1 Malthus and history

Human history, on average, has been, as said Hobbs, nasty, brutish, and short. Robert Lucas shows the graph in Figure 1 as what the annual per capita consumption has been over human history. For most of this history, consumption has been near the subsistance level, around US$500 per year. It has fluxuated a little: during the Roman empire it rose a bit, but it had always returned to the same level until beginning about 1800. Malthus noted that the human population grew over this long period as did the technology level: from small bands of simple hunter and gatherer, to settled farmers, to the rise of cities, the population density increased and humans spread out from Africa and settled most of the world. Although over the long run there was a relatively regular flow of new technology, new tools for hunting, new techniques for farming, new goods and handicraft industries to make them, per capita consumption remained steady at the subsistance level.

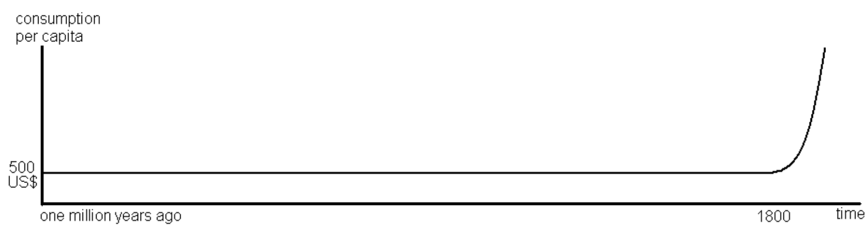

Figure 1: Per capita consumption over human history

Malthus looked for a theory to explain these data. He described a model where technology could increase, but the average benefits of these technological improvements were only temporary. During the long history of humans, most people lived very near the subsistance level of consumption.

1.1 The model

Goods production is a function of the level of technology, z_t, the amount of land that is being used in production, L, and the amount of labor (the population) that is working, N_t. We assume the amount of land is a constant so there is no need for a time subscript. Output of goods (which is also consumption of goods since there is no capital accumulation in this model) is equal to

$$C_t = Y_t = z_t F\left(L, N_t\right).$$

The production function displays constant returns to scale. We can write the consumption per person as

$$c_t = \frac{C_t}{N_t} = \frac{Y_t}{N_t} = \frac{z_t F\left(L, N_t\right)}{N_t} = z_t F\left(\frac{L}{N_t}, 1\right) = z_t F\left(l_t, 1\right) = z_t f\left(l_t\right).$$

For a given level of technology, consumption per person as a function of the amount of land per person looks like Figure 2. Notice that this production function has the characteristic that the marginal product of land and labor decline as each increases relative to the other. Figure 2, the graph shows that the marginal produce of land per unit of labor declines as the amount of land increases. The amount of production (and consumption) increase with more land per unit of labor, but the marginal increases are smaller.

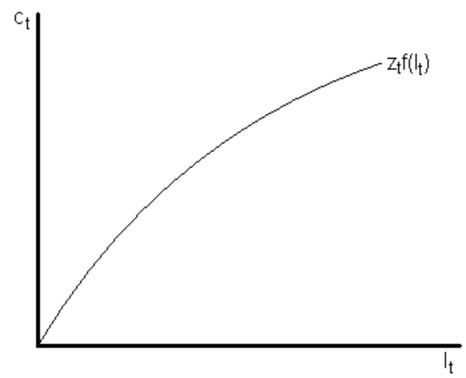

Figure 2: per capita consumption as a function of per capita land

Population growth is determined by consumption per person. If individuals consume more, the death rate is likely to be lower and the live birth rate (and the survival of infants) is likely to be higher. Both of these effects imply that more consumption implies that the population will grow more. Define the growth rate of the population as

$$\gamma_t^n = \frac{N_{t+1}}{N_t} = g(c_t)$$

and the $g(c_t)$ function looks like Figure 3.

Here c^* is the amount of consumption per person that just keeps the population constant.

We can combine the two graphs to show how they are related. In Figure 4, the graph of Figure 3 has been flipped so that consumption per capita is on the vertical axis in both graphs. Growth rates in the left hand side graph is measured on the horizontal axis, increasing to the left. The graph shows the amount of land per capita, l^*, that is a stationary state: a point where the population does not grow, where $\gamma_t^n = 1$, and land per capita is constant. At this point, consumption per capita is c^*.

Suppose that there is an increase in technology: z_t goes up to z_2. This is shown if Figure 5. For each unit of land per unit of labor, output and consumption go up. The technology increase means that at at each level of land per unit of labor, more goods are produced and consumed.

