Macroeconomics II
Staggered pricing models
(part 1)

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1 Staggered Pricing model

Staggered pricing

- Method of Calvo
- Reference: Calvo, Guillermo (1983) "Staggered prices in a utility-maximizing framework," Journal of Monetary Economics, 12, p. 383-398.
- In each period, a fraction $1-\rho$ of firms can choose their price
- In other periods, the price either
  - stays fixed
  - is adjusted by some rule of thumb
- Firms have some market power they can exploit

Structure of economy

- Households
  - buy final good for consumption
  - buy final good for investment
- Final good firms
  - Buy intermediate goods from intermediate goods firms
  - bundles goods together to make final good
  - are perfectly competitive

- Intermediate goods firms
  - Produce a differentiated intermediate good
  - Have some market power to set price
  - Use capital and labor from factor market
  - Can produce excess profits or losses

Final good firm

- Combines the $k \in [0,1]$ intermediate goods to make final good
- Production technology (bundler) is

$$Y_t = \left[\int_0^1 Y_t(k)^{\frac{\psi-1}{\psi}} dk \right]^{\frac{\psi}{\psi-1}}$$

- ψ is the elasticity of substitution in production
- Firm maxes profits

$$profits_t = P_t Y_t - \int_0^1 P_t(k) Y_t(k) dk$$

Final good firm

- Final good firm maximizes

$$\max_{\{Y_t(k)\}} P_t \left[\int_0^1 Y_t(k)^{\frac{\psi-1}{\psi}} dk \right]^{\frac{\psi}{\psi-1}} - \int_0^1 P_t(k) Y_t(k) di$$

- first order condition

$$P_t \left[\int_0^1 Y_t(k)^{\frac{\psi-1}{\psi}} dk \right]^{\frac{1}{\psi-1}} Y_t(k)^{\frac{-1}{\psi}} = P_t(k)$$

- this simplifies to a demand function for good k

$$Y_t(k) = Y_t \left(\frac{P_t}{P_t(k)} \right)^{\psi}$$

Final good firm

- Putting demand into the bundler function

$$Y_t = \left[\int_0^1 \left[Y_t \left(\frac{P_t}{P_t(k)} \right)^{\psi} \right]^{\frac{\psi-1}{\psi}} dk \right]^{\frac{\psi}{\psi-1}} = Y_t \left[\int_0^1 \left(\frac{P_t}{P_t(k)} \right)^{\psi-1} dk \right]^{\frac{\psi}{\psi-1}}$$

- which can be written as

$$\frac{1}{P_t} = \left[\int_0^1 \left(\frac{1}{P_t(k)} \right)^{\psi - 1} dk \right]^{\frac{1}{\psi - 1}}$$

- final goods pricing rule

$$P_t = \left[\int_0^1 P_t(k)^{1-\psi} dk \right]^{\frac{1}{1-\psi}}$$

Intermediate goods firms

- Some common rules of thumb
  - 1) keeping prices the same as the last updating

$$P_t(k) = P_{t-1}(k)$$

  - 2) updating prices by a stationary state gross inflation rate $\overline{\pi}$,

$$P_t(k) = \overline{\pi} P_{t-1}(k)$$

  - 3) updating prices by the one period lagged realized gross inflation rate, $\pi_{t-1} = P_{t-1}/P_{t-2}$,

$$P_t(k) = \pi_{t-1} P_{t-1}(k)$$

Intermediate goods firms

- Choose $P_t^*(k)$, to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \rho^i \left[P_t^*(k) Y_{t+i} \left(\frac{P_{t+i}}{P_t^*(k)} \right)^{\psi} \right. \\ \left. - P_{t+i} r_{t+i} K_{t+i}(k) - P_{t+i} w_{t+i} H_{t+i}(k) \right]$$

- subject to the production technology,

$$Y_{t+i} \left(\frac{P_{t+i}}{P_t^*(k)} \right)^{\psi} = \lambda_{t+i} K_{t+i}^{\theta}(k) H_{t+i}^{1-\theta}(k)$$

Intermediate goods firms

- A firm that is maximizing above is also minimizing costs

- Minimization problem

$$\min_{K_{t+i}(k),H_{t+i}(k)} r_{t+i}K_{t+i}(k) + w_{t+i}H_{t+i}(k)$$

subject to the production technology

$$Y_{t+i}(k) = \lambda_{t+i}K^{\theta}_{t+i}(k)H^{1-\theta}_{t+i}(k)$$

- First order conditions

$$\frac{(1-\theta)\, r_t}{\theta w_t} = \frac{H_t(k)}{K_t(k)}$$

Intermediate goods firms: Costs

- Combine FOC with production function
- Demand equations for factors

$$H_{t+i}(k) = \frac{Y_{t+i}(k)}{\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta}$$

and

$$K_{t+i}(k) = \frac{Y_{t+i}(k)}{\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta-1}$$

