Economic Growth
Class 8
Two more models of the AK type

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May 5, 2009

.Our three models type AK

- The AK model: $Y_t = AK_t$
- The Romer model with externality of aggregate per worker capital:
  - Production of firm i is equal to $Y(i)_t = AK(i)_t^{\theta} H(i)_t^{1-\theta} \kappa_t^{\eta}$
  - where $\kappa = K_t/H_t$
  - and where $\theta + \eta = 1$
- Government production model
  - production function is $Y_t = K_t^{\theta} G_t^{1-\theta}$
  - and the government budget constraint is $\tau Y_t = G_t$
- All of these function like AK models
  - All fail to have decreasing marginal product to capital
  - All fail the Inada conditions

.The Sobelow model: Combining Solow and AK

- Assume that the production function is

$$Y_t = AK_t + BK_t^{\theta} H_t^{1-\theta}$$

- This model gives constant returns to scale

$$A\lambda K_t + B(\lambda K_t)^{\theta} (\lambda H_t)^{1-\theta} = A\lambda K_t + \lambda BK_t^{\theta} H_t^{1-\theta} = \lambda Y_t$$

- Returns to capital and labor are positive

$$\frac{\partial Y_t}{\partial K_t} = A + \theta B K_t^{\theta-1} H_t^{1-\theta} > 0$$

$$\frac{\partial Y_t}{\partial H_t} = (1-\theta)\, B K_t^{\theta} H_t^{-\theta} > 0$$

- and are decreasing (check out the second derivatives)

The Sobelow model: Combining Solow and AK

- The model does not fulfill the Inada conditions (one of them)

$$\lim_{K\to\infty} \frac{\partial Y_t}{\partial K_t} = A \neq 0$$

$$\lim_{K\to 0} \frac{\partial Y_t}{\partial K_t} = \infty$$

$$\lim_{H\to\infty} \frac{\partial Y_t}{\partial H_t} = 0$$

$$\lim_{H\to 0} \frac{\partial Y_t}{\partial H_t} = \infty$$

- Fails in one of the conditions

The Sobelow model: Combining Solow and AK

- Find the per worker production function

$$y_t = A k_t + B k_t^{\theta}$$

- Use the law of motion of capital with the conditions for savings added

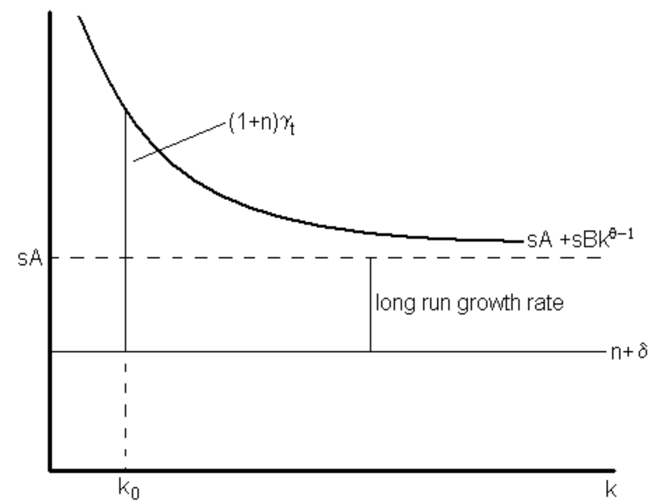
$$\begin{aligned}(1+n)\, k_{t+1} &= (1-\delta)\, k_t + s f(k_t) \\ &= (1-\delta)\, k_t + sA k_t + sB k_t^{\theta}\end{aligned}$$

