Macroeconomics II
Staggered pricing: part 2

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1 Staggered pricing: Continued

Log-linearization: Final goods pricing rule

- Equation
$$P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1-\rho) P_t^*(k)^{1-\psi}$$
- Using Uhlig's method for log-linearization gives
$$\overline{P} e^{(1-\psi)\widetilde{P}_t} = \rho \overline{P} e^{(1-\psi)\widetilde{P}_{t-1}} + (1-\rho)\overline{P} e^{(1-\psi)\widetilde{P}_t^*(k)}$$
since all firms have the same price, $\overline{P}^*(k)$ and applying the price bundler results in $\overline{P} = \overline{P}^*(k)$.
- Using the standard approximation rule gives
$$\begin{aligned} 1 + (1-\psi)\widetilde{P}_t &\approx \rho\left(1 + (1-\psi)\widetilde{P}_{t-1}\right) \\ &\quad + (1-\rho)\left(1 + (1-\psi)\widetilde{P}_t^*(k)\right) \end{aligned}$$
or
$$\widetilde{P}_t \approx \rho \widetilde{P}_{t-1} + (1-\rho)\widetilde{P}_t^*(k)$$

Log-linearization: intermediate goods firms

- Begin with pricing rule
$$\begin{aligned} & P_t^*(k) \\ = \quad & \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}}{w_{t+i}}\right]^{\theta}}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k)} \end{aligned}$$
.

- Multiplying out the denominator gives

$$
\begin{aligned}
& P_t^*(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k) \\
= \quad & \frac{\psi}{\psi-1} E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}}{w_{t+i}}\right]^{\theta}
\end{aligned}
$$

Log-linearization: intermediate goods firms

- Right hand side gives

$$
\begin{aligned}
& E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_t^*(k) Y_{t+i}(k) \\
= \quad & E_t \sum_{i=0}^{\infty} (\beta\rho)^i \overline{P}^*(k) \overline{Y}(k) e^{\widetilde{P}_t^*(k) + \widetilde{Y}_{t+i}(k)} \\
\approx \quad & \overline{P}^*(k) \overline{Y}(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left(1 + \widetilde{P}_t^*(k) + \widetilde{Y}_{t+i}(k)\right) \\
= \quad & \frac{\overline{P}^*(k) \overline{Y}(k)}{1-\beta\rho} \left(1 + \widetilde{P}_t^*(k)\right) \\
& + \overline{P}^*(k) \overline{Y}(k) E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left(\widetilde{Y}_{t+i}(k)\right)
\end{aligned}
$$

Log-linearization: intermediate goods firms

- Left hand side gives

$$
\begin{aligned}
& \frac{\psi}{\psi-1} E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta} \\
= \quad & \frac{\psi}{\psi-1} \overline{PY}(k) \frac{\overline{w}}{(1-\theta)\overline{\lambda}} \left[\frac{(1-\theta)\overline{r}}{\theta\overline{w}}\right]^{\theta} \\
& \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i e^{\widetilde{P}_{t+i} + \widetilde{Y}_{t+i}(k) + (1-\theta)\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}} \\
\approx \quad & \frac{\psi}{\psi-1} \overline{PY}(k) \frac{\overline{w}}{(1-\theta)\overline{\lambda}} \left[\frac{(1-\theta)\overline{r}}{\theta\overline{w}}\right]^{\theta} \\
& \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \Big[1 + \widetilde{P}_{t+i} + \widetilde{Y}_{t+i}(k) \\
& + (1-\theta)\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\Big]
\end{aligned}
$$

Log-linearization: intermediate goods firms

- In the stationary state

$$\frac{\psi}{\psi - 1} = \frac{1}{\frac{\overline{w}}{(1-\theta)\overline{\lambda}}\left[\frac{(1-\theta)\overline{r}}{\theta\overline{w}}\right]^{\theta}}$$

- The right side becomes simply

$$\begin{aligned} \approx \quad & \overline{PY}\,(k)\,E_t \sum_{i=0}^{\infty} (\beta\rho)^i \Big[1 + \widetilde{P}_{t+i} + \widetilde{Y}_{t+i}\,(k) \\ & + (1-\theta)\,\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\Big] \end{aligned}$$

- We have shown that $\overline{P} = \overline{P}^*$, so stationary state prices and $\overline{Y}(k)$ cancel out on both sides of the equation. Both sides contain $E_t \sum_{i=0}^{\infty} (\beta\rho)^i \widetilde{Y}_{t+i}\,(k)$

Log-linearization: intermediate goods firms

- the log-linear pricing rule for the intermediate goods firm is therefore

$$\begin{aligned} & \widetilde{P}_t^*(k) \\ = \quad & (1-\beta\rho)\,E_t \sum_{i=0}^{\infty} (\beta\rho)^i \Big[\widetilde{P}_{t+i} + (1-\theta)\,\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\Big] \end{aligned}$$

Price equation

- Combine

$$\begin{aligned} \widetilde{P}_t^*(k) \quad = \quad & (1-\beta\rho) \\ & \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \Big[\widetilde{P}_{t+i} + (1-\theta)\,\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\Big] \end{aligned}$$

