

Macroeconomics II

Staggered pricing: part 2

Prof. McCandless
UCEMA

Prof. McCandless
UCEMA

October 31, 2006

1 Staggered pricing: Continued

Log-linearization: Final goods pricing rule

- Equation

$$P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1-\rho) P_t^*(k)^{1-\psi}$$

- Using Uhlig's method for log-linearization gives

$$\bar{P} e^{(1-\psi)\tilde{P}_t} = \rho \bar{P} e^{(1-\psi)\tilde{P}_{t-1}} + (1-\rho) \bar{P} e^{(1-\psi)\tilde{P}_t^*(k)}$$

since all firms have the same price, $\bar{P}^*(k)$ and applying the price bundler results in $\bar{P} = \bar{P}^*(k)$.

- Using the standard approximation rule gives

$$1 + (1-\psi)\tilde{P}_t \approx \rho \left(1 + (1-\psi)\tilde{P}_{t-1} \right) + (1-\rho) \left(1 + (1-\psi)\tilde{P}_t^*(k) \right)$$

or

$$\tilde{P}_t \approx \rho \tilde{P}_{t-1} + (1-\rho)\tilde{P}_t^*(k)$$

Log-linearization: intermediate goods firms

- Begin with pricing rule

$$P_t^*(k) = \frac{\psi}{\psi-1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}}{w_{t+i}} \right]^\theta}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k)}$$

- Multiplying out the denominator gives

$$\begin{aligned}
& P_t^*(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k) \\
&= \frac{\psi}{\psi-1} E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}}{w_{t+i}} \right]^\theta
\end{aligned}$$

Log-linearization: intermediate goods firms

- Right hand side gives

$$\begin{aligned}
& E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_t^*(k) Y_{t+i}(k) \\
&= E_t \sum_{i=0}^{\infty} (\beta\rho)^i \bar{P}^*(k) \bar{Y}(k) e^{\tilde{P}_t^*(k) + \tilde{Y}_{t+i}(k)} \\
&\approx \bar{P}^*(k) \bar{Y}(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left(1 + \tilde{P}_t^*(k) + \tilde{Y}_{t+i}(k) \right) \\
&= \frac{\bar{P}^*(k) \bar{Y}(k)}{1-\beta\rho} \left(1 + \tilde{P}_t^*(k) \right) \\
&\quad + \bar{P}^*(k) \bar{Y}(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left(\tilde{Y}_{t+i}(k) \right)
\end{aligned}$$

Log-linearization: intermediate goods firms

- Left hand side gives

$$\begin{aligned}
& \frac{\psi}{\psi-1} E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta \\
&= \frac{\psi}{\psi-1} \bar{P}\bar{Y}(k) \frac{\bar{w}}{(1-\theta)\bar{\lambda}} \left[\frac{(1-\theta)\bar{r}}{\theta\bar{w}} \right]^\theta \\
&\quad \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i e^{\tilde{P}_{t+i} + \tilde{Y}_{t+i}(k) + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i}} \\
&\approx \frac{\psi}{\psi-1} \bar{P}\bar{Y}(k) \frac{\bar{w}}{(1-\theta)\bar{\lambda}} \left[\frac{(1-\theta)\bar{r}}{\theta\bar{w}} \right]^\theta \\
&\quad \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[1 + \tilde{P}_{t+i} + \tilde{Y}_{t+i}(k) \right. \\
&\quad \left. + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right]
\end{aligned}$$

Log-linearization: intermediate goods firms

- In the stationary state

$$\frac{\psi}{\psi - 1} = \frac{1}{\frac{\bar{w}}{(1-\theta)\bar{\lambda}} \left[\frac{(1-\theta)\bar{r}}{\theta\bar{w}} \right]^\theta}$$

- The right side becomes simply

$$\begin{aligned} \approx & \bar{P}\bar{Y}(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[1 + \tilde{P}_{t+i} + \tilde{Y}_{t+i}(k) \right. \\ & \left. + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \end{aligned}$$

- We have shown that $\bar{P} = \bar{P}^*$, so stationary state prices and $\bar{Y}(k)$ cancel out on both sides of the equation. Both sides contain $E_t \sum_{i=0}^{\infty} (\beta\rho)^i \tilde{Y}_{t+i}(k)$

Log-linearization: intermediate goods firms

- the log-linear pricing rule for the intermediate goods firm is therefore

$$\begin{aligned} & \tilde{P}_t^*(k) \\ = & (1 - \beta\rho) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+i} + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \end{aligned}$$

