AK model

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Problems with Solow type models

- How to explain different growth rates among countries
- How to explain that leader country grows at 2% or so a year
- How to handle technology growth
  - Solow residual is simply given
  - In competitive market – no research and development
  - because it gets captured by all immediately
  - no one can benefit from research
- Endogenous growth

Technology AK

- Assume that the production function is linear

$$Y_t = AK_t$$

- Doesn't have diminishing returns to capital as in Solow model (and variations)
- How to make sense of a production function like this
  - Think about labor
  - Comprised of a bit of physical labor and lots of education and training
  - This education and training is human capital
  - Suppose we sum all capital: physical and human $= K_t$
- Then we are implicitly including labor as a kind of capital
- Note: rich countries have lots of physical and human capital

Characteristics of Technology AK

- Exhibits constant returns to scale: suppose $Y_t = AK_t$

$$A(\lambda K_t) = \lambda A K_t = \lambda Y_t$$

- Returns to capital are positive but not decreasing

$$\frac{\partial Y_t}{\partial K_t} = A \text{ and } \frac{\partial^2 Y_t}{\partial K_t^2} = 0$$

- Does not meet the Inada conditions

$$\lim_{K \to 0} F(K) = A \neq \infty \text{ and } \lim_{K \to \infty} F(K) = A \neq 0$$

Full model

- Production function at the per wokerl level

$$y_t = \frac{Y_t}{L_t} = A\frac{K_t}{L_t} = Ak_t$$

- Savings is a constant fraction of output

$$s_t = sy_t$$

- Investment equals savings

$$s_t = i_t$$

- Law of capital growth (in per worker terms)

$$(1+n)\,k_{t+1} = (1-\delta)\,k_t + i_t$$

- combine to get

$$\left(1+n\right)k_{t+1} = \left(1-\delta\right)k_t + sAk_t$$

- to get growth in per worker capital

$$\left(1+n\right)k_{t+1} - \left(1+n\right)k_t = \left(1-\delta\right)k_t - \left(1+n\right)k_t + sAk_t$$

$$\left(1+n\right)\left(k_{t+1} - k_t\right) = sAk_t - \left(\delta+n\right)k_t$$

But

- sA is a constant
- $-(\delta + n)$ is a constant
- The model gives

$$(1+n)(k_{t+1} - k_t) = [sA - (\delta + n)]\, k_t$$

- If $sA > (\delta + n)$ then capital grows at the rate γ_t, where

$$\gamma_t = \frac{k_{t+1} - k_t}{k_t} = \frac{sA - (\delta + n)}{(1+n)}$$

- FOREVER

Some results of the model with technology AK

- Economies grow without need for some variable to grow exogenously (such as technology)
- However, improvements in technology increase the rate of growth of an economy
- Higher savings results in higher growth rates (for same depreciation and population growth rates)
- Lowering depreciation or population growth causes faster per worker growth in capital (and output)
- Negative shocks to the economy are permanent (a one period technology shock)
  - The loss caused by the shock never disappears compared to path without the shock
- Rate of growth of output is not dependent on *level* of output (rich countries do not need to grow slower than poor countries)