Intermediate goods firms: Costs

- Put these demand equations into cost equation

$$r_{t+i}\frac{Y_{t+i}(k)}{\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta-1} + w_{t+i}\frac{Y_{t+i}(k)}{\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta}$$

- Real costs equal

$$\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta} Y_{t+i}(k)$$

- Marginal costs are

$$\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta}$$

Intermediate goods firms: maximization problem

- Rewriting costs get

$$\max_{P^*_t(k)} E_t\sum_{i=0}^{\infty}(\beta\rho)^i\left[P^*_t(k)Y_{t+i}\left(\frac{P_{t+i}}{P^*_t(k)}\right)^{\psi}\right.$$
$$\left.-P_{t+i}\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}}\left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^{\theta} Y_{t+i}\left(\frac{P_{t+i}}{P^*_t(k)}\right)^{\psi}\right]$$

- or

$$\max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i} \left(\frac{P_{t+i}}{P_t^*(k)} \right)^{\psi}$$
$$\times \left[P_t^*(k) - \frac{P_{t+i} w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta} \right]$$

Intermediate goods firms: maximization problem

- FOC is

$$0 = E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k) \left[1 - \psi + \frac{\psi P_{t+i} w_{t+i}}{P_t^*(k)(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta} \right]$$

- Price setting rule is

$$P_t^*(k) = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta}}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k)}$$

- $\frac{\psi}{\psi-1}$ is the markup of discounted stream of expected nominal costs divided by discounted stream of expected output

Price process

- All firms fixing prices at date t set the same price
- All set to P_t^* ($1-\rho$ firms)
- Recall that firms that don't fix price keep old one (ρ firms)
- Prices follow updating process of

$$P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1-\rho)(P_t^*)^{1-\psi}$$

The households

- Unit mass of households
- Max

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + B h_t^i \right]$$

- subject to a cash in advance constraint

$$P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1}$$

- and a real budget constraint,

$$k_{t+1}^i + \frac{m_t^i}{P_t} = w_t h_t^i + r_t k_t^i + \xi_t^i + (1-\delta)k_t^i$$

The households

- First order conditions for family i are

$$\frac{B}{w_t} = E_t \left[\frac{B\beta}{w_{t+1}} \left(r_{t+1} + (1-\delta) \right) \right]$$

and

$$-E_t \left[\frac{\beta}{c_{t+1}^i P_{t+1}} \right] = \frac{B}{w_t P_t}$$

Equilibrium conditions

- Aggregation across families gives

$$C_t = c_t^i,$$

$$K_t = k_t^i,$$

$$H_t = h_t^i,$$

and

$$M_t = m_t^i.$$

- Aggregating labor across intermediate firms

$$H_t = \int_0^1 H_t(k)dk.$$

Equilibrium conditions

- Use the demand for labor

$$H_t(k) = \frac{Y_t(k)}{\lambda_t} \left[\frac{r_t(1-\theta)}{w_{ti}\theta} \right]^\theta,$$

- equilibrium condition for the labor market in period t is

$$H_t = \int_0^1 H_t(k)dk = \frac{1}{\lambda_t} \left[\frac{r_t(1-\theta)}{w_t\theta} \right]^\theta \int_0^1 Y_t(k)dk.$$

Equilibrium conditions

- For the market for capital,

$$K_t = \int_0^1 K_t(k)dk$$

- Using the demand for capital

$$K_t(k) = \frac{Y_t(k)}{\lambda_t}\left[\frac{r_t(1-\theta)}{w_t\theta}\right]^{\theta-1}$$

- the equilibrium condition for the capital market in period t is

$$K_t = \int_0^1 K_t(k)dk = \frac{1}{\lambda_t}\left[\frac{r_t(1-\theta)}{w_t\theta}\right]^{\theta-1}\int_0^1 Y_t(k)dk$$

- Note integral of output (not bundler)

Equilibrium conditions

- Aggregate excess profits paid to the families in period t are

$$\begin{aligned} P_t\xi_t &= P_t\int_0^1 \xi_t^i di = P_t\int_0^1 profits(k)dk \\ &= \int_0^1 P_t(k)Y_t(k)dk - P_t\frac{w_t}{(1-\theta)\lambda_t}\left[\frac{r_t(1-\theta)}{w_t\theta}\right]^{\theta}\int_0^1 Y_t(k)dk \end{aligned}$$

- Since the final good firms are perfectly competitive and make no profits,

$$P_tY_t = \int_0^1 P_t(k)Y_t(k)dk$$

- Substituting this into the total profits equation for period t, and removing the price level P_t, gives excess profits as

$$\xi_t = \left(Y_t - \frac{w_t}{(1-\theta)\lambda_t}\left[\frac{r_t(1-\theta)}{w_t\theta}\right]^{\theta}\int_0^1 Y_t(k)dk\right)$$

