1 The knowledge economy (research and development)

1.1 Intro

The problem with competitive economies: there are no incentives for research and development. If the one who invents something cannot benefit from it, because the technology is acquired by everyone and everyone increases their productivity, reducing prices (or increasing output, with the same effect), then there are no incentives for improving technology.

In learning by doing models, there is no *intentional* or directed improvement in technology, technological improvement just comes from the act of production: the more you produce the better you get at it.

What is technological improvement: the ability to combine capital and labor better to get more output from the same amount of factors. The improvement may be embodied in a new machine, it might be in the improved skills of the worker, or it might be in a better organization of production. For example, people discovered that if one waited a bit before plowing the fields, the weeds would begin to grow up and plowing managed to kill off a lot of them by cutting off their growth cycle.

Concepts of rival goods and excludable goods. Rival goods mean that because one person consumes the good, others are not able to consume it. A steak, potatoes and other food items are classic examples of rival goods. Once I eat a steak, no one else can consume it. A book is parcially rival, while I am reading it, no one else can, but after I have read it, I can lend it to others and they can read it. Or I could read it aloud, as I do with books for my son, and several people can enjoy it at the same time. Watching a play is only slightly rival, since the fact that others are watching only slightly impinges on my ability to watch it (their head may block my view of the stage or they may make noise that causes it to be difficult for me to hear the words). Driving a car into the city of Buenos Aires from the suburbs is not a rival good during much of the day, but during rush hour it is. The fact that I am using the highway does not normally reduce the ability of others to use it unless the density of cars gets too big. Then delays in human reaction time cause traffic to move in waves.

Excludable goods are those that one person can prevent another from using. The owner of a kiosk excludes me from consuming what he or she has until I pay for them. Likewise in other stores (note that restaurants are somewhat strange in this regard: you consume first and then you pay). Dance halls exclude customers they view as undesirable. I can exclude other from reading the copy of the book I own by storing it away. Private property rights are the rights to exclude others from the use of something that you "own". That is exactly what owning means: the right of exclusion. You can exclude others from using your labor because you cannot be made a slave. That is the first property right. You can keep others from moving into you home or from taking your cows or from planting on your land. Some good are difficult to exclude others from enjoying: a fireworks display, an open air band concert, fishing in the sea, hunting in the woods (although in the past, hunting rights were limited to the aristocracy and so were excludable). One of the difficulties with non-exclusion is overuse: the tragedy of the commons, which gets its name because in some societies, portions of land were declared common and everyone could use them for their animals. These lands were most often overgrazed and left bare. The tragedy of the commons is seen in the depletion of fishing grounds were fishing is a rival good (if I catch a fish, you can't catch it) but it is difficult to exclude other fishermen.

Inventions are goods that are normally non-rival and non-excludible. Some exclusion can be obtained by keeping the ideas secret: this works if the idea is a process but not if the idea is a product (since by selling one unit of the good, you make knowledge of the good available to others.[1]) If I invent a formula, except for needing the knowledge to be able to interpret the formula and have the preliminary skills to utilize it, if the knowledge of the formula gets out, many people can make it (the formula for Coca-Cola, for example). This is the basic problem with inventions (research and development) it is much easier (less costly) to back-engineer something than to invent it the first time and once a good is invented, it must be seen to be sold (and the person who bought it can back-engineer it unless that is intentionally (and at high costs) designed to be difficult to take apart.

The book suggests that we can think of a range of goods with two dimensions, one for rivalry and the other for excludability.

	rival	**non-rival**
easily excludable	food, clothes	cable tv
parcially excludable	books, seats in theaters	computer programs
difficult to exclude	fish in the sea	defense, ideas

This table is only parcially correct, since some goods fall in the cracks: for traveling on a highway, rivalry depends on the number of other cars, if low, non-rival, if high, rival. The main point of all this is that ideas, research and development, technology are non-rival goods that are difficult to exclude others from using. This is why countries have patent laws, laws that give inventors limited monopoly rights on their inventions in exchange for making the invention public (through the patent application). Economic history suggests that it was only after the development of patent rights that research and development became important.

Some qualifications on the last statement are in order. Inventions take place at a certain time with a certain amount of previous knowledge available. It is frequently the case that important inventions are developed more or less simultaneously in different places (countries or parts of the world). Indeed, many of the basic developments need for the steam engine came out of a competition held by the British Navy to develope a method for measuring longetude (the prize

[1] The supposed secret of the atomic bomb was whether it could work or not. Once it was set off in Japan, it was clear to all that it could work, so by using it, the secret was given away.

was finally given to the inventor of an accurate portable clock: cronometer).

1.2 A simple model of Research and Development

The model imagines an increasing number of intermediate capital goods that have to be invented to be used. It is assumed that the inventor gets an infinitely lived patent on the design and can sell the intermediate good at a monopoly price. Old intermediate goods keep being used. This is different from more complicated models of creative destruction where older technologies stop being used once they are replaced with more productive new technologies.

