Staggered wage setting

George McCandless
CEMA

August 5, 2008

1 Staggered wage setting model

Staggered wage setting model

- Apply the Calvo price setting rules to wages
- Labor is differentiated
- a fraction $(1-\rho_w)$ of households can set their wages each period
- Group is randomly chosen
- ρ_w follow a rule of thumb

The labor bundler

- A perfectly competitve firm (or firms) bundle labor
- They then sell this labor to the goods producing firms
- Bundling technology is

$$H_t = \left[\int_0^1 \left(h_t^i \right)^{\frac{\psi_w - 1}{\psi_w}} di \right]^{\frac{\psi_w}{\psi_w - 1}}$$

- Cost minimization on the part of the bundler implies

$$W_t = \left[\int_0^1 W_t(i)^{1-\psi_w} di \right]^{\frac{1}{1-\psi_w}}$$

The labor bundler

- Labor is differentiated
- The bundling firm demands different types of labor

- The demand for each type of labor is

$$h_t^i = H_t \left(\frac{W_t}{W_t(i)} \right)^{\psi_w}$$

.Rules of thrumb

- The households that do not optimize follow a rule of thumb
- Some potential rules of thumb are
  - Wages stay constant until optimization is possible:

$$W_t(i) = W_{t-1}(i)$$

  - Wages are updated by stationary state inflation rate

$$W_t(i) = \overline{\pi} W_{t-1}(i)$$

  - Wages are updated by lagged inflation

$$W_t(i) = \frac{P_{t-1}}{P_{t-2}} W_{t-1}(i)$$

- Choice of rule of thumb matters
- Note: we assume an insurance plan for labor to keep household incomes the same

.Optimization problem of the family

- A household i chooses $\left\{ c_{t+k}^i, m_t^i, k_{t+1}^i \right\}$ to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\ln c_{t+k}^i + A \ln(1 - h_{t+j}^i) \right]$$

  subject to the cash in advance constraint

$$P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1}$$

  and the flow budget constraint

$$k_{t+1}^i + \frac{m_t^i}{P_t} = \frac{W_t(i)}{P_t} h_t^i + r_t k_t^i + (1 - \delta) k_t^i + b_t^i$$

- b_t^i are the lumb sum transfers from the insurance plan

.Optimization problem of the family

- A household that can set its wage also chooses $W_t^*(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} (\beta\rho_w)^j \left[\ln c_{t+j}^i + A\ln(1-h_{t+j}^i)\right]$$

- subject to the sequence of budget constraints

$$P_{t+j}c_{t+j}^i = m_{t+j-1}^i + (g_{t+j}-1)\, M_{t+j-1}$$

- and

$$k_{t+j+1}^i + \frac{m_{t+j}^i}{P_{t+j}} = \frac{W_t^*(i)}{P_{t+j}}h_{t+j}^i + r_{t+j}k_{t+j}^i + (1-\delta)k_{t+j}^i + b_{t+j}^i$$

- where

$$h_{t+j}^i = H_{t+j}\left(\frac{W_{t+j}}{W_t^*(i)}\right)^{\psi_w}$$

- Notice the change in the discount factor on utility: $\beta\rho_w$

.Optimization problem of the family

- First order condition from the basic problem

$$E_t\frac{P_t}{P_{t+1}c_{t+1}^i} = \beta E_t\frac{P_{t+1}}{P_{t+2}c_{t+2}^i}\left(r_{t+1}+(1-\delta)\right)$$

  - Note the inclusion of $P_{t+2}c_{t+2}^i$

- First order condition from the wage setting problem (after some algebra)

$$W_t^*(i) = \frac{\psi_w}{(\psi_w-1)}\frac{A}{\beta}\frac{E_t\sum_{j=0}^{\infty}(\beta\rho_w)^j\frac{1}{1-h_{t+j}^i}h_{t+j}^i}{E_t\sum_{j=0}^{\infty}(\beta\rho_w)^j\frac{1}{P_{t+1+j}c_{t+1+j}^i}h_{t+j}^i}$$

