{p_1(t+k-1)}{p_2(t+k-2)} = \frac{1}{p_2(t+k-2)r(t+k-1)};\; p_2(t+k-2)$$
$$= \frac{1}{r(t+k-2)r(t+k-1)} \;\ldots\ldots\; p_k(t) = \frac{1}{r(t)r(t+1).....r(t+k-2)r(t+k-1)};$$
$$r_k(t) = \left(\frac{1}{p_k(t)}\right)^{1/k} = (r(t)r(t+1).....r(t+k-2)r(t+k-1))^{1/k}$$

b. If perfect foresight is not assumed, $p^e_{k-1}(t+1) \neq p_{k-1}(t+1)$, and the above relationship need not hold.

5.10 $r(t+1) = 8.05$. This test imply that our perfect foresight version of the expectations hypothesis of the term structure of interest rates has failed.

5.11 $r(1) = 0.67$; $r(2) = 0.6$; $p_1(1) = 1.5$; $p_2(2) = 1.67$; $c^h_0(1) = 1.25$; $c^h_1 = [1.75, 1.17]$; $c^h_2 = [1.83, 1.1]$; $c^h_3 = [1.9, 1]$; $s^h_1(1) = 0.25$; $s^h_2(2) = 0.17$; $s^h_t(t) = 0$ for all $t \geqslant 3$.

5.12 Hint page 141.

6.1 $c^h_t(t) = \omega^h_t(t) - l^h(t) - p(t)a^h(t) + a^h(t)d(t)$; $c^h_t(t+1)$
$= \omega^h_t(t+1) + r(t)l^h(t) + a^h(t)p^{h,e}(t+1)$.

6.2 $l^h(t) = \omega^h_t(t) - c^h_t(t) - p(t)a^h(t)$; $c^h_t(t+1)$
$= \omega^h_t(t+1) + r(t)[\omega^h_t(t) - c^h_t(t) - p(t)a^h(t)] + a^h(t)d(t+1) + a^h(t)p^{h,e}(t+1)$;
$r(t)c^h_t(t) + c^h_t(t+1) = r(t)\omega^h_t(t) + \omega^h_t(t+1) - a^h(t)[r(t)p(t) - d(t+1) - p^{h,e}(t+1)]$.

6.3 If $r(t)p(t) > d(t+1) + p^e(t+1)$ $\quad a^h(t) = 0$ for all h of generation t. If $r(t)p(t) < d(t+1) + p^e(t+1)$ $\quad a^h(t) = \infty$ for all h of generation t. Neither of these cases can be an equilibrium.

6.4 a. $r(t) = 2.5$; $p(t) = 0.8$; $s^h_t(t) = 0.8$ for all $t \geqslant 1$.

b. $r(t) = 2$; $p(t) = 0.75$; $s^h_t(t) = 0.75$ for all $t \geqslant 1$.

c. $r(t) = 3$; $p(t) = 0.83$ if t is odd; $r(t) = 2$; $p(t) = 0.75$ if t is even.

d. $r(t) = 3.33$; $p(t) = 0.6$ for all $t \geqslant 1$.

e. $r(t) = 2.5$; $p(t) = 0.8$ if t is odd; $r(t) = 5$; $p(t) = 0.4$ if t is even.

6.5 1. $100 - \dfrac{50p(t)}{1+p(t+1)} = p(t)100$; 2. $75 - \dfrac{50p(t)}{1+p(t+1)} = p(t)100$;
3. $N(t)\left(1 - \dfrac{p(t)}{2+2p(t+1)}\right) = p(t)100$; $N(t) = (1.1)^{t-1}100$.

6.6 2. $p(t+1) = \dfrac{p(t)150 - 75}{75 - p(t)100}$; 3. $p(t+1) = \dfrac{p(t)(200+N(t)) - 2N(t)}{2N(t) - p(t)200}$; $N(t) = (1.1)^{t-1}100$.

6.7 $p(t) = 0.569$; $r(t) = 2.76$ for all $t \geqslant 1$.

6.8 $p(t) = 0.707$; $r(t) = 1.707$ for all $t \geqslant 1$.