Price equation

- Combine

$$\begin{aligned} \tilde{P}_t^*(k) &= (1 - \beta\rho) \\ & \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+i} + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \end{aligned}$$

- and

$$\tilde{P}_t \approx \rho\tilde{P}_{t-1} + (1-\rho)\tilde{P}_t^*(k)$$

- we get

$$\begin{aligned} \tilde{P}_t &\approx \rho\tilde{P}_{t-1} + (1-\rho)(1-\beta\rho) \\ & \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+i} + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \end{aligned}$$

Price equation: quasi differencing

- Quasi-differencing entails multiplying both sides of the equation by the polynomial in the lag operator $1 - \beta\rho L^{-1}$, where L stands for the lag operator
- Lag operator

$$LX_t = X_{t-1}$$

or

$$L^{-1}X_t = X_{t+1}$$

- Applying this polynomial to the left side of the equation gives

$$(1 - \beta\rho L^{-1})\tilde{P}_t = \tilde{P}_t - \beta\rho\tilde{P}_{t+1}$$

Price equation: quasi differencing

- applying it to the right side gives

$$\begin{aligned} & (1 - \beta\rho L^{-1}) \left(\rho\tilde{P}_{t-1} + (1 - \rho)(1 - \beta\rho) \right. \\ & \quad \left. \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+i} + (1 - \theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \right) \\ = & \rho\tilde{P}_{t-1} + (1 - \rho)(1 - \beta\rho) \\ & \quad \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+i} + (1 - \theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \\ & \quad - \beta\rho\rho\tilde{P}_t + \beta\rho(1 - \rho)(1 - \beta\rho) \\ & \quad \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\tilde{P}_{t+1+i} + (1 - \theta)\tilde{w}_{t+1+i} - \tilde{\lambda}_{t+1+i} + \theta\tilde{r}_{t+1+i} \right] \end{aligned}$$

Price equation: quasi differencing

- Removing the parts that cancel out gives and putting in both sides

$$\begin{aligned} \tilde{P}_t - \beta\rho E_t\tilde{P}_{t+1} & \approx \rho\tilde{P}_{t-1} - \beta\rho\rho\tilde{P}_t \\ & \quad + (1 - \rho)(1 - \beta\rho) \left[\tilde{P}_t + (1 - \theta)\tilde{w}_t - \tilde{\lambda}_t + \theta\tilde{r}_t \right] \end{aligned}$$

- Sometimes this is written as a Phillips curve:

$$\begin{aligned} \left[\tilde{P}_t - \tilde{P}_{t-1} \right] & \approx \beta \left[E_t\tilde{P}_{t+1} - \tilde{P}_t \right] \\ & \quad + \frac{(1 - \rho)(1 - \beta\rho)}{\rho} \left[(1 - \theta)\tilde{w}_t - \tilde{\lambda}_t + \theta\tilde{r}_t \right] \end{aligned}$$

Log-linear version of model

- FOCs of households

$$-\tilde{w}_t = \beta \bar{r} E_t \tilde{r}_{t+1} - E_t \tilde{w}_{t+1}$$

$$E_t \tilde{C}_{t+1} + E_t \tilde{P}_{t+1} = \tilde{w}_t + \tilde{P}_t$$

- The cash-in-advance and budget constraints become

$$(\tilde{P}_t + \tilde{C}_t) = (\tilde{g}_t + \tilde{M}_{t-1})$$

and

$$\bar{K} \tilde{K}_{t+1} + \frac{\bar{M}}{\bar{P}} (\tilde{M}_t - \tilde{P}_t) = \bar{Y} \tilde{Y}_t + (1 - \delta) \bar{K} \tilde{K}_t$$

- Combine the second first order condition and the cash in advance constraint to get

$$E_t [\tilde{g}_{t+1} + \tilde{M}_t] = \tilde{w}_t + \tilde{P}_t$$

- using the law of motion for \tilde{g}_{t+1} this can be written as

$$\pi \tilde{g}_t + \tilde{M}_t = \tilde{w}_t + \tilde{P}_t$$

Log-linear version of model

- Pricing equation

$$\begin{aligned} [\tilde{P}_t - \tilde{P}_{t-1}] &\approx \beta [E_t \tilde{P}_{t+1} - \tilde{P}_t] \\ &+ \frac{(1 - \rho)(1 - \beta\rho)}{\rho} [(1 - \theta) \tilde{w}_t - \tilde{\lambda}_t + \theta \tilde{r}_t] \end{aligned}$$