- Plus budget constraints

.Rest of the model

- Production function

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta}$$

- the aggregate real wage

$$w_t = \frac{W_t}{P_t} = (1-\theta)\,\lambda_t K_t^{\theta} H_t^{-\theta} = (1-\theta)\,\frac{Y_t}{H_t}$$

- real rental rate

$$r_t = \theta\lambda_t K_t^{\theta-1} H_t^{1-\theta} = \theta \frac{Y_t}{K_t}$$

- labor bundler

$$H_t = \left[\int_0^1 \left(h_t^i\right)^{\frac{\psi_w - 1}{\psi_w}} di\right]^{\frac{\psi_w}{\psi_w - 1}}$$

- capital is summed,

$$K_t = \int_0^1 k_t^i di$$

Equilibrium conditions

- From identical households

$$K_t = k_t^i$$

- consumption

$$C_t = c_t^i$$

- Aggregating the real budget constraints

$$K_{t+1} + \frac{M_t}{P_t} = \int_0^1 \frac{W_t(i)}{P_t} h_t^i di + r_t K_t + (1-\delta) K_t$$

- the insurance payments or premiums drop out

$$\int_0^1 b_t^i di = 0$$

Equilibrium conditions

- labor bundling firms are also perfectly competitive

$$W_t H_t = \int_0^1 W_t(i) h_t^i di$$

- aggregated family real budget constraint gives

$$K_{t+1} + \frac{M_t}{P_t} = \frac{W_t}{P_t} H_t + r_t K_t + (1-\delta) K_t.$$

The full model

- The variables: C_t, r_t, K_t, M_t, W_t, H_t, W_t^*, P_t, h_t^*, Y_t, and shocks λ_t and g_t
- Equations

– aggregate version of the first order condition,

$$E_t \frac{P_t}{P_{t+1} c_{t+1}^i} = \beta E_t \frac{P_{t+1}}{P_{t+2} c_{t+2}^i} \left(r_{t+1} + (1-\delta) \right)$$

– aggregate budget constraint

$$K_{t+1} + \frac{M_t}{P_t} = \frac{W_t}{P_t} H_t + r_t K_t + (1-\delta) K_t$$

– cash-in-advance constraint

$$P_t C_t = g_t M_{t-1}$$

– money supply growth rule,

$$M_t = g_t M_{t-1}$$

.The full model

- Equations

  – nominal wage setting equation

$$W_t^*(i) = \frac{\psi_w}{(\psi_w - 1)} \frac{A}{\beta} \frac{E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{1-h_{t+j}^i} h_{t+j}^i}{E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{P_{t+1+j} c_{t+1+j}^i} h_{t+j}^i}$$

  – demand from the bundler for labor of the families that can fix their wages in period t

$$h_t^* = H_t \left(\frac{W_t}{W_t^*(i)} \right)^{\psi_w}$$

  – aggregate nominal wage equation from the labor bundler

$$W_t^{1-\psi} = (1-\rho_w) (W_t^*)^{1-\psi} + \rho_w (W_{t-1})^{1-\psi}$$

  – demand for bundled labor from the firms

$$\frac{W_t}{P_t} = (1-\theta) \frac{Y_t}{H_t}$$

.The full model

- Equations

  – demand for capital by the firms

$$r_t = \theta \frac{Y_t}{K_t}$$

– the production function,

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta}$$

– two stochastic process for technology and money growth,

$$\ln \lambda_t = \gamma \ln \lambda_{t-1} + \varepsilon_t^{\lambda}$$

and

$$\ln g_t = \pi \ln g_{t-1} + \varepsilon_t^{g}$$

Stationary state

- aggregate wage equation gives the result that

$$\overline{W}^* = W$$

- This implies

$$\overline{h}^* = \overline{H}$$

- wage setting equations gives the real wage as

$$\overline{W/P} = \frac{\psi_w}{(\psi_w - 1)} \frac{A}{\beta} \frac{\overline{C}}{\left(1 - \overline{H}\right)}$$

