Creciemiento Economico
Creative destruction

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Today's model is that of Aghion and Howitt: A model of Growth Through Creative Destruction, Econometrica, 1992.

This model adds two elements to the growth model: the Schumpeterian idea of creative destruction and a simple version of uncertainty. It is a model of product development through research and development, with firms that specialize in research, other firms that specialize in production of an intermediate good, and a set of firms that specialize in pruduction of the final good using the intermediate good and unskilled labor. As in the previous model, once a technology is invented, the inventor gets a patent for the invention that gives them a monopoly on production. As before, the patent has an infinite life, but here that isn't all that important. The reason that the infinite life of the patent isn't important is because the next patented invention will replace the old one in the production process.

Creative destruction is the name that Schumpeter gave to the process of capitalist economies whereby new inventions either as processes or as goods drive older inventions out of the market. The automobile has pretty much replaced horses for transport, the computer has replaced quite a number of items: the slide rule, the typewriter (how many of you have seen a typewriter?), typesetting machines, film and photoprocessing equipment, to name just a few. The search for new inventions and new markets is, according to Schumpeter, the source of growth of the capitalist system. One cost of this process is that older inventions lose their value once they are replaced by something that does the job better.

0.1 The ideas behind the model

The model has three types of labor, although two of them are not very interesting from the point of view of solving the model, they do make the model more realistic. There is unskilled labor that is used only in the production of the final good. Much final good production is pretty routine so this assumption is not unreasonable. Skilled labor can be used in both the production of intermediate goods and in the production of a new technology. This assumption allows

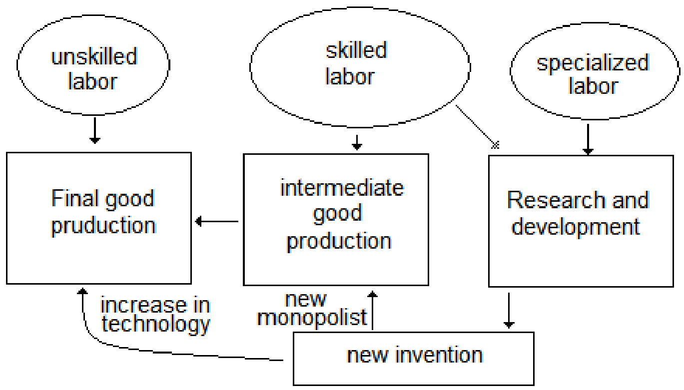

Figure 1: Drawing of the model

for competition between the two sectors for the services of skilled labor and means that increases in research and development have costs in terms of the amount of intermediate good (and, since intermediate goods are inputs into the final goods producions, in the amount of the final good) that will be available. Research needs specialized labor plus skilled labor. The specialized labor are the researchers who aren't really good at other things (to me this is pretty realistic). All three types come in fixed supply. The only labor decision needed in each period is how much of the skilled labor goes to intermediate goods production and how much to research.

Figure 1 shows the basic structure of the model. Some skilled labor and all the specialized labor work in research. When they invent a new technology, it gets a patent, becomes the only intermediate good in production (a monopoly) and increases the technology level of the final good production. The rest of the skilled labor produces the intermediate goods which go to the final goods production where they are used by the unskilled labor to produce the final goods.

The model tries to be realistic about the development of new technologies. Firms can do research for a long time without getting much in terms of results or other firms can be lucky and have a good result from their research quickly. Some technological developments can be marginal and not destroy or make obsolete the existing technology. Others are drastic in the sense that they do make the existing technologies obsolete. In this model, all technological developments make older technologies obsolete. This is done to make the model simplier. Near the end of their paper, Aghion and Howitt do tackle the problem of less than drastic technological changes. The details change but not the basic result. In the version here, all technological change exhibits creative destruction, completely wiping out older technologies and making the value of the patent on that technology equal to zero.