- The first order condition for the intermediate goods firms

$$\tilde{H}_t + \tilde{w}_t = \tilde{r}_t + \tilde{K}_t$$

- The aggregate production function

$$\tilde{Y}_t = \tilde{\lambda}_t + \theta \tilde{H}_t + (1 - \theta) \tilde{K}_t$$

- The process for the aggregate money stock i

$$\tilde{M}_t = \tilde{M}_{t-1} + \tilde{g}_t$$

- The two stochastic process equations

$$\tilde{\lambda}_t = \gamma \tilde{\lambda}_{t-1} + \varepsilon_t^\lambda \quad \text{and} \quad \tilde{g}_t = \pi \tilde{g}_{t-1} + \varepsilon_t^g$$

Log-linearization

- An interesting characteristic of the model after log-linearization
- Using the bundler technology of the final firm,

$$Y_t^{\frac{\psi-1}{\psi}} = \int_0^1 Y_t(k)^{\frac{\psi-1}{\psi}} dk,$$

- writing it in log-linear form gives

$$\begin{aligned} & \bar{Y}^{\frac{\psi-1}{\psi}} \left(1 + \frac{\psi-1}{\psi} \tilde{Y}_t \right) \\ \approx & \bar{Y}^{\frac{\psi-1}{\psi}} \left[\int_0^1 \left(1 + \frac{\psi-1}{\psi} \tilde{Y}_t(k) \right) dk \right], \end{aligned}$$

- or

$$\tilde{Y}_t \approx \left[\int_0^1 \tilde{Y}_t(k) dk \right].$$

Full model

$$\begin{aligned} 0 &= \tilde{w}_t + \beta \bar{r} E_t \tilde{r}_{t+1} - E_t \tilde{w}_{t+1}, \\ 0 &= \tilde{M}_t - \tilde{M}_{t-1} - \tilde{g}_t, \\ 0 &= \beta E_t \tilde{P}_{t+1} + \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho} \tilde{w}_t - \frac{(1-\rho)(1-\beta\rho)}{\rho} \tilde{\lambda}_t \\ & \quad + \frac{\theta(1-\rho)(1-\beta\rho)}{\rho} \tilde{r}_t - (1+\beta) \tilde{P}_t + \tilde{P}_{t-1}, \\ 0 &= \tilde{g}_t + \tilde{M}_{t-1} - \tilde{P}_t - \tilde{C}_t, \\ 0 &= \bar{Y} \tilde{Y}_t + (1-\delta) \bar{K} \tilde{K}_t - \bar{K} \tilde{K}_{t+1} - \frac{\bar{M}}{\bar{P}} \tilde{M}_t + \frac{\bar{M}}{\bar{P}} \tilde{P}_t, \\ 0 &= \tilde{w}_t + \tilde{P}_t - \tilde{M}_t - \pi \tilde{g}_t, \\ 0 &= \tilde{\lambda}_t + (1-\theta) \tilde{H}_t + \theta \tilde{K}_t, -\tilde{Y}_t, \\ 0 &= \tilde{H}_t + \tilde{w}_t - \tilde{K}_t - \tilde{r}_t. \end{aligned}$$

Solving the model

- Define

- $x_t = [\tilde{K}_{t+1}, \tilde{M}_t, \tilde{P}_t]'$

- $y_t = [\tilde{r}_t, \tilde{w}_t, \tilde{C}_t, \tilde{Y}_t, \tilde{H}_t]'$

- $z_t = [\tilde{\lambda}_t, \tilde{g}_t]'$,

- one can write the model as

$$\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}$$

- where

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & -1 \\ -\bar{K} & -\bar{C} & \bar{C} \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & B &= \begin{bmatrix} 0 & 1 & 0 \\ (1-\delta)\bar{K} & 0 & 0 \\ 0 & 0 & 0 \\ \theta & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \\
C &= \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \bar{Y} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1-\theta \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \\
D &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & -\pi \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \\
\text{widthheightequation* F} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \beta \end{bmatrix} \\
G &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -(1+\beta) \end{bmatrix} \\
H &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
J &= \begin{bmatrix} \bar{r} & -(\bar{r}+1-\delta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\text{widthheightequation* K} &= \begin{bmatrix} 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\theta(1-\rho)(1-\beta\rho)}{\rho} & \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho} & 0 & 0 & 0 \end{bmatrix} \\
L &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}$$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-(1-\rho)(1-\beta\rho)}{\rho} & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \gamma & 0 \\ 0 & \pi \end{bmatrix}$$