The full model

- FOCs of household

$$\frac{1}{w_t} = E_t\left[\frac{\beta}{w_{t+1}}\left(r_{t+1} + (1-\delta)\right)\right],$$

$$-E_t\left[\frac{\beta}{C_{t+1}P_{t+1}}\right] = \frac{B}{w_tP_t},$$

- the aggregated cash-in-advance constraint,

$$P_t C_t = g_t M_{t-1},$$

- aggregated family's real budget constraint,

$$K_{t+1} + \frac{M_t}{P_t} = w_t H_t + r_t K_t + \xi_t + (1-\delta) K_t.$$

.The full model

- all income goes to the families and sums to output

$$Y_t = w_t H_t + r_t K_t + \xi_t,$$

- family's real budget constraint will be used as

$$K_{t+1} + \frac{M_t}{P_t} = Y_t + (1-\delta) K_t.$$

- Price setting equation

$$P_t^*(k) = \frac{\psi}{\psi-1} \frac{E_t \sum_{i=0}^{\infty} (\beta\rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[\frac{r_{t+i}(1-\theta)}{w_{t+i}\theta}\right]^\theta}{E_t \sum_{i=0}^{\infty} (\beta\rho)^i Y_{t+i}(k)}$$

.The full model

- cost minimization,

$$\frac{(1-\theta) r_t}{\theta w_t} = \frac{H_t(k)}{K_t(k)}$$

- aggregate production equation,

$$\int_0^1 Y_t(k) dk = \lambda_t H_t^\theta K_t^{1-\theta}$$

- rule determining the final good price in period t,

$$P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1-\rho) (P_t^*)^{1-\psi}$$

- money supply growth rule

$$M_t = g_t M_{t-1}$$

stochastic process for money growth and technology,

$$\ln \lambda_t = \gamma \ln \lambda_{t-1} + \varepsilon_t^\lambda$$

$$\ln g_t = \pi \ln g_{t-1} + \varepsilon_t^\lambda$$

Interesting points of stationary state

- Much is the same as before, except
- The rule for determining the final goods price gives

$$\overline{P}^{1-\psi} = \rho\overline{P}^{1-\psi} + (1-\rho)\left(\overline{P}^{*}(k)\right)^{1-\psi}$$

- or that

$$\overline{P} = \overline{P}^{*}(k) = \overline{P}(k)$$

- Putting this into the demand function for intermediate good k gives

$$\overline{Y}(k) = \overline{Y}\left(\frac{\overline{P}}{\overline{P}(k)}\right)^{\psi} = \overline{Y}$$

Interesting points of stationary state

- The price setting rule gives

$$\begin{aligned}\overline{P}^{*}(k) &= \frac{\psi}{\psi-1}\frac{\frac{1}{1-\beta\rho}\overline{PY}\frac{\overline{w}}{(1-\theta)\overline{\lambda}}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta}}{\frac{1}{1-\beta\rho}\overline{Y}} \\ &= \frac{\psi}{\psi-1}\overline{P}\frac{\overline{w}}{(1-\theta)\overline{\lambda}}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta}\end{aligned}$$

- or a relation for wages and rentals

$$\frac{\psi}{\psi-1} = \frac{1}{\frac{\overline{w}}{(1-\theta)}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta}}$$

- wages are

$$\overline{w} = \left[\frac{(\psi-1)(1-\theta)^{1-\theta}\theta^{\theta}}{\psi\overline{r}^{\theta}}\right]^{\frac{1}{1-\theta}}$$

Interesting points of stationary state

- Dividends are

$$\overline{\xi} = \overline{Y}\left(1 - \frac{\overline{w}}{(1-\theta)}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta}\right) = \frac{\overline{Y}}{\psi}.$$

- Output is not yet determined. Using the real budget constraint, the first order condition for consumption, and the results above

$$-\frac{\beta\overline{w}}{B} = \overline{w}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta}\overline{Y} + \frac{\overline{Y}}{\psi} + (\overline{r}-\delta)\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta-1}\overline{Y},$$

- This can be rewritten as

$$\overline{Y} = \frac{-\beta\overline{w}}{B\left(\overline{w}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta} + \frac{1}{\psi} + (\overline{r}-\delta)\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{\theta-1}\right)}.$$

Stationary states

- $\beta = .99$, $B = -2.5805$, $\delta = .025$, $\theta = .36$. Galí argues that $\rho = .75$ and $\psi = 11$, a value that gives a 10% markup in the stationary state.

- Stationary states for this model are

Variable	$\overline{r}$	$\overline{w}$	$\overline{Y}$	$\overline{H}$	$\overline{K}$	$\overline{\xi}$	$\overline{C}$
SS value	.0351	2.0426	1.0218	.2901	9.5271	.0929	.7836