- and

$$\widetilde{P}_t \approx \rho\widetilde{P}_{t-1} + (1-\rho)\widetilde{P}_t^*(k)$$

- we get

$$\begin{aligned} \widetilde{P}_t \quad \approx \quad & \rho\widetilde{P}_{t-1} + (1-\rho)\,(1-\beta\rho) \\ & \times E_t \sum_{i=0}^{\infty} (\beta\rho)^i \Big[\widetilde{P}_{t+i} + (1-\theta)\,\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\Big] \end{aligned}$$

Price equation: quasi differencing

- Quasi-differencing entails multiplying both sides of the equation by the polynomial in the lag operator $1-\beta\rho L^{-1}$, where L stands for the lag operator
- Lag operator
$$LX_t = X_{t-1}$$
or
$$L^{-1}X_t = X_{t+1}$$
- Applying this polynomial to the left side of the equation gives
$$\left(1-\beta\rho L^{-1}\right)\widetilde{P}_t = \widetilde{P}_t - \beta\rho\widetilde{P}_{t+1}$$

Price equation: quasi differencing

- applying it to the right side gives
$$\begin{aligned} & \left(1-\beta\rho L^{-1}\right)\Big(\rho\widetilde{P}_{t-1} + (1-\rho)(1-\beta\rho) \\ & \times E_t\sum_{i=0}^{\infty}(\beta\rho)^i\left[\widetilde{P}_{t+i} + (1-\theta)\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\right]\Big) \\ = \; & \rho\widetilde{P}_{t-1} + (1-\rho)(1-\beta\rho) \\ & \times E_t\sum_{i=0}^{\infty}(\beta\rho)^i\left[\widetilde{P}_{t+i} + (1-\theta)\widetilde{w}_{t+i} - \widetilde{\lambda}_{t+i} + \theta\widetilde{r}_{t+i}\right] \\ & -\beta\rho\rho\widetilde{P}_t + \beta\rho(1-\rho)(1-\beta\rho) \\ & \times E_t\sum_{i=0}^{\infty}(\beta\rho)^i\left[\widetilde{P}_{t+1+i} + (1-\theta)\widetilde{w}_{t+1+i} - \widetilde{\lambda}_{t+1+i} + \theta\widetilde{r}_{t+1+i}\right] \end{aligned}$$

Price equation: quasi differencing

- Removing the parts that cancel out gives and putting in both sides
$$\begin{aligned} \widetilde{P}_t - \beta\rho E_t\widetilde{P}_{t+1} \;\approx\; & \rho\widetilde{P}_{t-1} - \beta\rho\rho\widetilde{P}_t \\ & +(1-\rho)(1-\beta\rho)\left[\widetilde{P}_t + (1-\theta)\widetilde{w}_t - \widetilde{\lambda}_t + \theta\widetilde{r}_t\right] \end{aligned}$$
- Sometimes this is written as a Phillips curve:
$$\begin{aligned} \left[\widetilde{P}_t - \widetilde{P}_{t-1}\right] \;\approx\; & \beta\left[E_t\widetilde{P}_{t+1} - \widetilde{P}_t\right] \\ & +\frac{(1-\rho)(1-\beta\rho)}{\rho}\left[(1-\theta)\widetilde{w}_t - \widetilde{\lambda}_t + \theta\widetilde{r}_t\right] \end{aligned}$$

Log-linear version of model

- FOCs of households

$$-\widetilde{w}_t = \beta\overline{r}E_t\widetilde{r}_{t+1} - E_t\widetilde{w}_{t+1}$$

$$E_t\widetilde{C}_{t+1} + E_t\widetilde{P}_{t+1} = \widetilde{w}_t + \widetilde{P}_t$$

- The cash-in-advance and budget constraints become

$$(\widetilde{P}_t + \widetilde{C}_t) = (\widetilde{g}_t + \widetilde{M}_{t-1})$$

and

$$\overline{K}\widetilde{K}_{t+1} + \frac{\overline{M}}{\overline{P}}(\widetilde{M}_t - \widetilde{P}_t) = \overline{Y}\widetilde{Y}_t + (1-\delta)\overline{K}\widetilde{K}_t$$

- Combine the second first order condition and the cash in advance constraint to get

$$E_t\left[\widetilde{g}_{t+1} + \widetilde{M}_t\right] = \widetilde{w}_t + \widetilde{P}_t$$

- using the law of motion for $\widetilde{g}_{t+1}$ this can be written as

$$\pi\widetilde{g}_t + \widetilde{M}_t = \widetilde{w}_t + \widetilde{P}_t$$

Log-linear version of model

- Pricing equation

$$\begin{aligned}\left[\widetilde{P}_t - \widetilde{P}_{t-1}\right] \quad &\approx \quad \beta\left[E_t\widetilde{P}_{t+1} - \widetilde{P}_t\right] \\ &+\frac{(1-\rho)(1-\beta\rho)}{\rho}\left[(1-\theta)\widetilde{w}_t - \widetilde{\lambda}_t + \theta\widetilde{r}_t\right]\end{aligned}$$