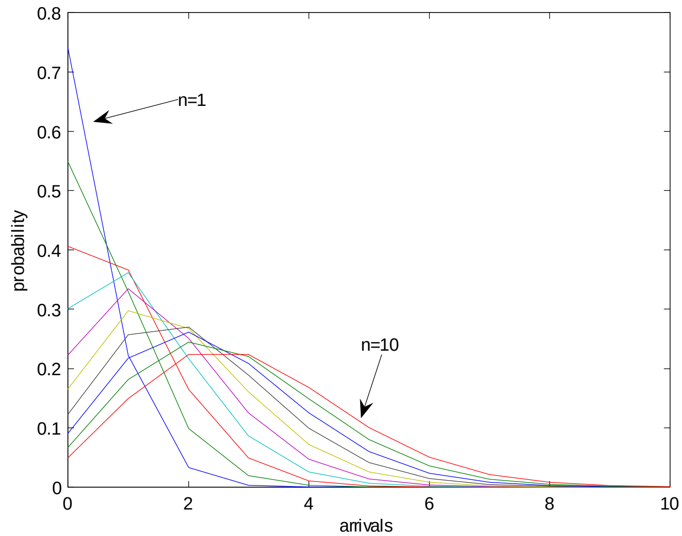

Figure 2: Poisson arrival rates

The model assumes that technological advances take place according to a Poisson arrival process. Technological changes occur to a reseacher at an arrival rate λ. Poisson processes are additive so that if there are n_t researchers, the arrival rate of $n_t\lambda$. Every time a new invention is made, total factor procuctivity, A_t in a production function of the final good,

$$Y_t = A_t x_t^{\alpha} L_u^{1-\alpha},$$

increases by the factor γ so that $A_{t+1} = \gamma A_t$. The amount of technological improvement that will come from a new invention is assumed to be known. What is not known is how long it will take for that invention to arrive. That is where the Poisson process enters. If there are n_t people working on research and development then the arrival rate is equal to $n_t\lambda$. The Poisson distribution has the density function

$$f\left(k|n_t\lambda\right) = \frac{\left(n_t\lambda\right)^k e^{-n_t\lambda}}{k!}$$

which gives the probability of exacty k occurances of the event in one period when the arrival parameter is λ. Figure 2 shows the probability densiy functions for $\lambda = .3$ and for $n = 1$ to 10.As one can see from the graph, increasing the number of researchers working on inventing the next technology shifts the probability distribution to the right, the probability of more inventions per period goes up as n goes up.

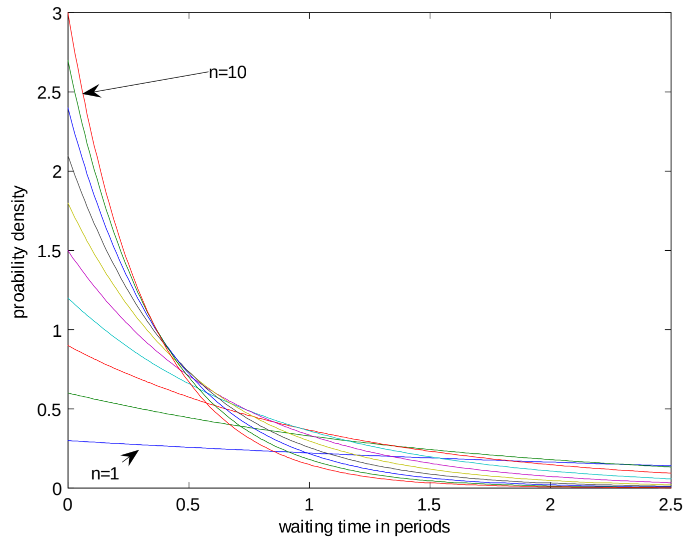

Figure 3: Waiting times from exponential distribution

The expected waiting time between new inventions is $1/(n_t\lambda)$ and the distribution of the waiting times (as the time between inventions is called) is an exponential density function with parameter $1/(n_t\lambda)$ of the form

$$f\left(x|\frac{1}{n_t\lambda}\right) = \frac{1}{n_t\lambda}e^{-\frac{x}{n_t\lambda}}.$$

The density functions for waiting times for $\lambda = .3$ and $n_t = 1$ to 10 are shown in Figure 3.

0.2 The model

0.2.1 Households

Households provide labor of the three types. The amount of each kind of labor, unskilled, skilled, and specialized are given and constant. All the unskilled labor, $M = 1$, is used in the production of the final good. In the paper by Aghion and Howitt, all of the specialized labor, R, is used in research and development. We assume that $R = 0$ in this presentation to keep things simplier. Skilled labor, N, can be used in both the intermediate good production and research so

$$N = L_t + n_t$$

where L_t is the amount of skilled labor used in intermediate good production and n_t is the amount used in research. The households maximize the discounted utility function

$$u = \int_0^\infty e^{-rt} y_t dt,$$

where $r < 0$ is the constant rate of time preference, y_t is the consumption of the final good at time t. Note that there is no utility loss from providing any type of labor to production.