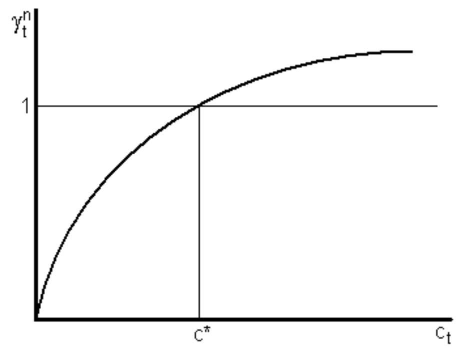

Figure 3: growth rate as a function of per capita consumption

We have assume that increased consumption per capita results in a decline in mortality and an increase in the birth rate. These result in an increase in the population growth rate as shown in Figure 6, where the population growth rate has increased to $\gamma_2^n > 1$. As time passes, the increase in the population growth rate causes the population to increase and, since the amount of land available is a constant, the amount of land per capita declines. The population keeps growing and the amount of land per capita keeps declining. This causes mortality to rise and the birth rate to decline until the output (and consumption) per capita returns to c^* and the population growth rate returns to $\gamma_t^n = 1$. This new stationary state occurs when the amount of land per capital is at l_2^* as shown in Figure 7.

The result of Malthus' theory is that output and consumption per capita always returns to the subsistance level, where the subsistance level of consumption is defined as that where the population is constant (where the population just sustains itself). As technology improves, the size of the population that is the subsistance level grows. The higher population drives down the amount of land per unit of labor and therefore, the marginal product of labor, until it is just what is needed to keep the population constant. At that point, the population need not fall any further, it can just sustain itself.

1.2 Simulation

To show how the model works, we can run a simulation of Malthus' economy. Let the production function be

$$Y_t = z_t L^\theta N_t^{1-\theta},$$

where z_t is the level of technology, L is the fixed amount of land, and N_t is the population. This can be written in per capita terms as

$$y_t = \frac{Y_t}{N_t} = z_t \left(\frac{L}{N_t}\right)^\theta = z_t l_t^\theta.$$

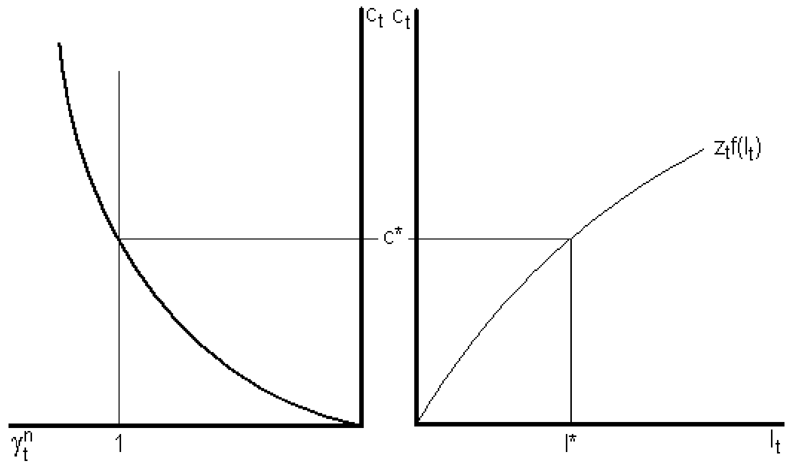

Figure 4: Stationary state of Malthus model

Let the $g(c_t)$ function be

$$g(c_t) = \beta c_t^{\alpha}.$$

Choose values for the variables: $z_0 = 100, \theta = .4, \beta = .045$, and $\alpha = .5$. Suppose that the initial population is $p_0 = 100$. We start the economy in a stationary state, where $g(c_t) = 1$, and let it run for four periods. In period 5, we let the technology go up to 120 and let the economy run to period 30. These numbers were chosen so that the initial stationary state equilibrium would have consumption near 500. Figure 8 shows the results of this simulation.

In period 5, output (which equals consumption) and the growth rate of population jump. Given that this is a model in discrete time, the population growth doesn't show up until period 6. The population grows and with the land fixed, the amount of land that each worker can use declines and output per worker drops. Since output per worker is consumption per worker, the decline in consumption per worker reduces the rate of population growth. This process continues until the population stops growing. At this point, the population has grown to 157.5 persons living on the same amount of land as before. The 20% increase in total factor productivity resulted in and increase in population of 57.5%.

1.3 Conclusion

This theory of Malthus could explain the pre-1800 economic history of the world reasonably well. Output per capita is constant even though there is a continual, if slow, advance in technology. What Malthus is not able to explain is the rapid growth in per capita income and consumption since the beginning of the industrial revolution.