- The first order condition for the intermediate goods firms

$$\widetilde{H}_t + \widetilde{w}_t = \widetilde{r}_t + \widetilde{K}_t$$

- The aggregate production function

$$\widetilde{Y}_t = \widetilde{\lambda}_t + \theta\widetilde{H}_t + (1-\theta)\widetilde{K}_t$$

- The process for the aggregate money stock i

$$\widetilde{M}_t = \widetilde{M}_{t-1} + \widetilde{g}_t$$

- The two stochastic process equations

$$\widetilde{\lambda}_t = \gamma\widetilde{\lambda}_{t-1} + \varepsilon_t^{\lambda} \text{ and } \widetilde{g}_t = \pi\widetilde{g}_{t-1} + \varepsilon_t^{g}$$

Log-linearization

- An interesting characteristic of the model after log-linearization
- Using the bundler technology of the final firm,

$$Y_t^{\frac{\psi-1}{\psi}} = \int_0^1 Y_t(k)^{\frac{\psi-1}{\psi}} dk,$$

- writing it in log-linear form gives

$$\begin{aligned} & \overline{Y}^{\frac{\psi-1}{\psi}} \left(1 + \frac{\psi-1}{\psi}\widetilde{Y}_t\right) \\ \approx \ & \overline{Y}^{\frac{\psi-1}{\psi}} \left[\int_0^1 \left(1 + \frac{\psi-1}{\psi}\widetilde{Y}_t(k)\right) dk\right], \end{aligned}$$

- or

$$\widetilde{Y}_t \approx \left[\int_0^1 \widetilde{Y}_t(k) dk\right].$$

Full model

$$\begin{aligned} 0 &= \widetilde{w}_t + \beta\overline{r}E_t\widetilde{r}_{t+1} - E_t\widetilde{w}_{t+1}, \\ 0 &= \widetilde{M}_t - \widetilde{M}_{t-1} - \widetilde{g}_t, \\ 0 &= \beta E_t\widetilde{P}_{t+1} + \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho}\widetilde{w}_t - \frac{(1-\rho)(1-\beta\rho)}{\rho}\widetilde{\lambda}_t \\ &\quad + \frac{\theta(1-\rho)(1-\beta\rho)}{\rho}\widetilde{r}_t - (1+\beta)\widetilde{P}_t + \widetilde{P}_{t-1}, \\ 0 &= \widetilde{g}_t + \widetilde{M}_{t-1} - \widetilde{P}_t - \widetilde{C}_t, \\ 0 &= \overline{Y}\widetilde{Y}_t + (1-\delta)\overline{K}\widetilde{K}_t - \overline{K}\widetilde{K}_{t+1} - \frac{\overline{M}}{\overline{P}}\widetilde{M}_t + \frac{\overline{M}}{\overline{P}}\widetilde{P}_t, \\ 0 &= \widetilde{w}_t + \widetilde{P}_t - \widetilde{M}_t - \pi\widetilde{g}_t, \\ 0 &= \widetilde{\lambda}_t + (1-\theta)\widetilde{H}_t + \theta\widetilde{K}_t, -\widetilde{Y}_t, \\ 0 &= \widetilde{H}_t + \widetilde{w}_t - \widetilde{K}_t - \widetilde{r}_t. \end{aligned}$$

Solving the model

- Define
- $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]'$
- $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{C}_t, \widetilde{Y}_t, \widetilde{H}_t\right]'$
- $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]'$,

- one can write the model as

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}
$$

.

- where

$$
A = \begin{bmatrix} 0 & 0 & -1 \\ -\overline{K} & -\overline{C} & \overline{C} \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ (1-\delta)\,\overline{K} & 0 & 0 \\ 0 & 0 & 0 \\ \theta & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}
$$

$$
C = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \overline{Y} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1-\theta \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix}
$$

$$
D = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & -\pi \\ 1 & 0 \\ 0 & 0 \end{bmatrix}
$$

widthheightequation* F= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \beta \end{bmatrix}$

$$
G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -(1+\beta) \end{bmatrix}
$$

$$
H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$
J = \begin{bmatrix} \overline{r} & -(\overline{r}+1-\delta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

widthheightequation* K= $\begin{bmatrix} 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\theta(1-\rho)(1-\beta\rho)}{\rho} & \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho} & 0 & 0 & 0 \end{bmatrix}$

$$
L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-(1-\rho)(1-\beta\rho)}{\rho} & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \gamma & 0 \\ 0 & \pi \end{bmatrix}$$