- Rest pretty normal
- Results

	$\overline{r}$	$\overline{W/P}$	$\overline{C} = \overline{M/P}$	$\overline{H}$	$\overline{K}$	$\overline{Y}$
values	.0351	2.3706	0.8830	0.3206	12.1795	1.1875

Log-linearization

- Wage setting equation

$$W_t^*(i) = \frac{\psi_w}{(\psi_w - 1)} \frac{A}{\beta} \frac{E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{1 - h_{t+j}^i} h_{t+j}^i}{E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{P_{t+1+j} c_{t+1+j}^i} h_{t+j}^i}$$

- can be written as

$$W_t^*(i) E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{P_{t+1+j} c_{t+1+j}^i} h_{t+j}^i$$

$$= \frac{\psi_w}{(\psi_w - 1)} \frac{A}{\beta} E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \frac{1}{1 - h_{t+j}^i} h_{t+j}^i$$

Log-linearization

- Log linearization of the left side gives

$$\overline{W/P}\frac{\overline{H}}{\overline{C}}E_t\sum_{j=0}^{\infty}(\beta\rho_w)^j\left(1+\widetilde{W}_t^*(i)+\widetilde{h}_{t+j}^i-\widetilde{P}_{t+1+j}-\widetilde{C}_{t+1+j}\right)$$

- of the right side is

$$\frac{\psi_w}{(\psi_w-1)}\frac{A}{\beta}\frac{\overline{H}}{1-\overline{H}}E_t\sum_{j=0}^{\infty}(\beta\rho_w)^j\left(1+\frac{1}{1-\overline{H}}\widetilde{h}_{t+j}^i\right)$$

Log-linearization (an interesting detail)

- in doing the log-linearization of the right side, one needs to take several approximations
- First one finds that

$$\begin{aligned}\frac{h_t^i}{1-h_t^i} &= \frac{\overline{H}e^{\widetilde{h}_t^i}}{1-\overline{H}e^{\widetilde{h}_t^i}}\approx\frac{\overline{H}\left(1+\widetilde{h}_t^i\right)}{1-\overline{H}\left(1+\widetilde{h}_t^i\right)}\\ &= \frac{\overline{H}}{1-\overline{H}}\frac{\left(1+\widetilde{h}_t^i\right)}{\left(1-\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i\right)}\end{aligned}$$

- The last item in the denominator, $\left(1-\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i\right)$
- is approximately equal to

$$e^{-\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i}$$

Log-linearization (an interesting detail)

- Substituting in, one gets

$$\begin{aligned}\frac{\overline{H}}{1-\overline{H}}\frac{\left(1+\widetilde{h}_t^i\right)}{e^{-\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i}} &= \frac{\overline{H}}{1-\overline{H}}\left(1+\widetilde{h}_t^i\right)e^{\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i}\\ &\approx \frac{\overline{H}}{1-\overline{H}}\left(1+\widetilde{h}_t^i\right)\left(1+\frac{\overline{H}}{1-\overline{H}}\widetilde{h}_t^i\right)\\ &= \frac{\overline{H}}{1-\overline{H}}\left(1+\frac{1}{1-\overline{H}}\widetilde{h}_{t+j}^i\right)\end{aligned}$$

Log-linearization (continued)

- stationary state of the above equation is

$$\overline{W/P}\frac{\overline{H}}{\overline{C}}=\frac{\psi_w}{(\psi_w-1)}\frac{A}{\beta}\frac{\overline{H}}{1-\overline{H}}$$

- so

$$E_t \sum_{j=0}^{\infty} (\beta \rho_w)^j \left(\widetilde{W}_t^*(i) - \frac{H}{1-\overline{H}} \widetilde{h}_{t+j}^i - \widetilde{P}_{t+1+j} - \widetilde{C}_{t+1+j} \right) = 0$$