Solutions are

•

$$x_{t+1} = Px_t + Qz_t$$

and

$$y_t = Rx_t + Sz_t$$

ρ	.1			.3		
P	.931	.020	-.020	.889	.096	-.096
	0	1	0	0	1	0
	-.509	.989	.011	-.486	.948	.052
Q	.151	.055		.117	.147	
	0	1		0	1	
	-.501	1.433		-.483	1.383	
R	-1.088	.318	-.318	-1.735	1.507	-1.507
	.509	.011	-.011	.486	.052	-.052
	.509	.011	-.011	.486	.052	-.052
	-.022	.197	-.197	-.421	.931	-.931
	-.597	.308	-.308	-1.221	1.455	-1.455
S	1.735	.328		1.200	1.783	
	.501	.047		.483	.098	
	.501	-.433		.483	-.383	
	1.789	.180		1.459	1.079	
	1.233	.281		.717	1.685	

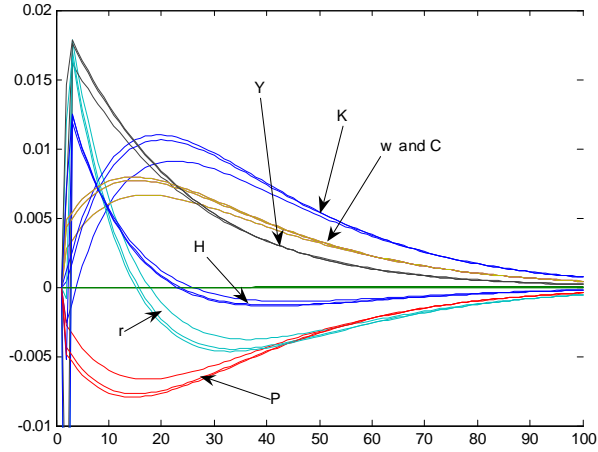


Figure 1: Responses of the model's variables to a technology shock: $\rho = .3, .5, .75$

ρ	.5			.75		
P	.792	.276	-.276	.446	.915	-.915
	0	1	0	0	1	0
	-.433	.849	.151	-.244	.500	.500
Q	.035	.372		-.258	1.216	
	0	1		0	1	
	-.438	1.260		-.278	.799	
R	-3.283	4.355	-4.355	-8.738	14.427	-14.427
	.433	.151	-.151	.244	.499	-.499
	.433	.151	-.151	.244	.499	-.499
	-1.378	2.691	-2.691	-4.748	8.913	-8.913
	-2.716	4.204	-4.204	-7.982	13.927	-13.927
S	-.087	5.323		-4.718	18.629	
	.438	.220		.278	.681	
	.438	-.260		.278	.201	
	.664	3.266		-2.197	11.487	
	-.525	5.102		-4.996	17.948	

Impulse response functions: technology shock

Impulse response functions: technology shock

Impulse response functions: technology shock

Impulse response functions: money growth shock

Impulse response functions: money growth shock

Impulse response functions: money growth shock

Inflation adjustment for non-optimizing firms

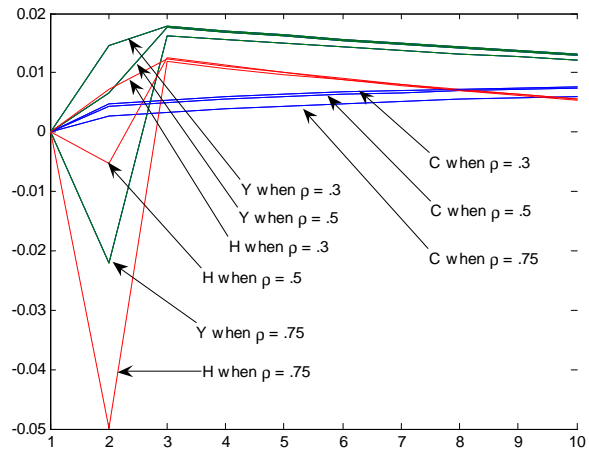


Figure 2: Responses of \tilde{C} , \tilde{Y} , and \tilde{H} to a technology shock