Solutions are

- $$x_{t+1} = Px_t + Qz_t$$

  and

  $$y_t = Rx_t + Sz_t$$

ρ	.1	.3
P	$\begin{bmatrix} .931 & .020 & -.020 \\ 0 & 1 & 0 \\ -.509 & .989 & .011 \end{bmatrix}$	$\begin{bmatrix} .889 & .096 & -.096 \\ 0 & 1 & 0 \\ -.486 & .948 & .052 \end{bmatrix}$
Q	$\begin{bmatrix} .151 & .055 \\ 0 & 1 \\ -.501 & 1.433 \end{bmatrix}$	$\begin{bmatrix} .117 & .147 \\ 0 & 1 \\ -.483 & 1.383 \end{bmatrix}$
R	$\begin{bmatrix} -1.088 & .318 & -.318 \\ .509 & .011 & -.011 \\ .509 & .011 & -.011 \\ -.022 & .197 & -.197 \\ -.597 & .308 & -.308 \end{bmatrix}$	$\begin{bmatrix} -1.735 & 1.507 & -1.507 \\ .486 & .052 & -.052 \\ .486 & .052 & -.052 \\ -.421 & .931 & -.931 \\ -1.221 & 1.455 & -1.455 \end{bmatrix}$
S	$\begin{bmatrix} 1.735 & .328 \\ .501 & .047 \\ .501 & -.433 \\ 1.789 & .180 \\ 1.233 & .281 \end{bmatrix}$	$\begin{bmatrix} 1.200 & 1.783 \\ .483 & .098 \\ .483 & -.383 \\ 1.459 & 1.079 \\ .717 & 1.685 \end{bmatrix}$

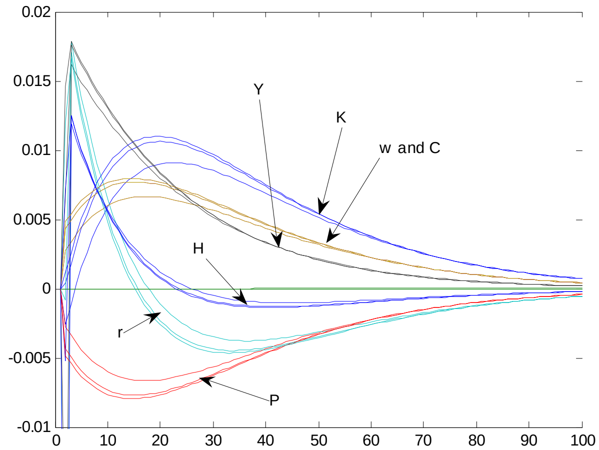

Figure 1: Responses of the model's variables to a technology shock: $\rho = .3, .5, .75$

ρ	.5	.75
P	$\begin{bmatrix} .792 & .276 & -.276 \\ 0 & 1 & 0 \\ -.433 & .849 & .151 \end{bmatrix}$	$\begin{bmatrix} .446 & .915 & -.915 \\ 0 & 1 & 0 \\ -.244 & .500 & .500 \end{bmatrix}$
Q	$\begin{bmatrix} .035 & .372 \\ 0 & 1 \\ -.438 & 1.260 \end{bmatrix}$	$\begin{bmatrix} -.258 & 1.216 \\ 0 & 1 \\ -.278 & .799 \end{bmatrix}$
R	$\begin{bmatrix} -3.283 & 4.355 & -4.355 \\ .433 & .151 & -.151 \\ .433 & .151 & -.151 \\ -1.378 & 2.691 & -2.691 \\ -2.716 & 4.204 & -4.204 \end{bmatrix}$	$\begin{bmatrix} -8.738 & 14.427 & -14.427 \\ .244 & .499 & -.499 \\ .244 & .499 & -.499 \\ -4.748 & 8.913 & -8.913 \\ -7.982 & 13.927 & -13.927 \end{bmatrix}$
S	$\begin{bmatrix} -.087 & 5.323 \\ .438 & .220 \\ .438 & -.260 \\ .664 & 3.266 \\ -.525 & 5.102 \end{bmatrix}$	$\begin{bmatrix} -4.718 & 18.629 \\ .278 & .681 \\ .278 & .201 \\ -2.197 & 11.487 \\ -4.996 & 17.948 \end{bmatrix}$

Inpulse response functions: technology shock
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Inpulse response functions: technology shock
Inpulse response functions: money growth shock
Inpulse response functions: money growth shock
Inpulse response functions: money growth shock
Inflation adjustment for non-optimizing firms

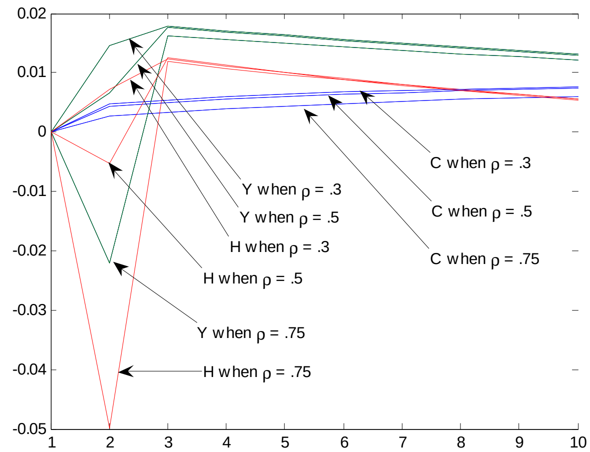

Figure 2: Responses of $\widetilde{C}$, $\widetilde{Y}$, and $\widetilde{H}$ to a technology shock