- the families that can set their wages choose

$$\widetilde{W}_t^* = (1 - \beta\rho_w)\, E_t \sum_{j=0}^{\infty} (\beta\rho_w)^j \left(\widetilde{P}_{t+1+j} + \widetilde{C}_{t+1+j} + \frac{H}{1-\overline{H}} \widetilde{h}_{t+j}^i \right)$$

Log-linearization (continued)

- Using the log-linear version of the wage evolution equation from the bundler,

$$\widetilde{W}_t = (1-\rho_w)\widetilde{W}_t^* + \rho_w \widetilde{W}_{t-1}$$

- gives

$$\begin{aligned} & \widetilde{W}_t - \rho_w \widetilde{W}_{t-1} \\ = \; & (1-\rho_w)\,(1-\beta\rho_w) \\ & \times E_t \sum_{j=0}^{\infty} (\beta\rho_w)^j \left(\widetilde{P}_{t+1+j} + \widetilde{C}_{t+1+j} + \frac{H}{1-\overline{H}} \widetilde{h}_{t+j}^i \right) \end{aligned}$$

Quasi-differencing (again)

- Operating on both sides of the above equation with $1 - \beta\rho_w L^{-1}$
- gives for the left side

$$(1 + \beta\rho_w\rho_w)\,\widetilde{W}_t - \rho_w \widetilde{W}_{t-1} - \beta\rho_w \widetilde{W}_{t+1}$$

- for the right side

$$\begin{aligned} & (1-\rho_w)\,(1-\beta\rho_w) \\ & \times E_t \sum_{j=0}^{\infty} (\beta\rho_w)^j \left(\widetilde{P}_{t+1+j} + \widetilde{C}_{t+1+j} + \frac{H}{1-\overline{H}} \widetilde{h}_{t+j}^i \right) \\ & -(1-\rho_w)\,(1-\beta\rho_w) \\ & \times E_t \sum_{j=0}^{\infty} (\beta\rho_w)^{j+1} \left(\widetilde{P}_{t+2+j} + \widetilde{C}_{t+2+j} + \frac{H}{1-\overline{H}} \widetilde{h}_{t+j+1}^i \right) \\ = \; & (1-\rho_w)\,(1-\beta\rho_w) \left(\widetilde{P}_{t+1} + \widetilde{C}_{t+1} + \frac{H}{1-\overline{H}} \widetilde{h}_t^i \right) \end{aligned}$$

Log-linearization (wage equation)

- Wage adjustment equation

$$\begin{aligned} & (1+\beta\rho_w\rho_w)\widetilde{W}_t - \rho_w\widetilde{W}_{t-1} - \beta\rho_w\widetilde{W}_{t+1} \\ = \quad & (1-\rho_w)(1-\beta\rho_w)\left(\widetilde{P}_{t+1} + \widetilde{C}_{t+1} + \frac{H}{1-\overline{H}}\widetilde{h}_t^*\right) \end{aligned}$$

- to remove the individual labor quantities from the equation, use the log linear versions of the demand function for labor and the bundler's aggregate wage rule

$$\begin{aligned} \widetilde{h}_t^* &= \widetilde{H}_t + \psi_w\widetilde{W}_t - \psi_w\widetilde{W}_t^*, \\ \widetilde{W}_t &= (1-\rho_w)\widetilde{W}_t^* + \rho_w\widetilde{W}_{t-1} \end{aligned}$$

- These give the result that

$$\widetilde{h}_t^* = \widetilde{H}_t - \frac{\psi_w\rho_w\widetilde{W}_t}{(1-\rho_w)} + \frac{\psi_w\rho_w\widetilde{W}_{t-1}}{(1-\rho_w)}$$