6.9 a. $p(t) = p$; time t $100\left(1 - \dfrac{p}{2+2p}\right) = 100p$; time t +1 $110\left(1 - \dfrac{p}{2+2p}\right) = 100p$, this can not be an equilibrium. b. $p(t+1) = 1.1p(t)$; time 1 $100\left(1 - \dfrac{p(1)}{2+2.2p(1)}\right) = 100p(1)$; time 2 $110\left(1 - \dfrac{1.1p(1)}{2+2.42p(1)}\right) = 100p(1)1.1$, this can not be an equilibrium.

6.10 $p = \dfrac{-(1-2d) - ((1-2d)^2 + 16d)^{1/2}}{-4}$; as d goes to 0, p goes toward 0.5.

6.11 $c_t^h(t) + \dfrac{c_t^h(t+1)}{(1-z_\gamma)r(t)}$
$= \dfrac{(1-z_\gamma)}{(1+z_c)}\left[\omega_t^h(t) + \dfrac{\omega_t^h(t+1)}{(1-z_\gamma)r(t)} - \dfrac{a^h(t)}{(1-z_\gamma)r(t)}\left(r(t)p(t) - d(t+1) - p(t+1)\right)\right]$;
$c_t^h(t) + \dfrac{c_t^h(t+1)}{(1-z_\gamma)r(t)} = g\left[\omega_t^h(t) + \dfrac{\omega_t^h(t+1)}{(1-z_\gamma)r(t)}\right]$. All nonnegative pairs (z_c, z_γ) satisfying $\dfrac{(1-z_\gamma)}{(1+z_c)} = g$ for a given, constant g satisfying $0 < g < 1$ are equivalent.

6.12 $c_t^h(t) = \omega_t^h(t) - l^h(t) - p(t)a^h(t)$; $c_t^h(t+1)$
$= \omega_t^h(t+1) + r(t)l^h(t) + a^h(t)[d(t+1)(1-\tau)] + a^h(t)p(t+1)$;
$r(t) = \dfrac{d(t+1)(1-\tau) + p(t+1)}{p(t)}$.
This tax affects the return and the price of the land.

6.13 PA. $p^V(t) = 0.78$; $r^V(t) = 2.28$; $c_0^{hV}(1) = 2.78$; $c_t^{hV} = [1.22, 2.78]$; $u_t^{hV} = 3.39$. $p^W(t) = 0.425$; $r^W(t) = 3.35$; $c_0^{hW}(1) = 3.85$; $c_t^{hW} = [1.15, 3.85]$; $u_t^{hW} = 4.43$.

LF. $p(t) = 0.549$; $r(t) = 2.82$; $c_t^h = [1.18, 3.31]$; $u_t^h = 3.9$.

People from country W is better off under PA. People from country V is better off under LF.

6.14 $p(t) = 0.549$; $r(t) = 2.82$; $c_t^h = [1.18, 3.31]$ for $t \geqslant 2$; $p^V(1) = 0.756$; $r^V(1) = 2.05$; $c_0^{hV}(1) = 2.756$; $c_1^{hV} = [1.24, 2.55]$; $p^W(1) = 0.43$; $r^W(1) = 3.6$; $c_0^{hW}(1) = 3.86$; $c_1^{hW} = [1.14, 4.1]$.

7.1 $p(t) = \dfrac{3+2p(t+1)}{4+2p(t+1)}$ for t odd; $p(t) = \dfrac{1+2p(t+1)}{2+2p(t+1)}$ for t even.

7.2 $p(t) = \dfrac{4+5p(t+2)}{5+6p(t+2)}$.

7.3 $p(t) = \begin{cases} 0.39 \text{ for } t \text{ odd} \\ 0.74 \text{ for } t \text{ even} \end{cases}$ $r(t) = \begin{cases} 4.46 \text{ for } t \text{ odd} \\ 1.88 \text{ for } t \text{ even} \end{cases}$ $c^h_{odd} = [0.61, 2.73]$; $c^h_{even} = [1.27, 2.38]$.