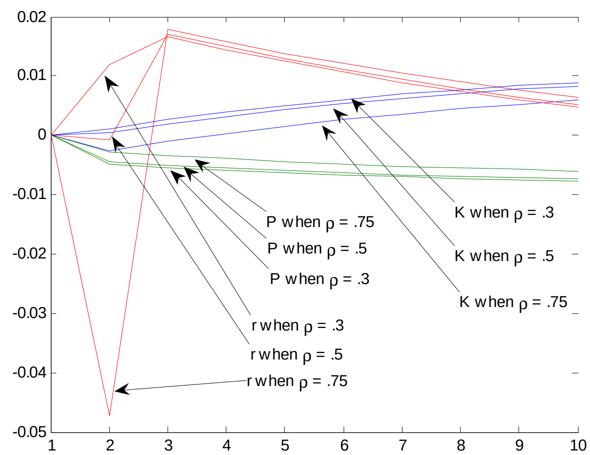

Figure 3: Responses of $\widetilde{r}$, $\widetilde{K}$, and $\widetilde{P}$ to a technology shock

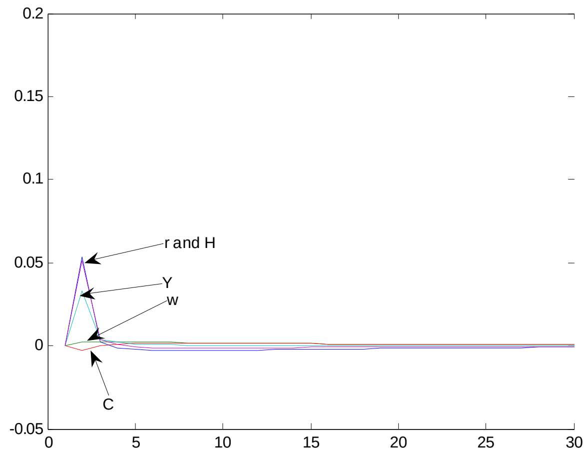

Figure 4: Response to a .01 money growth shock for economy where $\rho = .5$

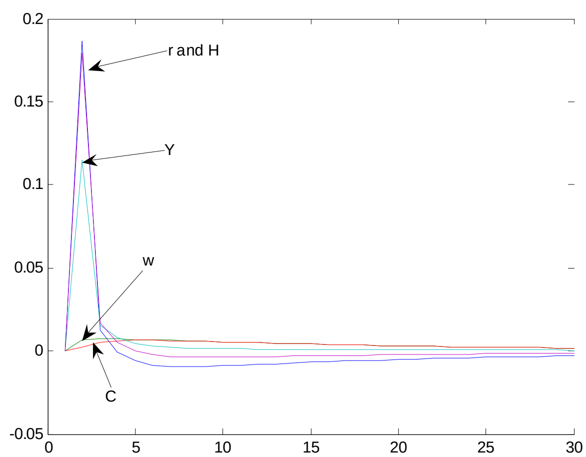

Figure 5: Response to a .01 money growth shock for economy where $\rho = .75$

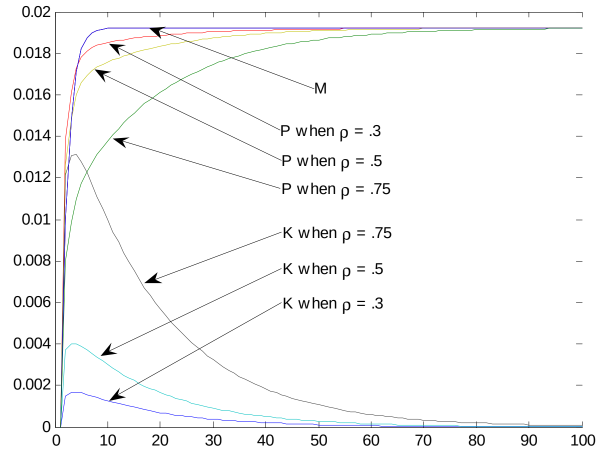

Figure 6: Responses of money, prices, and capital to a money growth shock for economies with $\rho = .3, .5$, and $.75$

- New rule of thumb

$$P_t(k) = \frac{P_{t-1}}{P_{t-2}} P_{t-1}(k)$$

- Price updating rule of

$$P_t^{1-\psi} = \rho \left(\frac{P_{t-1}}{P_{t-2}} P_{t-1} \right)^{1-\psi} + (1-\rho) P_t^*(k)^{1-\psi}$$

Inflation adjustment for non-optimizing firms

- Intermediate goods firms max

$$\max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta\rho)^i \left[\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k) Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^{\psi} \right.$$

$$\left. - P_{t+i} \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta} Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^{\psi} \right]$$