Log-linearization

- Final wage adjustment equation is

$$\begin{aligned} & \left[(1-\beta\rho_w)\frac{\psi_w\rho_w\overline{H}}{1-\overline{H}} + (1+\beta\rho_w\rho_w)\right]\widetilde{W}_t \\ & - \left[(1-\beta\rho_w)\frac{\psi_w\rho_w\overline{H}}{1-\overline{H}} + \rho_w\right]\widetilde{W}_{t-1} - \beta\rho_w\widetilde{W}_{t+1} \\ = \quad & (1-\rho_w)(1-\beta\rho_w)\left(\widetilde{P}_{t+1} + \widetilde{C}_{t+1} + \frac{H}{1-\overline{H}}\widetilde{H}_t\right) \end{aligned}$$

Log-linearization (first order condition)

- FOC

$$E_t\frac{P_t}{P_{t+1}c_{t+1}^i} = \beta E_t\frac{P_{t+1}}{P_{t+2}c_{t+2}^i}(r_{t+1} + (1-\delta))$$

- replace $P_{t+2}c_{t+2}^i$ with $g_{t+2}m_{t+1}^i$

$$E_t\frac{P_t}{P_{t+1}C_{t+1}} = \beta E_t\frac{P_{t+1}}{g_{t+2}M_{t+1}}(r_{t+1} + (1-\delta))$$

- The log-linearization of this equation yields, after some simplification,

$$E_t\left[\widetilde{P}_t - \widetilde{P}_{t+1} - \widetilde{C}_{t+1}\right] = E_t\left[\widetilde{P}_{t+1} - \widetilde{g}_{t+2} - \widetilde{M}_{t+1} + \beta\overline{r}\widetilde{r}_{t+1}\right]$$

- use the stochastic process for the growth rate of money

$$\widetilde{g}_t = \pi \widetilde{g}_{t-1} + \varepsilon_t^g$$

to find the expected value for $\widetilde{g}_{t+2}$ in terms of $\widetilde{g}_{t+1}$

- get

$$E_t \left[\widetilde{P}_t - \widetilde{P}_{t+1} - \widetilde{C}_{t+1} \right] = E_t \left[\widetilde{P}_{t+1} - \pi \widetilde{g}_{t+1} - \widetilde{M}_{t+1} + \beta \overline{r} \widetilde{r}_{t+1} \right]$$

.

$$\begin{aligned}
0 &= \widetilde{P}_t - 2E_t \widetilde{P}_{t+1} - E_t \widetilde{C}_{t+1} + \pi E_t \widetilde{g}_{t+1} + E_t \widetilde{M}_{t+1} - \beta \overline{r} E_t \widetilde{r}_{t+1}, \\
0 &= \overline{K} \widetilde{K}_{t+1} + \overline{M/P} \left[\widetilde{M}_t - \widetilde{P}_t \right] - \overline{Y} \widetilde{Y}_t - (1-\delta) \overline{K} \widetilde{K}_t, \\
0 &= \widetilde{P}_t + \widetilde{C}_t - \widetilde{g}_t - \widetilde{M}_{t-1}, \\
0 &= \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}, \\
0 &= \left[(1-\beta\rho_w) \frac{\psi_w \rho_w \overline{H}}{1-\overline{H}} + (1+\beta\rho_w\rho_w) \right] \widetilde{W}_t \\
&\quad - \left[(1-\beta\rho_w) \frac{\psi_w \rho_w \overline{H}}{1-\overline{H}} + \rho_w \right] \widetilde{W}_{t-1} - \beta\rho_w \widetilde{W}_{t+1} \\
&\quad -(1-\rho_w)(1-\beta\rho_w) \left(\widetilde{P}_{t+1} + \widetilde{C}_{t+1} + \frac{H}{1-\overline{H}} \widetilde{H}_t \right), \\
0 &= \widetilde{W}_t - \widetilde{P}_t - \widetilde{Y}_t + \widetilde{H}_t, \\
0 &= \widetilde{r}_t - \widetilde{Y}_t + \widetilde{K}_t, \\
0 &= \widetilde{Y}_t - \widetilde{\lambda}_t - \theta \widetilde{K}_t - (1-\theta) \widetilde{H}_t,
\end{aligned}$$