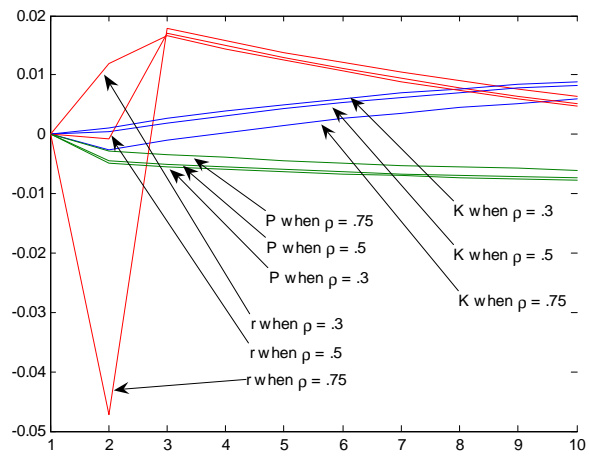


Figure 3: Responses of \tilde{r} , \tilde{K} , and \tilde{P} to a technology shock

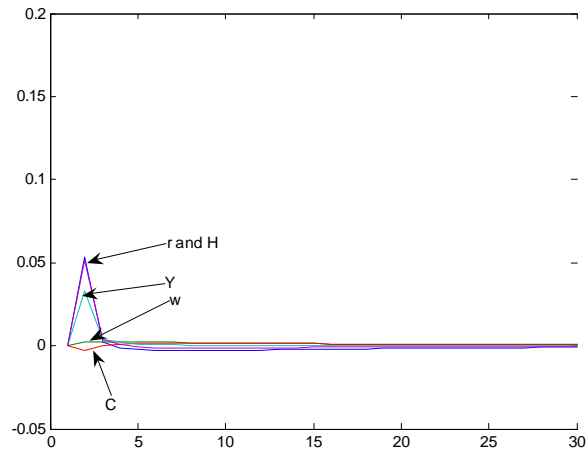


Figure 4: Response to a .01 money growth shock for economy where $\rho = .5$

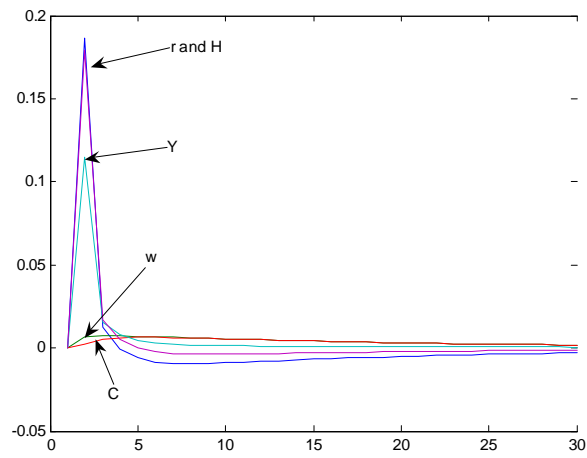


Figure 5: Response to a .01 money growth shock for economy where $\rho = .75$

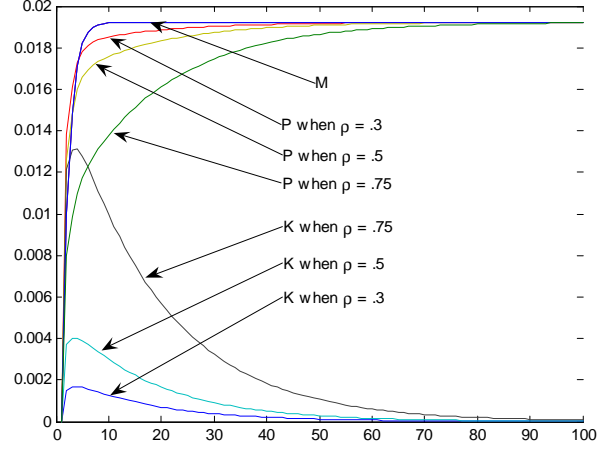


Figure 6: Responses of money, prices, and capital to a money growth shock for economies with $\rho = .3, .5$, and $.75$

- New rule of thumb

$$P_t(k) = \frac{P_{t-1}}{P_{t-2}} P_{t-1}(k)$$

- Price updating rule of

$$P_t^{1-\psi} = \rho \left(\frac{P_{t-1}}{P_{t-2}} P_{t-1} \right)^{1-\psi} + (1-\rho) P_t^*(k)^{1-\psi}$$

Inflation adjustment for non-optimizing firms

- Intermediate goods firms max

$$\begin{aligned} \max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta\rho)^i & \left[\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k) Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^\psi \right. \\ & \left. - P_{t+i} \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^\psi \right] \end{aligned}$$

- or

$$\begin{aligned} \max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta\rho)^i & Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^\psi \\ & \times \left[\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k) - \frac{P_{t+i} w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta \right] \end{aligned}$$

Inflation adjustment for non-optimizing firms

- First order conditions are

$$0 = E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k) \times \left[1 - \psi + \frac{\psi P_{t-1} P_{t+i} w_{t+i}}{P_{t+i-1} P_t^*(k) (1-\theta) \lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta \right]$$