1.2.1 Goods production

The production function is of the form

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

where L_t is the labor (which we will assume is fixed in this model) and K_t is the capital in use at time t. Capital is the aggregate of intermediate goods with a CES production function of

$$K_t = \left(\sum_{j=1}^{N_t} x_{jt}^{\alpha} \right)^{\frac{1}{\alpha}}$$

where N_t is the number of intermediate inputs at date t and x_{jt} is the amount of intermediate good j that is bought and used by the firms in period t. Putting this into the production function we get

$$Y_t = AL_t^{1-\alpha} \sum_{j=1}^{N_t} x_{jt}^{\alpha}.$$

Notice that the production depends on the amount and number of the intermediate goods available. Because of the additive property of capital, this production function has decreasing returns to each type of intermediate capital good but has constant returns to scale in aggregate capital and labor. All of the N_t intermediate goods are similar in productivity (in this model there is no productivity improvement from new capital goods) so the same amount of each of these goods is supplied (and demanded): $x_{jt} = x_t, \forall\ j$. In this case, the production function is

$$Y_t = AL_t^{1-\alpha} \left(N_t x_t \right)^{\alpha} N_t^{1-\alpha}.$$

Notice that $N_t x_t$ is the total amount of intermediate goods that are purchased. If this total amount is made up of more goods (i.e., if N_t is bigger for a fixed $N_t x_t$), then output increases with N_t.

Firms hire labor at a competitve wage and buy the intermediate good j at a price p_{jt}. These intermediate goods are completely used up in the production process. Firms want to maximize the present value of their flow of profits (discounted by the gross interest rate r_t), which is

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^i\left(AL_{t+i}^{1-\alpha}\sum_{j=1}^{N_t}x_{jt+i}^{\alpha}-w_{t+i}L_{t+i}-\sum_{j=1}^{N_t}p_{jt+i}x_{jt+i}\right).$$

Given that this maximization problem can be solved in a period by period manner, this means that in each period, firms maximize

$$AL_t^{1-\alpha}\sum_{j=1}^{N_t}x_{jt}^{\alpha}-w_tL_t-\sum_{j=1}^{N_t}p_{jt}x_{jt}.$$

First order conditions give

$$w_t=(1-\alpha)\,AL_t^{-\alpha}\sum_{j=1}^{N_t}x_{jt}^{\alpha}$$

and

$$p_{jt}=\alpha AL_t^{1-\alpha}x_{jt}^{\alpha-1}.$$

The last first order condition can be written as a demand function fog intermediate good x_{jt} as

$$x_{jt}=\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_tp_{jt}^{-\frac{1}{1-\alpha}}.$$

1.2.2 The firms that produce new intermediate goods (inventors)

A number of real world conditions are not included in this model. Normally, a patent on an invention only lasts a fixed and relatively small number of years (this is different from the time that an artist or writer owns a copywrite), 18 years for most goods in the US and as low as 8 years for medicines. In this model, to keep the math simple, the patent is forever. Second, in the real world, most inventions are eventually replaced by another that does the job better. The typewriter has been replaced by the computer, as has been the typesetting machine (linotype), postal services, some forms of books, slide rulers, and a number of other items. Jet planes have replaced ships for cross ocean travel. Gasoline powered cars replaced horses for local travel and these are likely to replaced by some kind of electric powered auto. A great many goods, and especially capital goods, have been replaced by others that do the job quicker, cheaper and probably better.

An inventor has two questions to decide: whether to build a new good or not, and once built, what price to charge for it. We will consider the pricing question first and then the decision whether to produce or not. We will assume that once invented, the intermediate good continues to be produced.

Assume that producing one unit of the intermediate good has a marginal cost equal to one unit of the final good, $MC = 1$. Given that, once invented, a firm that produces good j charges a sequence of prices $\{p_{jt}\}$ so as to maximize the discounted profits

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}\pi_{jt+i}=\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}(p_{jt+i}-MC)\,\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_{t+i}p_{jt+i}^{-\frac{1}{1-\alpha}}.$$

Given that the firm choose p_{jt}, the first order conditions are (after replace MC with 1)

$$0=\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_t p_{jt}^{-\frac{1}{1-\alpha}}+(p_{jt}-1)\left(-\frac{1}{1-\alpha}\right)\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_t p_{jt}^{-\frac{1}{1-\alpha}-1}$$

or

$$\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_t p_{jt}^{-\frac{1}{1-\alpha}}=(p_{jt}-1)\left(\frac{1}{1-\alpha}\right)\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_t p_{jt}^{-\frac{1}{1-\alpha}-1}.$$