Solving the model

- The state variables are $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t, \widetilde{W}_t \right]'$
- the jump variables are $y_t = \left[\widetilde{r}_t, \widetilde{C}_t, \widetilde{Y}_t, \widetilde{H}_t \right]'$
- the stochastic shocks are $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t \right]$
- Solve

$$\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t \left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t \right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}$$

- for policy functions

$$x_{t+1} = Px_t + Qz_t \text{ and } y_t = Rx_t + Sz_t$$

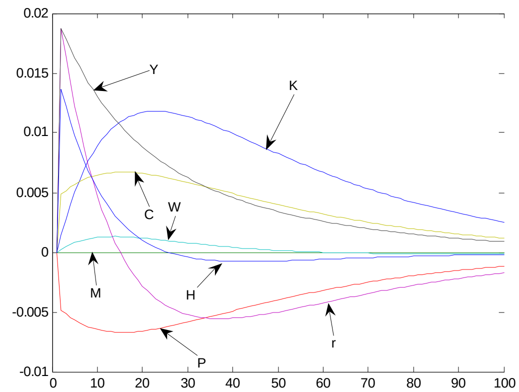

Figure 1: Response of staggered wage setting economy to a .01 technology shock

- Parameters used are $\rho_w = .7$ and $\psi_w = 21$

.

$$P = \begin{bmatrix} 0.9671 & 0.0939 & 0 & -0.0939 \\ 0 & 1 & 0 & 0 \\ -0.4310 & 0.7298 & 0 & 0.2702 \\ -0.0035 & 0.0750 & 0 & 0.9250 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.1475 & 0.0908 \\ 0 & 1 \\ -0.4836 & 0.7348 \\ 0.0254 & 0.1002 \end{bmatrix}$$

$$R = \begin{bmatrix} -0.7600 & 1.1641 & 0 & -1.1641 \\ 0.4310 & 0.2702 & 0 & -0.2702 \\ 0.2400 & 1.1641 & 0 & -1.1641 \\ -0.1875 & 1.8189 & 0 & -1.8189 \end{bmatrix},$$

$$S = \begin{bmatrix} 1.8728 & 1.1282 \\ 0.4836 & 0.2652 \\ 1.8728 & 1.1282 \\ 1.3637 & 1.7629 \end{bmatrix}.$$

Response to a .01 technology shock
Responses of real wages
Response to a .01 money growth shock

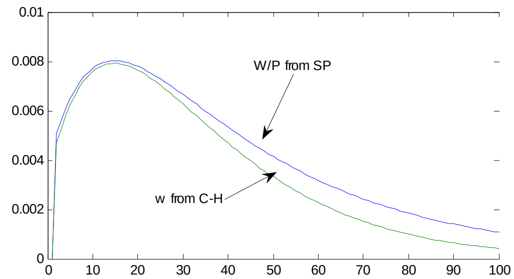

Figure 2: Responses of real wages in the Cooley-Hansen model and in the staggered wage setting model

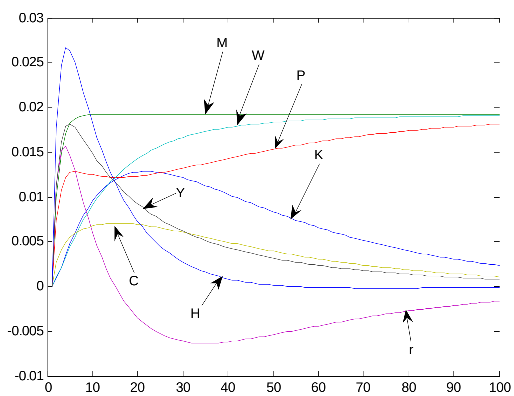

Figure 3: Response of staggered wage setting economy to a .01 money growth shock