0.2.2 Production

Production of the final good takes place with the technology

$$y_t = A_t x_t^\alpha.$$

with $0 < \alpha < 1$. The intermediate good production is monopolistic, given that only the most advanced technology good is produced, and the production technology is

$$x_t = L_t.$$

Research generates new technologies at an poisson arrival rate equal to

$$\lambda n_t.$$

When a technological innovation arrives, it is immediated adapted by an intermediate firm (which now has a monopoly on the production of that intermediate good) and the use of this new intermediate good causes the technology parameter in the final good production to increase by

$$A_{t+1} = \gamma A_t$$

where $\gamma > 1$.

0.2.3 Decision of the monopolist of the intermediate good

An intermediate good firm owns the currently most productive technology. We assume that this technology is drastic in that it completely replaces earlier technologies. The monopolist knows that the final good production function is

$$y_t = A_t x_t^\alpha$$

and uses that to determine what price it should charge the final good producer for the intermediate good. The final good producer maximizes the flow of profits as

$$\pi_t^{final} = y_y - p_t x_t = A_t x_t^\alpha - p_t x_t$$

by chosing the amount of intermediate good it wants to use. First order conditions are

$$\frac{\partial \pi_t^{final}}{\partial x_t} = \alpha A_t x_t^{\alpha-1} - p_t = 0,$$

so the (inverse) demand for intermediate goods is

$$p_t = \alpha A_t x_t^{\alpha-1}$$

Given that the time the intermediate good producer will own the most advanced technology is unknown and uncertain, it chooses to maximize the flow of profits for as long as it owns the lead patent, subject to the demand for its good by the final good producer. Therefore the intermediate good producer maximizes

$$\begin{aligned}\pi_t^{int} &= p_t x_t - w_t L_t = p_t x_t - w_t x_t \\ &= \alpha A_t x_t^{\alpha-1} x_t - w_t x_t \\ &= \alpha A_t x_t^{\alpha} - w_t x_t.\end{aligned}$$

First order conditions are

$$\frac{\partial \pi_t^{int}}{\partial x_t} = \alpha^2 A_t x_t^{\alpha-1} - w_t = 0$$

or

$$x_t = \left(\frac{\alpha^2}{\frac{w_t}{A_t}}\right)^{\frac{1}{1-\alpha}}.$$

If we define $\varpi_t = w_t/A_t$ as the technology-adjusted wage, x_t is a decreasing function of ϖ_t of the form

$$x_t = \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{1}{1-\alpha}}.$$

The intermediate good producer's profits are

$$\begin{aligned}\pi_t &= \alpha A_t x_t^{\alpha} - w_t x_t \\ &= \alpha A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}} - A_t \varpi_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{1}{1-\alpha}} \\ &= \alpha A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}} - \alpha^2 A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}} \\ &= (1-\alpha)\,\alpha A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}}. \\ &= A_t \widetilde{\pi}(\varpi_t),\end{aligned}$$

where $\widetilde{\pi}(\varpi_t)$ is a decreasing function of the technology adjusted wage. Taking the deriviative of profits with respect to wages gives

$$\begin{aligned}\frac{\partial \pi_t}{\partial \varpi_t} &= \frac{\alpha}{1-\alpha}(1-\alpha)\,\alpha A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}-1} \left(-\frac{\alpha^2}{\varpi_t^2}\right) \\ &= -\alpha^2 A_t \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\varpi_t}\right) < 0,\end{aligned}$$

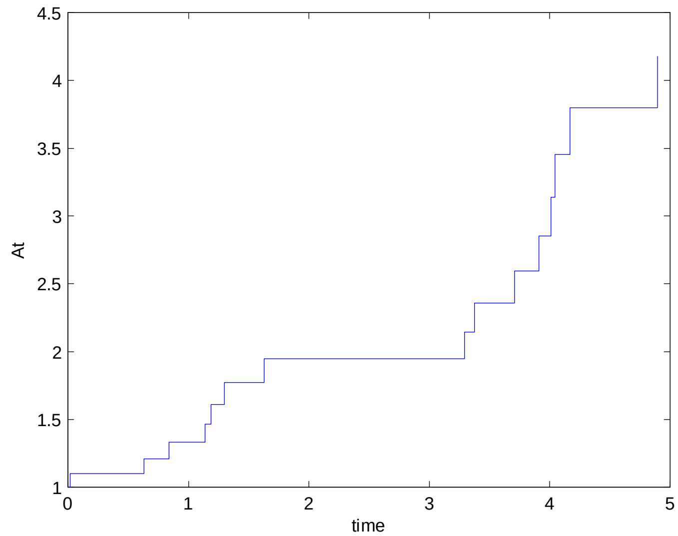

Figure 4: Innovations path

so that the intermediate producer's profits decline as ϖ_t rises. The implication of this result is that higher expected demand for **future** research drives up expected future wages and drives down the profits from future research that can be appropriated by the next inventor. This tends to discourage current research so that n_t comes down.