- or

$$\max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i} \left(\frac{P_{t+i}}{\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k)} \right)^{\psi}$$

$$\times \left[\frac{P_{t+i-1}}{P_{t-1}} P_t^*(k) - \frac{P_{t+i} w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta} \right]$$

Inflation adjustment for non-optimizing firms

- First order conditions are

$$
\begin{aligned}
0 &= E_t \sum_{i=0}^{\infty} (\beta\rho)^i \, Y_{t+i}(k) \\
&\times \left[1 - \psi + \frac{\psi P_{t-1} P_{t+i} w_{t+i}}{P_{t+i-1} P_t^*(k)(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta} \right]
\end{aligned}
$$

- which gives the pricing rule

$$
\begin{aligned}
& P_t^*(k) \\
= & \frac{\psi}{\psi-1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i \frac{P_{t-1}P_{t+i}}{P_{t+i-1}} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta}}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i \, Y_{t+i}(k)}
\end{aligned}
$$

Inflation adjustment for non-optimizing firms: Stationary states

- From the first order conditions of the households get (because $\overline{g} \neq 1$)

$$
\overline{C} = -\frac{\beta\overline{w}}{\overline{g}B}
$$

- That results in

$$
\begin{aligned}
& -\frac{\beta\overline{w}}{\overline{g}B} \\
= & \ \overline{w} \left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta} \right]^{\theta} \overline{Y} + \frac{\overline{Y}}{\psi} + (\overline{r} - \delta) \left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta} \right]^{\theta-1} \overline{Y}
\end{aligned}
$$

or

$$
\overline{Y} = \frac{-\beta\overline{w}}{\overline{g}B \left(\overline{w} \left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta} \right]^{\theta} + \frac{1}{\psi} + (\overline{r} - \delta) \left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta} \right]^{\theta-1} \right)}
$$

Inflation adjustment for non-optimizing firms: Stationary states

Variable	$\overline{r}$	$\overline{w}$	$\overline{Y}$	$\overline{H}$	$\overline{K}$	$\overline{\xi}$	$\overline{C}$
SS value	.0351	2.0426	$\frac{1.0218}{\overline{g}}$	$\frac{.2901}{\overline{g}}$	$\frac{9.5271}{\overline{g}}$	$\frac{.0929}{\overline{g}}$	$\frac{.7836}{\overline{g}}$

Inflation adjustment for non-optimizing firms: Log-linearization

- Aggregate price rule

$$
\begin{aligned}
\overline{P}^{1-\psi} e^{(1-\psi)\widetilde{P}_t} &= \rho\overline{P}^{1-\psi} e^{(1-\psi)\left(2\widetilde{P}_{t-1} - \widetilde{P}_{t-2}\right)} \\
&\quad + (1-\rho)\overline{P}^{1-\psi} e^{(1-\psi)\widetilde{P}_t^*(k)}
\end{aligned}
$$

or

$$
\widetilde{P}_t = \rho \left(2\widetilde{P}_{t-1} - \widetilde{P}_{t-2} \right) + (1-\rho)\widetilde{P}_t^*(k)
$$

- Price fixing rule for intermediate goods firms

$$
\begin{aligned}
& \overline{P}^*(k)e^{\widetilde{P}_t^*(k)} \\
= \quad & \frac{\psi}{\psi-1}\overline{P}\frac{\overline{w}}{(1-\theta)\lambda}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta} \\
& \times \frac{E_t\sum_{i=0}^{\infty}(\beta\rho)^i\, e^{\widetilde{P}_{t-1}+\widetilde{P}_{t+i}-\widetilde{P}_{t+i-1}+\widetilde{Y}_{t+i}(k)+(1-\theta)\widetilde{w}_{t+i}-\widetilde{\lambda}_{t+i}+\theta\widetilde{r}_{t+i}}}{E_t\sum_{i=0}^{\infty}(\beta\rho)^i\, e^{\widetilde{Y}_{t+i}(k)}}
\end{aligned}
$$

Inflation adjustment for non-optimizing firms: Log-linearization

- Simplifies to

$$
\begin{aligned}
\widetilde{P}_t^*(k) &= (1-\beta\rho)\, E_t\sum_{i=0}^{\infty}(\beta\rho)^i \\
&\quad \times \left[\widetilde{P}_{t-1}+\widetilde{P}_{t+i}-\widetilde{P}_{t+i-1}+(1-\theta)\widetilde{w}_{t+i}-\widetilde{\lambda}_{t+i}+\theta\widetilde{r}_{t+i}\right]
\end{aligned}
$$

- Combining gives

$$
\begin{aligned}
\widetilde{P}_t &= \rho\left(2\widetilde{P}_{t-1}-\widetilde{P}_{t-2}\right) \\
&\quad +(1-\rho)\,(1-\beta\rho)\, E_t\sum_{i=0}^{\infty}(\beta\rho)^i \\
&\quad \times \left[\widetilde{P}_{t-1}+\widetilde{P}_{t+i}-\widetilde{P}_{t+i-1}+(1-\theta)\widetilde{w}_{t+i}-\widetilde{\lambda}_{t+i}+\theta\widetilde{r}_{t+i}\right].
\end{aligned}
$$