7.4 If $d(t) = \begin{cases} 1/2 \text{ for } t \text{ odd} \\ 3/2 \text{ for } t \text{ even} \end{cases}$ $p(t) = \begin{cases} 0.8165 \text{ for } t \text{ odd} \\ 0.7247 \text{ for } t \text{ even} \end{cases}$

If $d(t) = 1/2$ for all t, $p(t) = 0.707$ for all t. If $d(t) = 3/2$ for all t, $p(t) = 0.823$ for all t.

7.5 $S(r(t)) = p(t)A$; $a_0 - \dfrac{a_1 p(t)}{d(t+1) + p(t)} = p(t)A$;

$$p(t) = \frac{-(a_0 - Ad(t+1) - a_1) - ((a_0 - Ad(t+1) - a_1)^2 + 4Aa_0 d(t+1))^{1/2}}{-2A}$$

7.6 $p(t) = 0.8$ for all t; $r(t) = 1$; $s^h_t(t) = 0.8$; $c^h_t = [1.2, 1.8]$; $u^h_t = -0.8$.

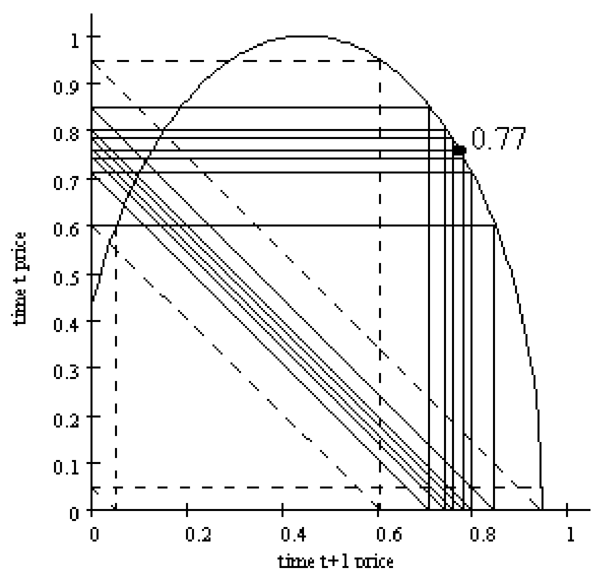
$$p(t) = \begin{cases} 0.3686 \text{ for } t \text{ odd} & r(t) = 2.62;\ s^h_t(t) = 0.37;\ c^h_t = [1.63, 1.97];\ u^h_t = -0.14 \\ 0.9648 \text{ for } t \text{ even} & r(t) = 0.38;\ s^h_t(t) = 0.96;\ c^h_t = [1.04, 1.36];\ u^h_t = -2.56 \end{cases}$$

$$p(t) = \begin{cases} 0.4784 \text{ for } t = 1, 4, ...;\ r(t) = 0.13;\ s^h_t(t) = 0.49;\ c^h_t = [1.51, 1.06];\ u^h_t = -3.77 \\ 0.0611 \text{ for } t = 2, 5, ...;\ r(t) = 16.35;\ s^h_t(t) = 0.06;\ c^h_t = [1.94, 1.99];\ u^h_t = -0.004 \\ 0.9991 \text{ for } t = 3, 6, ...;\ r(t) = 0.48;\ s^h_t(t) = 1;\ c^h_t = [1, 1.48];\ u^h_t = -2.08 \end{cases}$$

7.7 $p(t) = [5p(t+1) - 5(p(t+1))^2]^{1/2}$; $p(t) = \begin{cases} 0.264 \text{ for } t \text{ odd} \\ 0.986 \text{ for } t \text{ even} \end{cases}$

$$p(t) = \begin{cases} 0.2763 \text{ for } t = 1, 4, .. \\ 0.7262 \text{ for } t = 2, 5, .. \\ 0.9999 \text{ for } t = 3, 6, .. \end{cases}$$

7.8



7.9 $d(t) = 0.167$.