0.2.4 Research

Inventions occur with Poisson arrival rate λ per researcher or (which is the same thing) with an average waiting time of $1/\lambda$. Poisson processes are additive so with n_t researchers, the Poisson arrival rate for the economy is λn_t and the economy wide average waiting time is $1/\lambda n_t$. The arrival of a new technology increases productivity by a factor of γ, so $A_{t+1} = \gamma A_t$. A sample run with $\gamma = 1.1$, A0=1, $\lambda = .3$, and $n_t = 8$ is shown in Figure 4.

Because of the existence of patents for inventions and because inventions are drastic (they dominate old inventions), the inventor of a new innovation can sell it to an intermediate good firm which will have monopoly rights production of the intermediate good until another invention comes along. Notice from the graph that some inventions endured for only a short time but one (the 8th) endured for 1.66 time units. Because a new technology is known to all (although only the monopolist can use it), all continuing researchers know the new technology and make advances beginning there.

The object of a research firm is to hire labor so as to maximize the flow of expected profits from research. A research firm with n_t researchers has a flow of expected profits equal to

$$\lambda n_t V_{t+1} - w_t n_t,$$

where V_{t+1} is the value of the next $(t+1)$ innovation. Maximizing profits with respect to the number of resarchers gives that

$$\lambda V_{t+1} - w_t.$$

The intermediate good firm that own the time t technology will not engage in research. By doing so it will shorten the expected time it has a monopoly on the technology and if it is successful will have a gain from research equal to $V_{t+1} - V_t$ while other research firms will have a gain equal to V_{t+1}.

The value of the new technology is the expected discounted flow of profits it will produce. Notice that the flow of profits is constant over the time that the firm has the lead technology (since production of the final and intermediate goods are constant). Call these expected profits π_{t+1}. Then the value of the firm is

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}}.$$

This value comes from the fact that profit flows are discounted by the interest rat r and by the exponential expected arrival rate of the next technological breakthrough. This equation can be written as

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1}.$$

This form of writing the equation says that the expected income generated by one period of use of the new technology is equal to the profits during that time minus the expected "capital loss" that occurs if a new innovation occurs. This loss reflects Schumpeter's idea of creative destruction. More future research will (increases in n_{t+1}) reduce the expected value of the flow of profits from current inventions and make the current innovation worth less than otherwise.

Notice that there is an important technological externality. Each invention raises the productivity of all future inventions by γ and the inventor is not able to capture the benefits from these social gains. This would suggest that a social planner would want to have more research than would occur in a market economy because the social planner can internalize the externality.

0.2.5 Capital markets

Capital markets could be constructed in several different ways. An Arrow Debreu type world with frictionless credit markets discounting future consumption by the rate r is one possibility. Another is to have no credit markets: all workers consume their wages in every instant, monopoly firm owners consume their profits in every instant, and research workers receive no pay unless their research is successful and then they receive shares in the new intermediate good firm (this works because utility is linear in consumption, so people are risk neutral).

0.3 Equilibrium

There is only one decision for society to make: how much skilled labor to allocate between the intermediate good production and research, or

$$N = x_t + n_t.$$

Combining this equation with the equations

$$w_t = \lambda V_{t+1},$$

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}},$$

and

$$\pi_t = A_t \widetilde{\pi}\left(\varpi_t\right),$$

gives

$$\begin{aligned} w_t &= \lambda V_{t+1}, \\ \frac{w_t}{A_t} &= \lambda \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \frac{1}{A_t}, \\ \frac{w_t}{A_t} &= \lambda \frac{A_{t+1} \widetilde{\pi}\left(\varpi_{t+1}\right)}{r + \lambda n_{t+1}} \frac{1}{A_t}, \end{aligned}$$

or

$$\varpi_t = \lambda \gamma \frac{\widetilde{\pi}\left(\varpi_{t+1}\right)}{r + \lambda n_{t+1}}, \tag{1}$$

where

$$\widetilde{\pi}\left(\varpi_{t+1}\right) = (1-\alpha)\,\alpha A_{t+1} \left(\frac{\alpha^2}{\varpi_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}$$

