Macroeconomia 1
Class 14a revised
Diamond Dybvig model of banks

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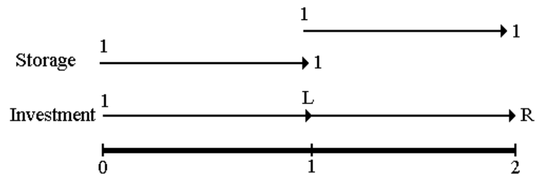
How to model (think about) liquidity

- Model of Diamond and Dybvig (Journal of Political Economy, 1983)
- Three possibilities
  - People can make independent investments
  - People can use a market
  - People deposit in a bank (mutual)
- Want to know
  - Which is best (highest utility), so: explains why banks exist.
  - What problems might there be with such a bank
- Want to model **Liquidity**
  - Need to define what liquidity means in a model

Time

- There are three periods: 0, 1, 2
- People have 1 unit of endownment in period 0
- Will want to consume in **either**
  - period 1
  - or period 2
  - **NOT** both

Preferences



- People can be type 1 or type 2
- Def: C_{ij} is the consumption of a type i in period j
  - Type 1
    - Only get utility from consuming in period 1
    - $u(C_{11}) > 0$ is what matters to a type 1
    - $u(C_{12}) = 0$
    - p_1 of the population will be type 1's
  - Type 2's
    - Only get utility from consuming in period 2
    - $u(C_{22}) > 0$ is what matters to a type 2
    - $u(C_{21}) = 0$
    - $p_2 = 1 - p_1$ of the population will be type2's

Problem 1 *In period* 0 *people don't know if they are going to be a type* 1 *or a type* 2 *Discover their type only in period* 1 *but must make investment decisions in period* 0

Technology available

- Investment technology
  - 1 unit of good put into investment at time 0
    - Pays L units in period 1 where $L < 1$
    - Pays R units in period 2 where $R > 1$
  - L is the payout when one liquidates an investment early
  - R is the payout when the investment matures normally
- Storage technology
  - Good can be put into storage at time 0 or 1
    - 1 unit stored in period i, $i = 0$ or 1
    - Pays 1 unit in period $i + 1$

Autarky

- Expected utility function to be maximized

$$U = p_1 u(C_{11}) + p_2 \rho u(C_{22})$$

  - ρ is a discount factor for second period consumption

- Budget restricitions

$$C_1 = 1 - I + L \cdot I = 1 - I(1 - L) \leq 1$$

  - $= 1$ only when $I = 0$
  - $1 - I$ is storage
  - $L \cdot I$ is return from liquidation of asset

- and

$$C_2 = 1 - I + R \cdot I = 1 + I(R - 1) \leq R$$

  - $= R$ only when $I = 1$
  - $1 - I$ is storage
  - $R \cdot I$ is return on long term investment

Autarky: equilibrium

- People choose I to maximize utility subject to these budget constraints
- If $0 < p_1 < 1$ and $U(\cdot)$ concave
- Then $0 < I < 1$
- $C_{11} < 1$
- $C_{22} < R$

Market equilibrium

- $p =$ price of a bond promising
  - 1 unit of good delivered in period 2
  - purchased in period 1
- Budget constraint for type 1
  - $C_{11} = 1 - I + pRI$
  - Type 1 sells investment for storage
- Budget constraint for type 2
  - $C_{22} = (1 - I)/p + RI = (1 - I + pRI)/p$

- Type 2 sell storage for investment

.

- For no arbitrage condition, $p = 1/R$
  - What happens if $p < 1/R$?
    * people prefer bond over investment
    * $I = 0$
    * Not an equilibrium: no investment to buy
  - What happens if $p > 1/R$?
    * people prefer investment over bond
    * $I = 1$
    * Not an equilibrium: no storage to sell

.Market equilibrium: continued

- When price is $p = 1/R$
- Budget constraint for type 1 yields
  - $C_{11} = 1 - I + pRI = 1 - I + (1/R)\, RI = 1 - I + I = 1$
- Budget constraint for type 2 yields
  - $C_{22} = (1 - I)/p + RI = (1 - I)/\,(1/R) + RI = (1 - I)R + RI = R$
- Optimum with market implies that $C_1 = 1$ and $C_2 = R$
- Market improves on Autarky

.Optimal allocation

- The market equilibrium is not necessarily optimal
- Optimal is
  - Choose a pair of desired consumptions (C_1^*, C_2^*)
  - To max
$$p_1 u(C_1) + p_2 u(C_2)$$
  - When
$$p_1 C_1 + p_2 C_2/R = 1$$
- Maximization implies that
$$u'(C_1^*) = Ru'(C_2^*)$$

- If $C \longrightarrow Cu'(C)$ is decreasing (assumption of D-D)
  - Then
$$R \cdot u'(R) < u'(1)$$
  - And this implies that the optimal $C_1^* > 1$ and $C_2^* < R$

Inventing a Financial Intermediary

- A "mutual" bank can
  - Take deposits
  - Promise to pay C_1^* to real type ones
  - Pay C_2^* to real type twos : $C_2^* > C_1^*$
  - Bank holds reserves of $p_1 C_1^*$
- Bank invests $1 - p_1 C_1^*$ and then $p_2 C_2^* = R(1 - p_1 C_1^*)$