8.1 a. $c^h_0(1) = 0.5$; $c^h_t = [0.5, 0.5]$ for all $t \geqslant 1$; $u^h_t = 0.25$ $(C(t) = 1 = Y(t)$ for all $t \geqslant 1)$

b. $c_0^h(1) = 0.5$; $c_t^h = [0.3, 0.9]$ for all $t \geqslant 1$; $u_t^h = 0.27$
Consumption allocation b is Pareto superior.

8.2 There is another allocation that gives person h greater utility and gives everyone else the same utility level. See chapter 1.

8.3 If $r(t) < \lambda \quad l^h(t) \to -\infty$, $k^h(t+1) \to \infty$ and this is not feasible. Therefore it can not be an equilibrium.

8.4 $S_t(r(t)) = K(t+1) = 0$; $\{r(t), K(t)\} = \{r, 0\}$ for all $t \geqslant 1$.

8.5 $S(\lambda) = K > 0$; $\{r(t), K(t)\} = \{\lambda, K\}$ for all $t \geqslant 1$.

8.6 $S_t(r(t)) = p(t)A$; $p(t) = \dfrac{p(t+1) + d(t+1)}{r(t)}$; $r(t) > \lambda$; $K(t+1) = 0$ for all $t \geqslant 1$.

8.7 $S_t(\lambda) = p(t)A + K(t+1)$; $p(t) = \dfrac{p(t+1) + d(t+1)}{r(t)}$; $r(t) = \lambda$ for all $t \geqslant 1$.

8.8 a. $c_t^h(t) + \dfrac{c_t^h(t+1)}{r(t)} = (1 - z_\omega)\left[\omega_t^h(t) + \dfrac{\omega_t^h(t+1)}{r(t)}\right]$
$-a^h(t)\left[p(t) - \dfrac{d(t+1) + p(t+1)}{r(t)}\right] - k^h(t+1)\left[1 - \dfrac{\lambda}{r(t)}\right]$

b. $c_t^h(t) + \dfrac{c_t^h(t+1)}{r(t)} = \omega_t^h(t) + \dfrac{\omega_t^h(t+1)}{r(t)}$
$-a^h(t)\left[p(t) - \dfrac{(1 - z_d)d(t+1) + p(t+1)}{r(t)}\right] - k^h(t+1)\left[1 - \dfrac{\lambda}{r(t)}\right]$.

A tax on land rents affects $r(t)$.

8.9 See page 222

9.1 $Y(t) = 66.24$; $Y'(t) = 88.32$; $Y'(t) = \dfrac{4}{3}Y(t)$.

9.2 $wage(t) = wage'(t) = 4.42$; $rental(t) = rental'(t) = 1.77$.
$wage(t)L(t) = 39.8$; $wage(t)L'(t) = 53$; $rental(t)K(t) = 26.6$; $rental(t)K'(t) = 35.4$.

9.3 $s_t^h(r(t)) = 4.71 - \dfrac{4.52}{r(t)}$.

9.4 $K = 694$; $Y = 5784$; $c^h = [19.1, 31.7]$.

9.5 $K(t+1) = 50.68K(t)^{0.4}$; $Y(t) = 422.34K(t)^{0.4}$.

9.6 $K(t+1) = (1.05)^t 50.68K(t)^{0.4}$; $Y(t) = (1.05)^t 422.34K(t)^{0.4}$.

10.1 $N(t) = 100$; $A = 100$; $\omega_t^h = [2, 1]$; $d(t+1) = 0$; $u_t^h = -(c_t^h(t) - b^\gamma)^2 - \beta(c_t^h(t+1) - b^o)^2$; where $\beta = 4$ and $b = [b^\gamma, b^o] = [2, 2]$.

10.2 $p^m(t) = 0.25$ stationary monetary equilibrium; $c_t^h = [1.5, 1.5]$; $c_0^h(1) = 1.5$. $p^m(t) = 0$ stationary nonmonetary equilibrium; $c_t^h = [2, 1]$; $c_0^h(1) = 1$. If $p^m(t)$ is between 0.25 and 0 for any t, then a nonstationary equilibrium that is consistent with that $p^m(t)$ is a series of prices that converge to a price of 0.