.But

$$n_{t+1} = N - x_{t+1}$$

and

$$x_{t+1} = \left(\frac{\alpha^2}{\varpi_{t+1}}\right)^{\frac{1}{1-\alpha}},$$

so equation 1 can be written as

$$\varpi_t = \lambda \gamma \frac{\widetilde{\pi}\left(\varpi_{t+1}\right)}{r + \lambda \left(N - \left(\frac{\alpha^2}{\varpi_{t+1}}\right)^{\frac{1}{1-\alpha}}\right)}. \tag{2}$$

If one prefers, the model can also be thought of in terms of a wage equation,

$$\varpi_t = \lambda \gamma \frac{\widetilde{\pi}\left(\varpi_{t+1}\right)}{r + \lambda n_{t+1}},$$

and a labor market equilibrium equation,

$$n_t = N - \left(\frac{\alpha^2}{\varpi_t}\right)^{\frac{1}{1-\alpha}},$$

that can be written as

$$\varpi_t = \frac{\alpha^2}{(N - n_t)^{1-\alpha}},$$

0.3.1 Balanced growth paths (or stationary states)

We restrict attention to equilibria that are particular perfect foresight equilibria. At time t, the wages and employment of time $t+1$ need to be known to find the solution for time t. A perfect foresight equilibrium is a sequence of known $\{n_s\}_{s=t}^{\infty}$ where the equilibrium conditions (the wage equation and the labor market equilibrium equation) hold in every period s.

A stationary state is a perfect foresight equilibrium where the technology adjusted wage is constant through time and the amount of skilled labor supplied to research is also constant. These conditions are values $\varpi_t = \varpi_{t+1} = \varpi$ and $n_t = n_{t+1} = n$ where the equilibrium conditions hold.

Taking the wage equation (with the function for profits put in)

$$\frac{\varpi_t}{\lambda} = \gamma \frac{(1-\alpha)\,\alpha A_{t+1}\left(\frac{\alpha^2}{\varpi_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}}{r + \lambda n_{t+1}},$$

and substituting in the labor market equilibrium equation for both time t and time $t+1$, after a bit of algebra, one gets

$$\frac{\alpha^2}{\lambda\,(N - n_t)^{1-\alpha}} = \gamma \frac{(1-\alpha)\,\alpha A_{t+1}\,(N - n_{t+1})^{\alpha}}{r + \lambda n_{t+1}}. \qquad (3)$$

As the equation is written, the left hand side is the marginal cost of research and the right hand side is the marginal benefit of research. This equation can be solved for a equation of the form

$$n_t = \Theta\,(n_{t+1})$$

and a stationary state is the case where $n_t = n_{t+1} = \overline{n}$.

Aghion and Howitt illustrate the model in terms of equation of the costs and benefits of research. A graph similar to theirs is shown in Figure 5. The figure gives the stationary state equilibrium and shows the time path of another perfect foresight equilibrium.

Aghion and Howitt give this proposition.

Proposition 1 *The amount of research employment $\overline{n}$ in a stationary perfect foresight equilibrium increases with: a) a decrease in the rate of interest, b) an increase in the size γ of each innovation, c) an increase in the total endowment N of skilled labor, or d) an increase in the arrival rate λ.*

The proposition comes directly from the definition of costs and benefits of research given in equation 3.

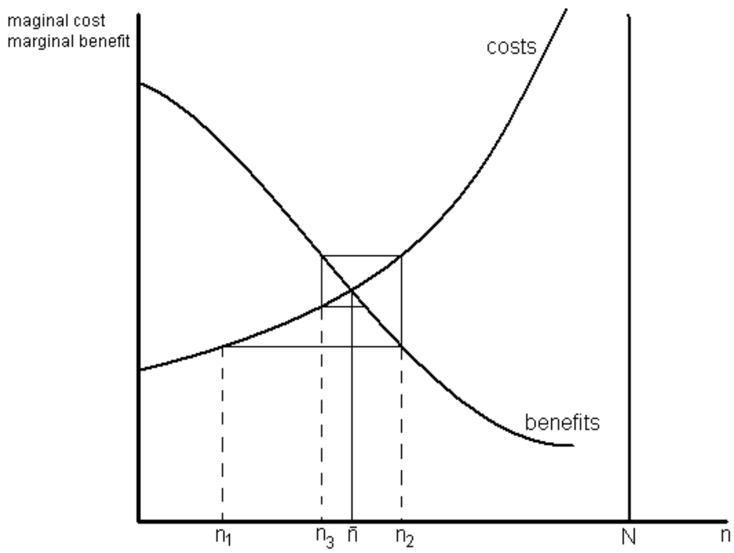

Figure 5: Costs and benefits of research: time path