Cancelling out (a whole lot of) common terms gives

$$p_{jt}=\frac{1}{\alpha}.$$

Putting this into the demand function for good x_{jt} gives

$$x_{jt}=\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}L_t\left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}}=A^{\frac{1}{1-\alpha}}L_t\alpha^{\frac{2}{1-\alpha}}.$$

If we put this into the production function for the final good, we get

$$\begin{aligned}Y_t &= AL_t^{1-\alpha}\left(N_tA^{\frac{1}{1-\alpha}}L_t\alpha^{\frac{2}{1-\alpha}}\right)^{\alpha}N_t^{1-\alpha}\\ &= A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}L_tN_t.\end{aligned}$$

Notice that Y_t grows with N_t if the labor supply is constant (as we have assumed).

Putting this price into the profit function gives

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}\pi_{jt}=\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}\left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}}A^{\frac{1}{1-\alpha}}L$$

and since the right hand side is a constant, this gives

$$\pi_{jt}=\pi=\left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}}A^{\frac{1}{1-\alpha}}L=\left(\frac{1}{\alpha}-1\right)x$$

and assuming that the interest rate stays constant

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}\pi_{jt}=\pi\frac{r}{r-1},$$

which is the present value that comes from owning the patent on a new intermediate good.

The decision to invent a new good or not depends on whether the cost of inventing an intermediate good, Φ, is less than or equal to the present value of the stream of profits that come from that invention. In the real world, inventing a good is an uncertain process where the costs may end up being much less or much more than originally planned. Here we leave out uncertainty (as adding substantial complication to the model) and assume the cost is simply Φ. The condition of free entry into the invention industry in that the cost of invention is equal to the expected return on inventing or that

$$\Phi = \sum_{i=1}^{\infty} \left(\frac{1}{r}\right)^i \pi = \pi \frac{r}{r-1} - \pi = \frac{\pi}{r-1}.$$

Since time is descrete and inventing a good today only brings in profits beginning in the next period, we need to sum up the future profits beginning in the next period. Whether the costs of inventing rise or fall with the number of inventions is an open question. Older inventions usually make new ones possible so they may reduce the costs. However, if there are only a fixed number of things to be invented, every time one is invented it would make it harder (costlier) to make the next. That would increase the costs of inventing with the number of inventions. We assume here that the cost of inventing is fixed and exactly equal to

$$\Phi = \frac{\pi}{r-1}.$$

1.2.3 Consumers

Consumers maximize

$$\sum_{i=0}^{\infty} \beta^i \frac{c_t^{1-\theta} - 1}{1-\theta}$$

subject to the budget constraint

$$b_{t+1} + c_t = w_t + r b_t$$

where b_tt are bonds held by the households at the beginning of the period t and w_t is the wage at time t. First order condition from the Lagrangian is

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = r\beta$$

so that with a constant growth rate

$$\gamma_c = \frac{c_{t+1}}{c_t} - 1 = (r\beta)^{\frac{1}{\theta}} - 1$$

Using the condition on free entry into inventing from above,

$$r = \frac{\pi}{\Phi} + 1,$$

the growth rate of consumption can be written as

$$\gamma_c = \left(\beta\left(\frac{\pi}{\Phi} + 1\right)\right)^{\frac{1}{\theta}} - 1$$

and substituting in the profits to get

$$\gamma_c = \left(\beta\left(\frac{\left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}{\Phi} + 1\right)\right)^{\frac{1}{\theta}} - 1.$$

In this model, the growth rate of consumption is a constant. The growth rate of consumption is larger, the larger is the population, L, and since the price is equla to $1/\alpha$, the higher the price for the intermediate goods, the lower is the growth rate.

Equilibrium in the financial markets imply that the total of the bonds held by the consumers, b_t, equal the total value of the only asset in the economy, which is the total value of the inventions held by the inventors, $N_t\Phi$, or

$$b_t = N_t\Phi.$$

Given that the salary is equal to

$$w_t = (1-\alpha)\frac{Y_t}{L_t}$$

we can write the budget constraint of the consumers as

$$N_{t+1}\Phi = (1-\alpha) Y_t + r\Phi N_t - c_t.$$

Subtracting ΦN_t from both sides, we get

$$(N_{t+1} - N_t)\Phi = (1-\alpha) Y_t + (r-1)\Phi N_t - c_t,$$

where $(N_{t+1} - N_t)\Phi$ is the investment being made in inventing new goods in this period. Using

$$(r-1)\Phi = \pi,$$

one can rewrite the expression as

$$(N_{t+1} - N_t)\Phi = (1-\alpha) Y_t + \pi N_t - c_t.$$

Since $\pi = \left(\frac{1}{\alpha} - 1\right)x$, this becomes

$$\begin{aligned}(N_{t+1} - N_t)\Phi &= (1-\alpha) Y_t + \left(\frac{1}{\alpha} - 1\right) x N_t - c_t \\ &= Y_t - xN_t + \frac{xN_t}{\alpha} - \alpha Y_t - c_t.\end{aligned}$$