Inflation adjustment for non-optimizing firms: Log-linearization

- After quasi differencing, this becomes

$$
\begin{aligned}
& (1+2\beta)\,\widetilde{P}_t-\beta E_t\widetilde{P}_{t+1}-(2+\beta)\,P_{t-1}+\widetilde{P}_{t-2} \\
= \quad & \frac{(1-\rho)\,(1-\beta\rho)}{\rho}\left[(1-\theta)\widetilde{w}_t-\widetilde{\lambda}_t+\theta\widetilde{r}_t\right]
\end{aligned}
$$

- Notice the $\widetilde{P}_{t-2}$

Inflation adjustment for non-optimizing firms: Solving the model

- Managing $\widetilde{P}_{t-2}$
- write model as

$$
\begin{aligned}
0 &= Ax_t+Bx_{t-1}+Cy_t+Dz_t \\
0 &= E_t\left[Fx_{t+1}+Gx_t+Hx_{t-1}+Jy_{t+1}+Ky_t+Lz_{t+1}+Mz_t\right] \\
z_{t+1} &= Nz_t+\varepsilon_{t+1}
\end{aligned}
$$

- where

$$x_t = [K_{t+1}, M_t, P_t, P_{t-1}]'$$

- With this definition of x_t, x_{t-1} is now

$$x_{t-1} = [K_t, M_{t-1}, P_{t-1}, P_{t-2}]'$$

.

- Matrices for the expectational variables

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1-2\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2+\beta & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

.

$$J = \begin{bmatrix} \overline{r} & -(\overline{r}+1-\delta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\theta(1-\rho)(1-\beta\rho)}{\rho} & \frac{(1-\theta)(1-\rho)(1-\beta\rho)}{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-(1-\rho)(1-\beta\rho)}{\rho} & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices A to D are

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -\overline{K} & -\overline{C} & \overline{C} & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (1-\delta)\,\overline{K} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \overline{Y} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1-\theta \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & -\pi \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

.

- The solution is of the form

$$x_{t+1} = Px_t + Qz_t$$

and

$$y_t = Rx_t + Sz_t$$

- where, for $\rho = .75$,

$$P = \begin{bmatrix} 0.3450 & 1.1358 & -1.8440 & 0.7082 \\ 0 & 1 & 0 & 0 \\ -0.1881 & 0.3788 & 1.0073 & -0.3862 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} -0.3926 & 1.5977 \\ 0 & 1 \\ -0.2040 & 0.5893 \\ 0 & 0 \end{bmatrix}$$

.

$$R = \begin{bmatrix} -10.3266 & 17.9123 & -29.0784 & 11.1662 \\ 0.1881 & 0.6212 & -1.0073 & 0.3862 \\ 0.1881 & 0.6212 & -1.0073 & 0.3862 \\ -5.7294 & 11.0663 & -17.9655 & 6.8992 \\ -9.5147 & 17.2911 & -28.0711 & 10.7800 \end{bmatrix}$$

$$S = \begin{bmatrix} -6.8340 & 24.6587 \\ 0.2040 & 0.8907 \\ 0.2040 & 0.4107 \\ -3.5044 & 15.2115 \\ -7.0381 & 23.7679 \end{bmatrix}$$

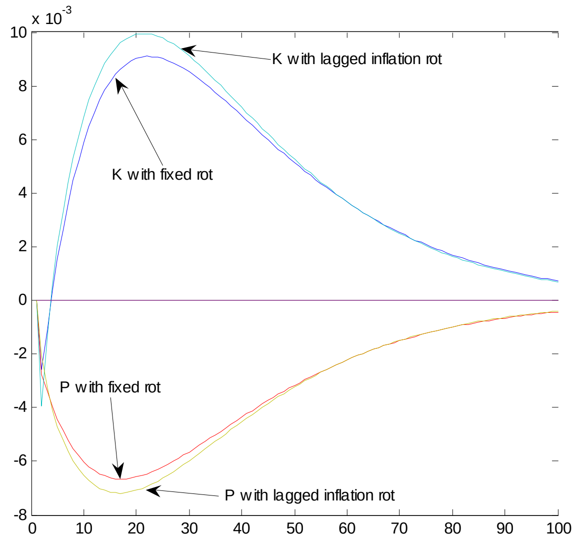

Figure 7: Response of $\widetilde{P}$ and $\widetilde{K}$ to a technology shock for two rules of thumb

Impulse response functions: Technology shock
Impulse response functions: Technology shock
Impulse response functions: Technology shock
Impulse response functions: Money growth shock
Impulse response functions: Money growth shock (fixed)
Impulse response functions: Money growth shock (lagged)