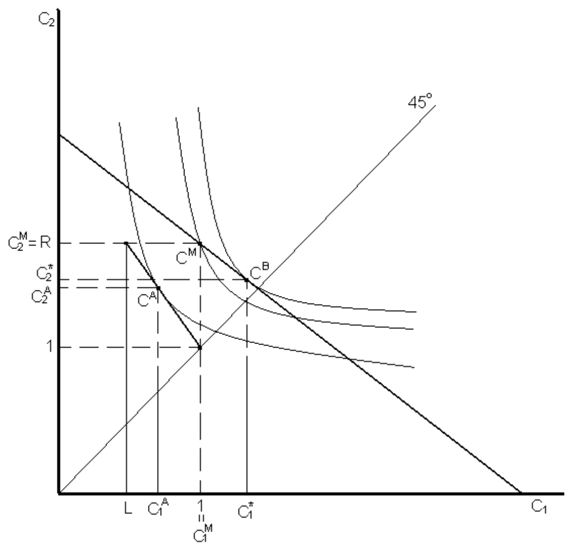
With Banks (mutual banks)

- Equilibrium # 1 (no run)
  - Banks promise C_1^* in period one
  - What remains is divided among those who remain
    * In deterministic model this is C_2^*
- This is a **Nash equilibrium**
  - everyone does what is best for them
  - given what the rest are doing

Graph of D-D model

Bank runs

- Equilibrium #2 (run)
  - Other Nash equilibrium
  - Sufficient type 2's attempt to withdraw their deposits
  - Suppose that $\delta > 0$ attempt to withdraw
    * Then banks must pay $(p_1 + \delta) C_1^*$ in period 1
    * Only have $p_1 C_1^*$ in reserves
    * Must liquidate
$$\frac{\delta C_1^*}{L} > \delta C_1^*$$
      of investment to pay them off



Bank runs

- The investment that remains for the type 2's is

$$\frac{p_2 C_2^*}{R} - \frac{\delta C_1^*}{L}$$

- This needs to be divided among type 2s who stay in bank
- Each type 2 who remains gets

$$\frac{\left[\frac{p_2 C_2^*}{R} - \frac{\delta C_1^*}{L}\right] R}{p_2 - \delta}$$

Bank runs

- With a bit of algebra, one can show that the
  - δ where the payment to the remaining type 2s
  - is less than what the type 1's get when

$$\delta > \delta^* = \frac{p_2 L}{R - L}\left[\frac{C_2^*}{C_1^*} - 1\right]$$

  - Note that at optimal solution $R - L > \frac{C_2^*}{C_1^*} - 1$
    - smaller difference between C_2^* and C_1^*, more likely run

    - larger difference between R and L, more likely run

- If less than δ^* run the bank
  - It is best for the remaining type 2's not to run the bank
  - Therefore it is not optimal for those who do run the bank
  - Nash Equilibrium: Then no one should run the bank

Bank runs

- If more than δ^* run the bank
  - It is optimal for the remaining type 2's to run the bank
    - The bank will be bankrupt
    - All run the bank
  - If everybody runs
    - Bank must close
    - Because

$$C_1^* > p_1 C_1^* + \frac{L}{R} p_2 C_2^*$$

    - where $\frac{L}{R} p_2 C_2^*$ comes from the liquidation of all investment

- Note that the bank was solvent in that if it were not run it could have paid
  - C_1^* to all the real type 1's
  - C_2^* to all the real type 2's

Bank runs

- Problem in equilibrium for Financial Intermediary
- If real type 1s and real type 2s can't be identified
  - There is the second Nash equilibrium
- If some type 2s believe that other type 2s will act like type 1s
  - They will fear they won't get paid in period 2
  - And will act like type 1s in period 1 and withdraw C_1^* in period 1
    - Remember they can store the good till period 2
  - If enough do this, the bank can't pay the rest
- This is a bank run equilibrium and bank fails
- Sequential servicing heightens the problem

.Bank runs

- The economy suffers real losses (in terms of goods)

$$p_1 C_1^* + \frac{L}{R} p_2 C_2^* < p_1 C_1^* + p_2 C_2^*$$

- With sequential servicing

$$\frac{C_1^* - \left[p_1 C_1^* + \frac{L}{R} p_2 C_2^*\right]}{C_1^*} = 1 - p_1 - \frac{L}{R} p_2 \frac{C_2^*}{C_1^*}$$

  end up getting 0

- Of those who do get paid
  - p_1 get paid out of reserves (storage)
  - $\frac{L}{R} p_2 \frac{C_2^*}{C_1^*}$ get paid out of liquidation of the assets of the bank

.What happens if you add uncertainty about the banks

- Version of model by Catena and McCandless
- Version with uncertain returns (on L and R)
- There is an equilibrium which depends on the returns to the bank
  - If returns are low enough, people run bank
  - If returns are high enough, they do not
- However, banks can and do get run and do fail

.Handling bank runs

- There are four techniques that have been proposed (and used) to solve this problem

1. Suspension of payments
   - This needs to be specified in the deposit contract
   - Sometimes it is directly in banking laws
   - For the lawyers: what was the problem with coralito?
2. Deposit insurance
   - The system will guarantee some fraction of deposits
   - Normally all deposits up to some limit
   - One needs to determine how these are paid

(a) Banks pay insurance premiums

- OK if only a few banks fail
- Difficult to calculate premium: risk based

(a) Paid by taxes (as in Savings and Loan crisis in USA)

Handling bank runs

3. Lender of last resort

- Central bank can lend to banks
  (a) Exchange for good loans
  (b) Can issue money for this purpose
- Large literature on this
- There is an important difference between
  (a) runs on one or a few banks and
  (b) runs on the entire banking system.

Handling bank runs

4. Narrow banking (Simon Banks)

- Most restrictive is one with 100% in liquid reserves
  (a) there are enough reserves to pay all
  $$C_1 \leq 1 - I$$
  (b) The investment alone can pay all
  $$C_2 \leq R \cdot I$$
  (c) here C_1 and C_2 are the maximum payments allowed
  (d) Returns of this bank are worse than autarky
- With sequential servicing and with C_1 not completely known, the best is $C_1 = C_1 = 1$ and $I = 0$.