10.3 If $p^m(t) = p^m(t+1)$, $r(t) = 1$; $\dfrac{p'(t+1) + d(t+1)}{p'(t)} = 1$; $p'(t) = p'(t+1) + d(t+1)$. For each $p(1)$, there is one n for which $p(n) < 0$. This can not be an equilibrium.

10.4 $r(t) = \dfrac{p^m(t+1)}{p^m(t)}$ and $r(t) \geqslant \lambda$. If $r(t) \geqslant \lambda > 1$ no monetary equilibrium exist. If $r(t) \geqslant \lambda < 1$ a monetary equilibrium exist.

10.5 The return on money in a monetary equilibrium is $r(t) = n > \lambda$, then $k^h(t+1) = 0$.There is a monetary equilibrium.

10.6 $u_t^h = 2.25$; $c_0^h(1) = 1.5$ stationary monetary equilibrium.

$2 < u_t^h < 2.25$; $1 < c_0^h(1) < 1.5$ nonstationary monetary equilibrium.

10.7 $u_t^h = 2.25$; $c_0^h(1) = 1.25$ $h = 1, ..., 50$ stationary monetary equilibrium

$u_t^h = 1$; $c_0^h(1) = 1.25$ $h = 51, ..., 100$

$u_t^h < 2.25$; $c_0^h(1) < 1.25$ $h = 1, ..., 50$ nonstationary monetary equilibrium

$u_t^h < 1$; $c_0^h(1) < 1.25$ $h = 51, ..., 100$

10.8 $r(t) = \dfrac{\frac{\partial u_t^h}{\partial c_t^h(t)}}{\frac{\partial u_t^h}{\partial c_t^h(t+1)}} = \dfrac{c_t^h(t+1)}{c_t^h(t)}$

$$= \frac{r(t)[\omega_t^h(t) - c_t^h(t)] + \omega_t^h(t+1) + \frac{p^m(t+1)(\mu-1)M(t)}{N(t)}}{c_t^h(t)}$$

$$c_t^h(t) = \frac{\omega_t^h(t)}{2} + \left[\omega_t^h(t+1) + \frac{p^m(t+1)(\mu-1)M(t)}{N(t)}\right]\left[\frac{1}{2r(t)}\right]$$

$$s_t^h(t) = \omega_t^h(t) - c_t^h(t)$$

$$= \frac{\omega_t^h(t)}{2} - \left[\omega_t^h(t+1) + \frac{p^m(t+1)(\mu-1)M(t)}{N(t)}\right]\left[\frac{1}{2r(t)}\right]$$

10.9 See page 282.

10.10 $g = \left(1 - \dfrac{1}{\mu}\right)\left(\dfrac{200 - 100(\mu)^{-1}}{1 + (\mu)^{-2}}\right)$.

10.11 $g = \left(1 - \dfrac{1}{\mu}\right)(50 - 100\mu)$.

10.12 See page 285.

10.13 See figure 10.4. A lump-sum tax can raise the same revenue as point c_2 but with higher utility.

10.14 Use the same graph as in exercise 10.13.

11.1a. $E = 2$; $p^A(t) = 0.19$; $p^B(t) = 0.38$; $c_t^{hi} = [1.5, 1.5]$ i equals A and B.

b. $E = 4$; $p^A(t) = 0.125$; $p^B(t) = 0.5$; $c_t^{hi} = [1.5, 1.5]$ i equals A and B.

11.2 $M^A(1) = 234; M^A(2) = 271; M^A(3) = 314; M^A(4) = 361; M^A(5) = 414$. $M^B(1) = 108; M^B(2) = 118; M^B(3) = 129; M^B(4) = 141; M^B(5) = 154$.

11.3 $M^A(1) = 220; M^A(2) = 242; M^A(3) = 269; M^A(4) = 299; M^A(5) = 332$. $M^B(1) = 115; M^B(2) = 132; M^B(3) = 152; M^B(4) = 173; M^B(5) = 197$.

11.4 $E(1) = 0.9$; $E(2) = 0.84$; $E(3) = 0.79$; $E(4) = 0.75$.