- which gives the pricing rule

$$P_t^*(k) = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i \frac{P_{t-1} P_{t+i}}{P_{t+i-1}} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k)}$$

Inflation adjustment for non-optimizing firms: Stationary states

- From the first order conditions of the households get (because $\bar{g} \neq 1$)

$$\bar{C} = -\frac{\beta\bar{w}}{\bar{g}B}$$

- That results in

$$-\frac{\beta\bar{w}}{\bar{g}B} = \bar{w} \left[\frac{\bar{r}(1-\theta)}{\bar{w}\theta} \right]^\theta \bar{Y} + \frac{\bar{Y}}{\psi} + (\bar{r} - \delta) \left[\frac{\bar{r}(1-\theta)}{\bar{w}\theta} \right]^{\theta-1} \bar{Y}$$

or

$$\bar{Y} = \frac{-\beta\bar{w}}{\bar{g}B \left(\bar{w} \left[\frac{\bar{r}(1-\theta)}{\bar{w}\theta} \right]^\theta + \frac{1}{\psi} + (\bar{r} - \delta) \left[\frac{\bar{r}(1-\theta)}{\bar{w}\theta} \right]^{\theta-1} \right)}$$

Inflation adjustment for non-optimizing firms: Stationary states

Variable	\bar{r}	\bar{w}	\bar{Y}	\bar{H}	\bar{K}	$\bar{\xi}$	\bar{C}
SS value	.0351	2.0426	$\frac{1.0218}{\bar{g}}$	$\frac{.2901}{\bar{g}}$	$\frac{9.5271}{\bar{g}}$	$\frac{.0929}{\bar{g}}$	$\frac{.7836}{\bar{g}}$

Inflation adjustment for non-optimizing firms: Log-linearization

- Aggregate price rule

$$\bar{P}^{1-\psi} e^{(1-\psi)\tilde{P}_t} = \rho \bar{P}^{1-\psi} e^{(1-\psi)(2\tilde{P}_{t-1} - \tilde{P}_{t-2})} + (1-\rho) \bar{P}^{1-\psi} e^{(1-\psi)\tilde{P}_t^*(k)}$$

or

$$\tilde{P}_t = \rho \left(2\tilde{P}_{t-1} - \tilde{P}_{t-2} \right) + (1-\rho) \tilde{P}_t^*(k)$$

- Price fixing rule for intermediate goods firms

$$\begin{aligned} & \bar{P}^*(k)e^{\tilde{P}_t^*(k)} \\ = & \frac{\psi}{\psi-1} \bar{P} \frac{\bar{w}}{(1-\theta)\lambda} \left[\frac{\bar{r}(1-\theta)}{\bar{w}\theta} \right]^\theta \\ & \times \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i e^{\tilde{P}_{t-1} + \tilde{P}_{t+i} - \tilde{P}_{t+i-1} + \tilde{Y}_{t+i}(k) + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i}}}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i e^{\tilde{Y}_{t+i}(k)}} \end{aligned}$$

Inflation adjustment for non-optimizing firms: Log-linearization

- Simplifies to

$$\begin{aligned} \tilde{P}_t^*(k) &= (1-\beta\rho) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \\ & \times \left[\tilde{P}_{t-1} + \tilde{P}_{t+i} - \tilde{P}_{t+i-1} + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right] \end{aligned}$$

- Combining gives

$$\begin{aligned} \tilde{P}_t &= \rho \left(2\tilde{P}_{t-1} - \tilde{P}_{t-2} \right) \\ & + (1-\rho)(1-\beta\rho) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \\ & \times \left[\tilde{P}_{t-1} + \tilde{P}_{t+i} - \tilde{P}_{t+i-1} + (1-\theta)\tilde{w}_{t+i} - \tilde{\lambda}_{t+i} + \theta\tilde{r}_{t+i} \right]. \end{aligned}$$

Inflation adjustment for non-optimizing firms: Log-linearization

- After quasi differencing, this becomes

$$\begin{aligned} & (1+2\beta)\tilde{P}_t - \beta E_t \tilde{P}_{t+1} - (2+\beta)\tilde{P}_{t-1} + \tilde{P}_{t-2} \\ = & \frac{(1-\rho)(1-\beta\rho)}{\rho} \left[(1-\theta)\tilde{w}_t - \tilde{\lambda}_t + \theta\tilde{r}_t \right] \end{aligned}$$

