Working capital

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1 Working capital models

Initial comments

- Model with neither staggered prices nor wages
- Add financial intermediaries
- Several ways to do it
  - Working capital
    - Capital used to finance in period production
    - In our case: the wage bill
    - could be other parts of production costs as well
  - Agency costs
    - Loans need collateral
    - Firm owners use their own wealth
    - and borrow from bank
    - Aggregate risk
    - Somewhat more complicated

Model with working capital

- Households max

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t^i, 1 - h_t^i)$$

- subject to a cash-in-advance constraint

$$P_t c_t^i \leq m_{t-1}^i - N_t^i$$

- the budget constraint

$$\frac{m_t^i}{P_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1-\delta) k_t^i + \frac{r_t^n N_t^i}{P_t}$$

- N_t^i = nominal deposits in a financial intermediary
- r_t^n = gross interest rate on deposits

Model with working capital

- Competitive firms
- production function

$$Y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

- the representative firm maximizes profits subject to the budget constraint

$$Y_t = r_t^f w_t H_t + r_t K_t$$

- r_t^f = gross interest rate on borrowing funds from FI

Model with working capital

- Financial intermediaries
- The budget constraints for the financial intermediary (a zero profit condition)

$$r_t^f \left(N_t + (g_t - 1) M_{t-1}\right) = \int_0^1 r_t^n N_t^i di = r_t^n N_t$$

- g_t is the gross growth rate of money in period t
- FI's receive money transfers from government (important)
- equilibrium condition for the financial market

$$\left(N_t + (g_t - 1) M_{t-1}\right) = P_t w_t H_t$$

Full model

- FOCs of household

$$\begin{aligned} \frac{B}{w_t} &= -\beta \frac{P_t}{E_t P_{t+1} C_{t+1}} \\ \frac{1}{w_t} &= \beta E_t \frac{r_{t+1} + 1 - \delta}{w_{t+1}} \\ r_t^n &= -\frac{w_t}{BC_t} = \frac{E_t P_{t+1} C_{t+1}}{\beta P_t C_t} \end{aligned}$$

- cash in advance constraint for household consumption

$$P_t C_t = M_{t-1} - N_t$$

- the real flow budget constraint

$$\frac{M_t}{P_t} + K_{t+1} = w_t H_t + r_t K_t + (1-\delta)K_t + \frac{r_t^n N_t}{P_t}$$

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- factor market conditions

$$r_t^f w_t = (1-\theta)\,\lambda_t K_t^{\theta} H_t^{-\theta}$$

$$r_t = \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta}$$

- The production function

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta}$$

- FI's zero profit condition

$$r_t^f\,(N_t + (g_t - 1)\,M_{t-1}) = r_t^n N_t$$

- Clearing of the credit market

$$(N_t + (g_t - 1)\,M_{t-1}) = P_t w_t H_t$$

- money growth rule

$$M_t = g_t M_{t-1}$$

Stationary state

$$\overline{r} = 1/\beta - 1 + \delta \quad \text{and} \quad \overline{r}^n = \overline{\pi}/\beta = \overline{g}/\beta$$

- Solve nuemerically five equations for $\overline{M/P}$, $\overline{N/P}$, $\overline{r}^f$, $\overline{C}$, and $\overline{H}$

$$\overline{r}^f = -\frac{\beta\,(1-\theta)\left(\frac{\theta}{\overline{r}}\right)^{\frac{\theta}{1-\theta}}}{\overline{C}\overline{g}B}$$

$$\left[\overline{r}^n - \overline{r}^f\right]\overline{N/P} = \overline{r}^f\left[1 - \frac{1}{\overline{g}}\right]\overline{M/P}$$

$$-\frac{\overline{C}\overline{g}B}{\beta}\overline{H} = \overline{N/P} + \left[1 - \frac{1}{\overline{g}}\right]\overline{M/P}$$

$$\overline{C} = \frac{\overline{M/P}}{\overline{g}} - \overline{N/P}$$

$$\overline{M/P} = \frac{\overline{g}}{\beta}\overline{N/P} + \left[(\overline{r}-\delta)\left[\frac{\theta}{\overline{r}}\right]^{\frac{1}{1-\theta}} - \frac{\overline{C}\overline{g}B}{\beta}\right]\overline{H}$$