12.1 a. $\omega_t^h = [1, 2]$; $\overline{s}_t^h(2, 0.4) = 0$; $\overline{s}_t^h(1, 0.4) = -0.4$; $\overline{s}_t^h(0.5, 0.4) = -0.4$; $\overline{s}_t^h(2, 0.2) = 0$; $\overline{s}_t^h(1, 0.2) = -0.2$; $\overline{s}_t^h(0.5, 0.2) = -0.2$. b. $\omega_t^h = [2, 1]$; $\overline{s}_t^h(2, 0.4) = 0.75$; $\overline{s}_t^h(1, 0.4) = 0.5$; $\overline{s}_t^h(0.5, 0.4) = 0$; $\overline{s}_t^h(2, 0.2) = 0.75$; $\overline{s}_t^h(1, 0.2) = 0.5$; $\overline{s}_t^h(0.5, 0.2) = 0$.

12.2 a. $x = 0$; $p(t) = 0.017$; $u_t^h = 2.25$; $c_0^h(1) = 1.17$ $h = 1, .., 20$; $u_t^h = 2.25$; $c_0^h(1) = 2.17$ $h = 21, .., 30$.

b. $x = 0.4$; $p(t) = 0.02$; $u_t^h = 2.25$; $c_0^h(1) = 1.2$ $h = 1, .., 20$; $u_t^h = 2.24$; $c_0^h(1) = 2.2$ $h = 21, .., 30$.

12.3 a. $x = 0$; $g = 0$; $u_t^h = 2.11$; $c_0^h(1) = 1$ $h = 1, .., 20$; $u_t^h = 2.45$; $c_0^h(1) = 2$ $h = 21, .., 30$.

b. $x = 0.4$; $g = 0.7$; $u_t^h = 2.11$; $c_0^h(1) = 1.093$ $h = 1, .., 20$; $u_t^h = 2.35$; $c_0^h(1) = 2.093$ $h = 21, .., 30$.

12.4 $g = \left(1 - \frac{1}{\mu}\right)(25{-}20\mu)$; $g_{x(t)=0.4} = \left(1 - \frac{1}{\mu}\right)\left[20\left(1 - \frac{\mu}{2}\right) - 4\right]$; $g_{x(t)=0.2} = \left(1 - \frac{1}{\mu}\right)\left[20\left(1 - \frac{\mu}{2}\right) - 2\right]$.

12.5 $r^m = 1$; $r^L = 1.5$; $r^a = 1.33$; $p(t) = 0.155$.

12.6 $r^m = 1$; $r^L = 2$; $r^a = 1.67$; $p(t) = 0.14$.

12.7 a. $r^m = 1$; $r^L = 1.5$; $r^a = 1.33$; $p(t) = 0.12$; $c_t^{even} = [1.88, 2.49]$; $c_t^{odd} = [1.16, 1.76]$; $u_t^{even} = 4.68$; $u_t^{odd} = 2.04$. b. $r^L = 1$; $p(t) = 0.0625$; $c_t^{even} = [2, 2]$; $c_t^{odd} = [1.5, 1.5]$; $u_t^{even} = 4$; $u_t^{odd} = 2.25$.

12.8 $r^m = 1$; $r^L = 1.5$; $r^a = 1.33$; $p(t) = 0.112$.

12.9 The marginal rates of substitucion are different. We can reallocate the resources and find a feasible allocation where the marginal rates of substitucion are the same. This allocation is Pareto superior.

12.10 Use graph page 281.

12.11 Use result of exercise 12.10 plus definition of Pareto superior.

12.12 All symmetric allocations where the government gets g units of the consumption good fall on the 45-degree line in figure 12.15. Use definition of Pareto superior.

12.13 If $n = N$ and $F = \omega_t(t) - c_t^*(t)$; everyone consume at point c^*.

12.14 $c_t^*(t) = \dfrac{\omega_t(t) + \omega_t(t+1) - \frac{g}{n}}{2}$; $s_t(t) = \dfrac{\omega_t(t) - \omega_t(t+1) + \frac{g}{n}}{2}$.

Yes, this is the F for the c^* consumption, when $g = 0$.