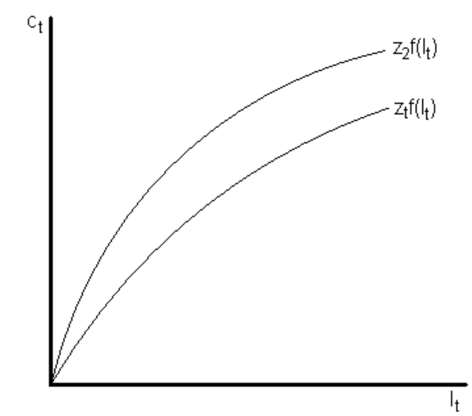

Figure 5: An increase in technology to z_2

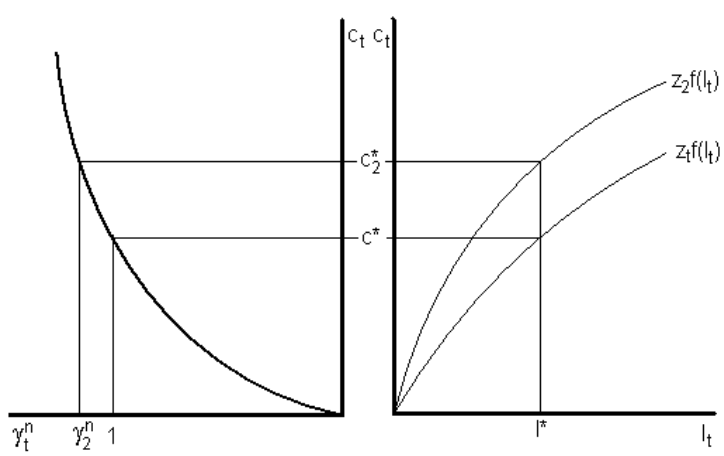

Figure 6: Immediately after technology increase

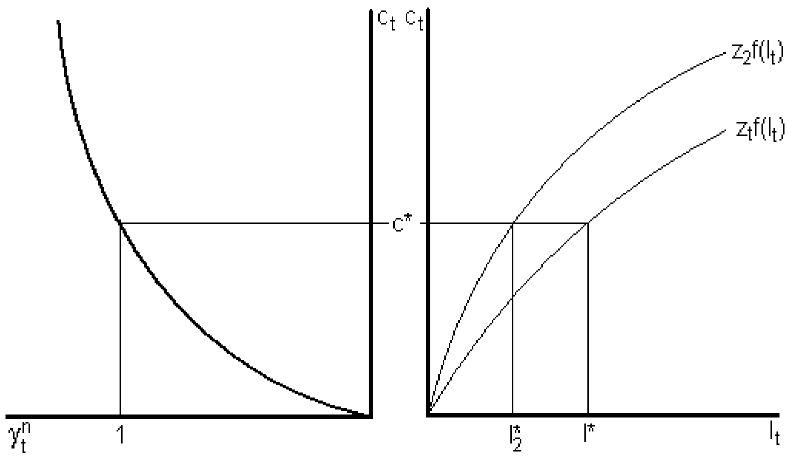

Figure 7: New stationary state with smaller amount of land per capita

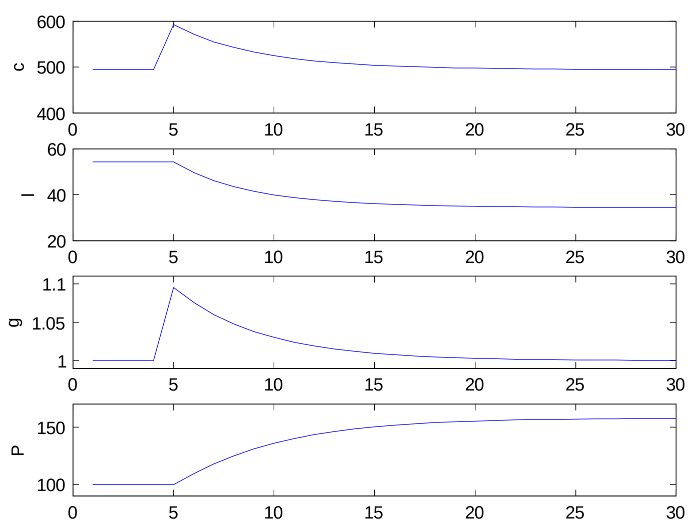

Figure 8: Results of simulation of Malthus economy