- Notice the \tilde{P}_{t-2}

Inflation adjustment for non-optimizing firms: Solving the model

- Managing \tilde{P}_{t-2}

- write model as

$$\begin{aligned} 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t \\ 0 &= E_t [Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t] \\ z_{t+1} &= Nz_t + \varepsilon_{t+1} \end{aligned}$$

- where

$$x_t = [K_{t+1}, M_t, P_t, P_{t-1}]'$$

- With this definition of x_t , x_{t-1} is now

$$x_{t-1} = [K_t, M_{t-1}, P_{t-1}, P_{t-2}]'$$

- Matrices for the expectational variables

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 - 2\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 + \beta & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} \bar{r} & -(\bar{r} + 1 - \delta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\theta(1-\rho)(1-\beta\rho)}{\rho} & \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-(1-\rho)(1-\beta\rho)}{\rho} & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices A to D are

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -\bar{K} & -\bar{C} & \bar{C} & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (1-\delta)\bar{K} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \bar{Y} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1-\theta \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & -\pi \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- The solution is of the form

$$x_{t+1} = Px_t + Qz_t$$

and

$$y_t = Rx_t + Sz_t$$

- where, for $\rho = .75$,

$$P = \begin{bmatrix} 0.3450 & 1.1358 & -1.8440 & 0.7082 \\ 0 & 1 & 0 & 0 \\ -0.1881 & 0.3788 & 1.0073 & -0.3862 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} -0.3926 & 1.5977 \\ 0 & 1 \\ -0.2040 & 0.5893 \\ 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} -10.3266 & 17.9123 & -29.0784 & 11.1662 \\ 0.1881 & 0.6212 & -1.0073 & 0.3862 \\ 0.1881 & 0.6212 & -1.0073 & 0.3862 \\ -5.7294 & 11.0663 & -17.9655 & 6.8992 \\ -9.5147 & 17.2911 & -28.0711 & 10.7800 \end{bmatrix}$$

$$S = \begin{bmatrix} -6.8340 & 24.6587 \\ 0.2040 & 0.8907 \\ 0.2040 & 0.4107 \\ -3.5044 & 15.2115 \\ -7.0381 & 23.7679 \end{bmatrix}$$

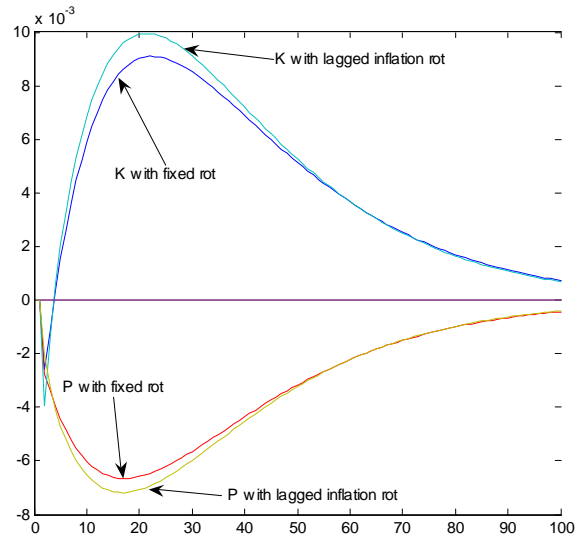


Figure 7: Response of \tilde{P} and \tilde{K} to a technology shock for two rules of thumb

- Impulse response functions: Technology shock
- Impulse response functions: Technology shock
- Impulse response functions: Technology shock
- Impulse response functions: Money growth shock
- Impulse response functions: Money growth shock (fixed)
- Impulse response functions: Money growth shock (lagged)

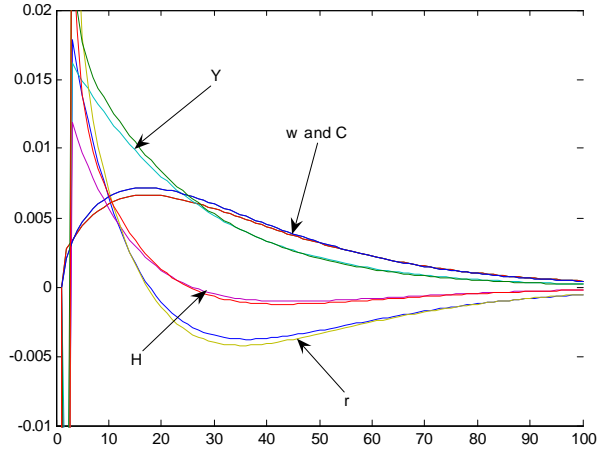


Figure 8: Response of \tilde{Y} , \tilde{C} , \tilde{H} , \tilde{r} and \tilde{w} to a technology shock for two rules of thumb