Stationary states

$Annual$ $inflation$	-4%	0	10%	100%	400%
$\overline{g}$	.99	1	1.024	1.19	1.41
$\overline{r}$	.035101	.035101	.035101	.035101	.035101
$\overline{r}^n$	1.0000	1.0101	1.0343	1.2020	1.4242
$\overline{r}^f$	1.0221	1.0101	0.9824	0.8259	0.6820
$\frac{M_t}{P_t} = \overline{M/P}$	1.6557	1.6675	1.6960	1.8896	2.1395
$\frac{N_t}{P_t} = \overline{N/P}$	.7736	.76715	0.7523	0.6626	0.5716
$\overline{C}$	.8988	.90038	0.9040	0.9253	0.9458
$\overline{Y}$	1.2087	1.2108	1.2158	1.2444	1.2720
$\overline{w}$	2.3193	2.3469	2.4130	2.8702	3.4762
$\overline{H}$	.3263	.32688	0.3282	0.3360	0.3434
$\overline{K}$	12.3967	12.418	12.4690	12.7627	13.0454
$utility$	-0.9488	-0.9485	-0.9479	-0.9445	-0.9418

A Phillips curve (for stationary states)

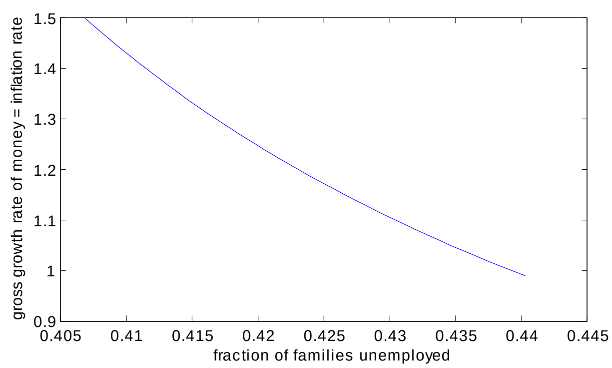

Figure 1: Stationary state Phillips curve

Log-linear version of the model

$$
\begin{aligned}
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t\widetilde{P}_{t+1} - E_t\widetilde{C}_{t+1}, \\
0 &= \widetilde{w}_t - E_t\widetilde{w}_{t+1} + \beta\overline{r}E_t\widetilde{r}_{t+1}, \\
0 &= \widetilde{r}_t^n - \widetilde{w}_t + \widetilde{C}_t, \\
0 &= \overline{C}\left[\widetilde{P}_t + \widetilde{C}_t\right] - \frac{\overline{M/P}}{\overline{g}}\widetilde{M}_{t-1} + \overline{N/P}\widetilde{N}_t, \\
0 &= \overline{M/P}\widetilde{M}_t + \left[\overline{r}^n\overline{N/P} - \overline{M/P}\right]\widetilde{P}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}(\widetilde{w}_t + \widetilde{H}_t) \\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - (\overline{r} + 1 - \delta)\overline{K}\widetilde{K}_t - \overline{r}^n\overline{N/P}\widetilde{N}_t - \overline{r}^n\overline{N/P}\widetilde{r}_t^n, \\
0 &= \widetilde{w}_t + \widetilde{r}_t^f - \widetilde{\lambda}_t - \theta\widetilde{K}_t + \theta\widetilde{H}_t, \\
0 &= \widetilde{r}_t - \widetilde{\lambda}_t - (\theta - 1)\widetilde{K}_t - (1 - \theta)\widetilde{H}_t, \\
0 &= \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1 - \theta)\widetilde{H}_t,
\end{aligned}
$$

Log-linear version of the model (continued)

$$
\begin{aligned}
0 &= \overline{r}^f\left[\overline{N/P} + \overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\right]\widetilde{r}_t^f + \left(\overline{r}^f - \overline{r}^n\right)\overline{N/P}\widetilde{N}_t \\
&\quad - \left[\left(\overline{r}^f - \overline{r}^n\right)\overline{N/P} + \overline{r}^f\overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\right]\widetilde{P}_t \\
&\quad + \overline{r}^f\overline{M/P}\widetilde{g}_t + \overline{r}^f\overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\widetilde{M}_{t-1} - \overline{r}^n\overline{N/P}\widetilde{r}_t^n, \\
0 &= \overline{N/P}\widetilde{N}_t + \overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\widetilde{M}_{t-1} - \left[\overline{N/P} + \overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\right]\widetilde{P}_t \\
&\quad + \overline{M/P}\widetilde{g}_t - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t \\
0 &= \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}.
\end{aligned}
$$

Solving the model

- state variables $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]'$
- jump variables $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{Y}_t, \widetilde{C}_t, \widetilde{H}_t, \widetilde{N}_t, \widetilde{r}_t^n, \widetilde{r}_t^f\right]'$
- stochastic variables $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]'$
- the system can be written as

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}
$$

- see book for matrices

Policy functions

- For the policy functions

$$x_{t+1} = Px_t + Qz_t$$

$$y_t = Rx_t + Sz_t$$

- The matrices P and Q are

$$P = \begin{bmatrix} 0.9430 & 0 & 0 \\ 0 & 1 & 0 \\ -0.3340 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.1490 & 0.2020 \\ 0 & 1 \\ -1.0337 & 0.6287 \end{bmatrix}$$