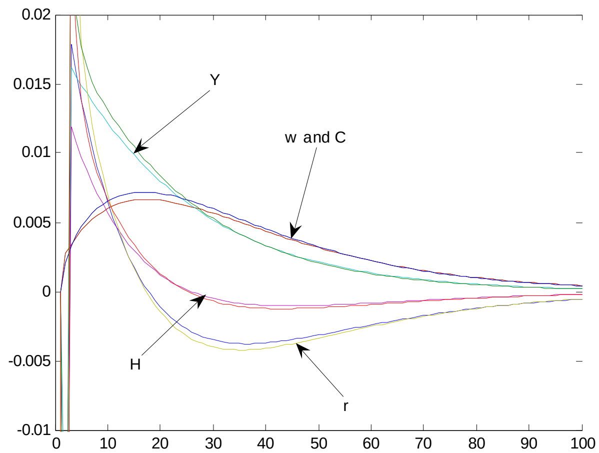

Figure 8: Response of $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{H}$, $\widetilde{r}$ and $\widetilde{w}$ to a technology shock for two rules of thumb

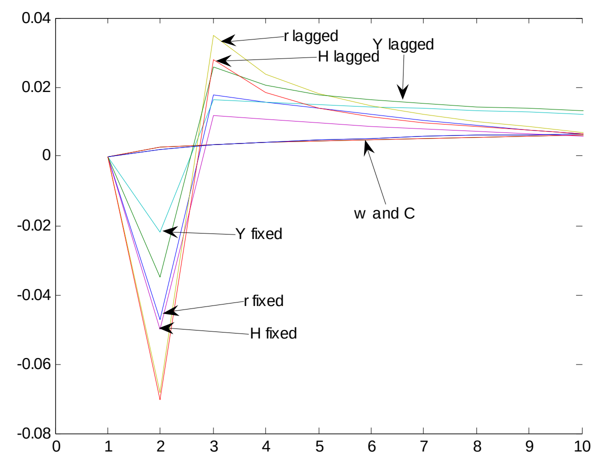

Figure 9: Response in the first 10 periods of $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{H}$, $\widetilde{r}$ and $\widetilde{w}$ to a technology shock for two rules of thumb

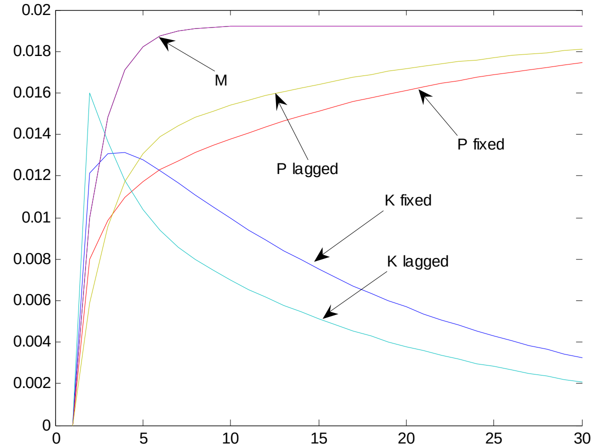

Figure 10: Response of $\widetilde{P}$ and $\widetilde{K}$ to a money growth shock for two rules of thumb

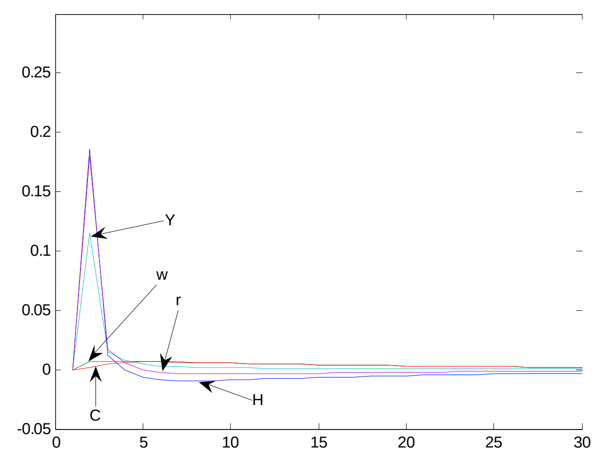

Figure 11: Response of $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{H}$, $\widetilde{r}$ and $\widetilde{w}$ to a money growth shock for rules of thumb with prices constant

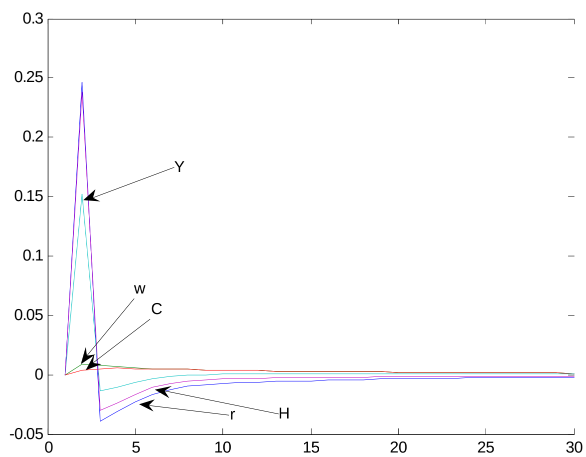

Figure 12: Response of $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{H}$, $\widetilde{r}$ and $\widetilde{w}$ to a money growth shock for rules of thumb with lagged inflation