It is fairly easy to show that $\frac{xN_t}{\alpha} - \alpha Y_t = 0$, so the equation gives

$$(N_{t+1} - N_t)\,\Phi = Y_t - xN_t - c_t.$$

Production is used for consumption, c_t, for inventing new goods, $(N_{t+1} - N_t)\,\Phi$, and for making the intermediate goods that will be used in this period's production, xN_t. Use this equation to write out the growth rate of the number of goods as

$$\begin{aligned}
\gamma_N \Phi &= \frac{(N_{t+1} - N_t)}{N_t}\Phi = \frac{Y_t}{N_t} - x - \frac{c_t}{N_t} \\
&= \frac{AL^{1-\alpha}\,(N_t x)^{\alpha}\,N_t^{1-\alpha}}{N_t} - x - \frac{c_t}{N_t} \\
&= AL^{1-\alpha}x^{\alpha} - x - \frac{c_t}{N_t}.
\end{aligned}$$

All the items in the last row are constants except c_t/N_t and for the growth rate of the number of goods to be a constant, N_t needs to grow at the same rate as c_t so that c_t/N_t is also a constant.

1.3 Is the market solution an optimum?

To see if the above market solution is an optimum, we need to compare it to that which could be obtained by a "social planner" who maximizes the same utility function subject to the resouce constraint

$$Y_t = AN_t x^{\alpha} L^{1-\alpha} = c_t + xN_t + \Phi\,(N_{t+1} - N_t)\,.$$

The planner divides the goods produced in period t between consumption, intermediate goods and goods to produce new inventions so as to maximize

$$\sum_{i=0}^{\infty} \beta^i \frac{c_t^{1-\theta} - 1}{1-\theta}.$$

The Lagrangian is

$$\mathcal{L} = \sum_{i=0}^{\infty} \beta^i \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + \lambda_t \left(AN_t x^{\alpha} L^{1-\alpha} - c_t + (\Phi - x)\,N_t - \Phi N_{t+1} \right) \right]$$

The first order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t} &= \frac{1}{c_t^{\theta}} - \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial x} &= \alpha AN_t x^{\alpha-1} L^{1-\alpha} - N_t = 0 \\
\frac{\partial \mathcal{L}}{\partial N_{t+1}} &= -\lambda_t \Phi + \beta \lambda_{t+1} \left(Ax^{\alpha} L^{1-\alpha} + \Phi - x \right) = 0
\end{aligned}$$

or that

$$
\begin{aligned}
1 &= \alpha A x^{\alpha-1} L^{1-\alpha} \\
\frac{1}{\beta}\Phi &= \left(\frac{c_t}{c_{t+1}}\right)^{\theta} \left(A x^{\alpha} L^{1-\alpha} + \Phi - x\right).
\end{aligned}
$$

In addition, there is the budget constraint

$$
A N_t x^{\alpha} L^{1-\alpha} = c_t + x N_t + \Phi \left(N_{t+1} - N_t\right).
$$

The first first order condition says that

$$
x_{sp} = A^{\frac{1}{1-\alpha}} L \alpha^{\frac{1}{1-\alpha}}.
$$

Compare this to the amount of each intermediate good used in the market solution,

$$
x_m = A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2}{1-\alpha}}.
$$

Since $\alpha < 1$, and first two items of the equations are identical, $x_m < x_{sp}$ and the amount of goods produced by the market is less than that produced bythe social planner. This is because in the market solution the price of the intermediate good is greater than its marginal cost while in the social planner solution it is set equal to its marginal cost.

The growth rate of consumption with the social planner is equal to

$$
\begin{aligned}
\gamma_c^{sp} &= \frac{c_{t+1}}{c_t} - 1 = \left[\frac{\beta\left(A x^{\alpha} L^{1-\alpha} + \Phi - x\right)}{\Phi}\right]^{\frac{1}{\theta}} - 1 \\
&= \left[\frac{\beta\left(A^{\frac{1}{1-\alpha}} L \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) + \Phi\right)}{\Phi}\right]^{\frac{1}{\theta}} - 1
\end{aligned}
$$

and this can be compared to the growth rate of consumption in the market economy,

$$
\gamma_c^{m} = \left(\beta \frac{\left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + \Phi}{\Phi}\right)^{\frac{1}{\theta}} - 1.
$$

Comparing the two growth rates everything cancels except

$$
\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \alpha^{\frac{1}{1-\alpha}} < \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right),
$$

where the first element comes from the market growth rate equation and the second from the social planner solution. Clearly, with $\alpha < 1$, the social planner economy grows faster than the market economy.