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- The matrices R and S are

$$R = \begin{bmatrix} -0.9236 & 0 & 0 \\ 0.5315 & 0 & 0 \\ 0.0764 & 0 & 0 \\ 0.5434 & 0 & 0 \\ -0.4432 & 0 & 0 \\ -0.2457 & 1 & 0 \\ -0.0119 & 0 & 0 \\ -0.0119 & 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 1.8309 & 1.2411 \\ 0.4701 & 0.1791 \\ 1.8309 & 1.2411 \\ 0.4077 & -1.1172 \\ 1.2982 & 1.9392 \\ 0.7347 & 0.5734 \\ 0.0625 & 1.2964 \\ 0.0625 & -0.8772 \end{bmatrix}.$$

Impulse response functions: real variables to a technology shock
Impulse response functions: real variables to a money growth shock
Impulse response functions: nominal variables to a technology shock
Impulse response functions: nominal variables to a money growth shock

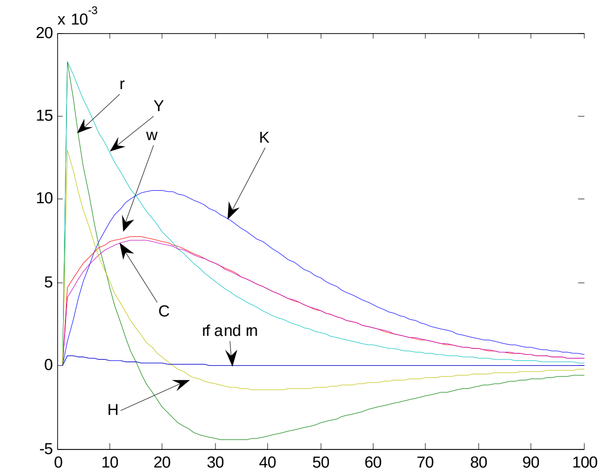

Figure 2: Responses of real variables to a technology shock

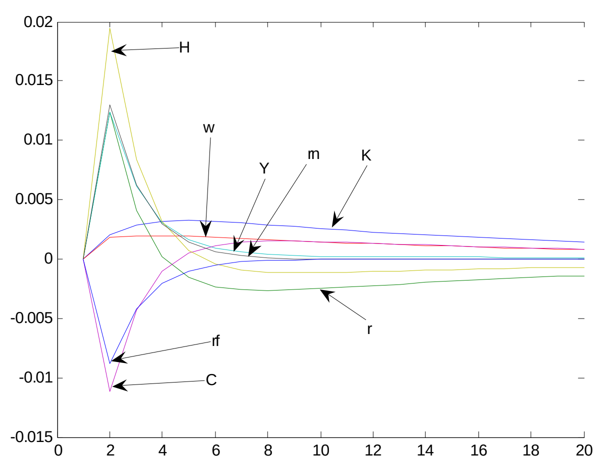

Figure 3: Response of real variables to a money growth shock

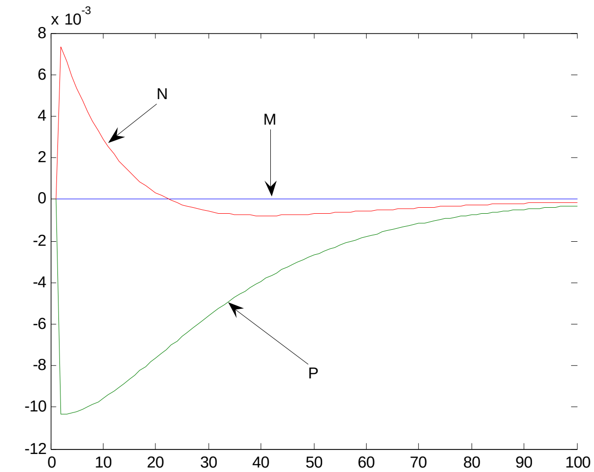

Figure 4: Response of nominal variables to a technology shock

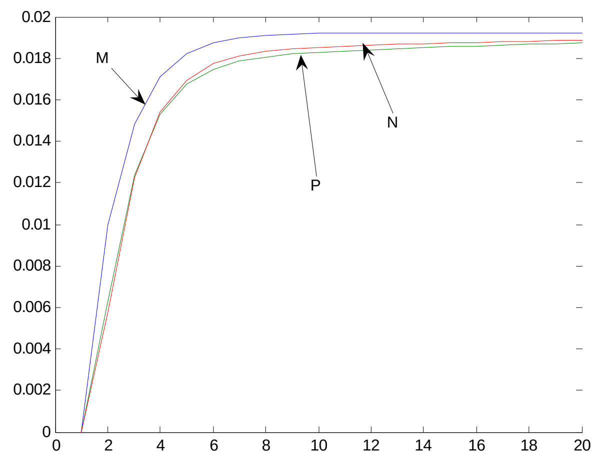

Figure 5: Response of nominal variables to a money growth shock