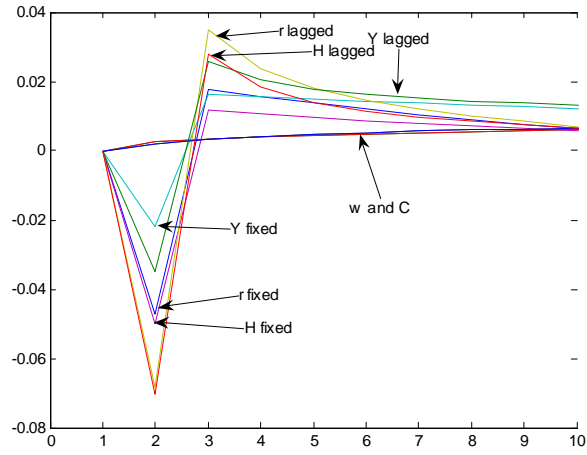


Figure 9: Response in the first 10 periods of \tilde{Y} , \tilde{C} , \tilde{H} , \tilde{r} and \tilde{w} to a technology shock for two rules of thumb

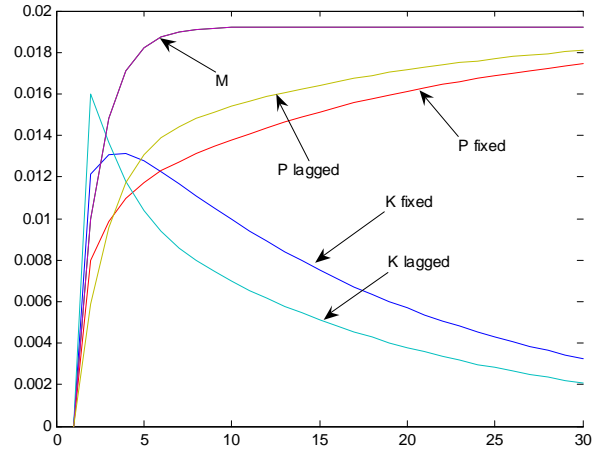


Figure 10: Response of \tilde{P} and \tilde{K} to a money growth shock for two rules of thumb

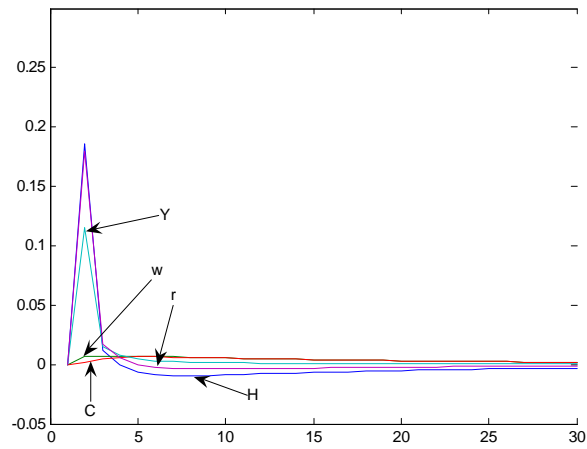


Figure 11: Response of \tilde{Y} , \tilde{C} , \tilde{H} , \tilde{r} and \tilde{w} to a money growth shock for rules of thumb with prices constant

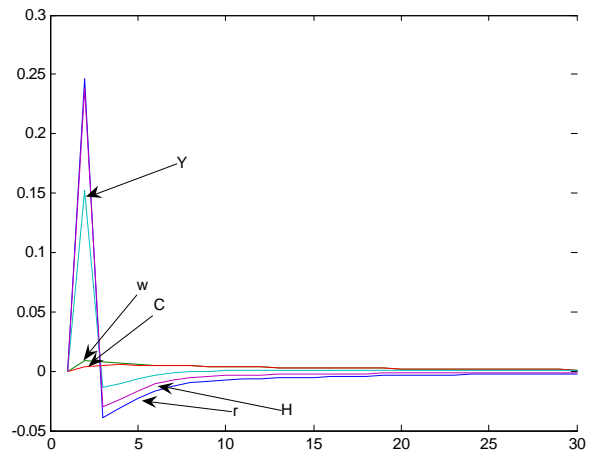


Figure 12: Response of \tilde{Y} , \tilde{C} , \tilde{H} , \tilde{r} and \tilde{w} to a money growth shock for rules of thumb with lagged inflation