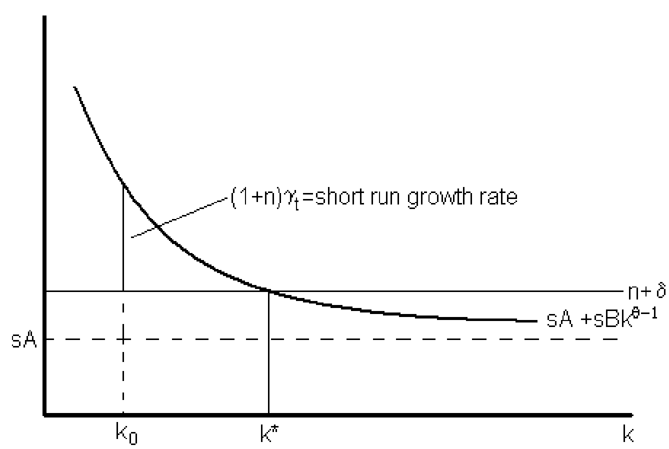
- Write as the growth rate of capital

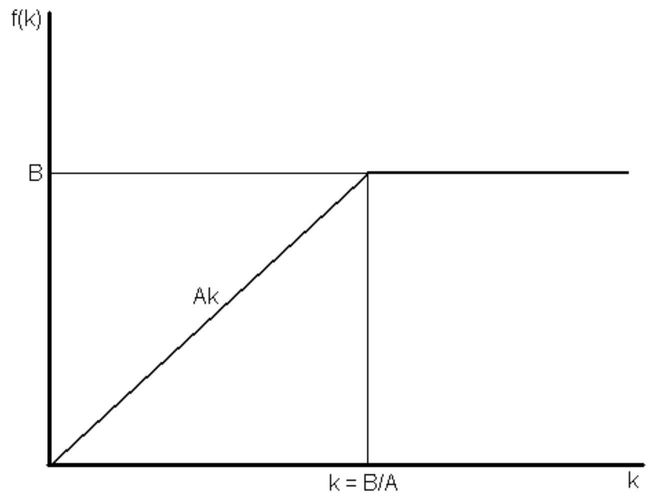
$$(1+n)\,\gamma_t = (1+n)\,\frac{(k_{t+1}-k_t)}{k_t} = sA + sB k_t^{\theta-1} - (n+\delta)$$

The Sobelow model: Combining Solow and AK
The Sobelow model: Combining Solow and AK
Harrod-Domar Model

- Very early growth model
  - Harrod (1939), Domar (1946)


(1+n)γt
sA +sBk^{θ-1}
sA
long run growth rate
n+δ
k0
k


(1+n)γt=short run growth rate
n+δ
sA +sBk^{θ-1}
sA
k0
k*
k



- Based on Leontief production functions

$$Y_t = \min\left[AK_t, BH_t\right]$$

- which in per worker terms is

$$y_t = \min\left[Ak_t, B\right]$$

- this says that
  - if k_t is small enough so that $Ak_t < B$, then $y_t = Ak_t$
  - if k_t is big enough so that $Ak_t \geq B$, then $y_t = B$
- The dividing point is when $k_t = B/A$

Harrod-Domar Model (the production function)
Harrod-Domar Model

- The usual law of motion of capital is

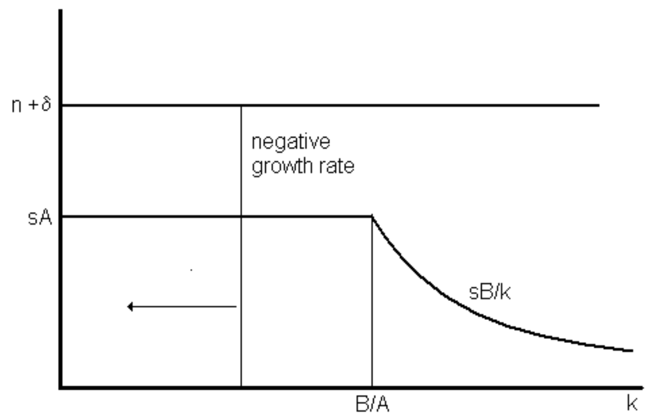
$$(1+n)\,k_{t+1} = (1-\delta)\,k_t + sf(k_t)$$

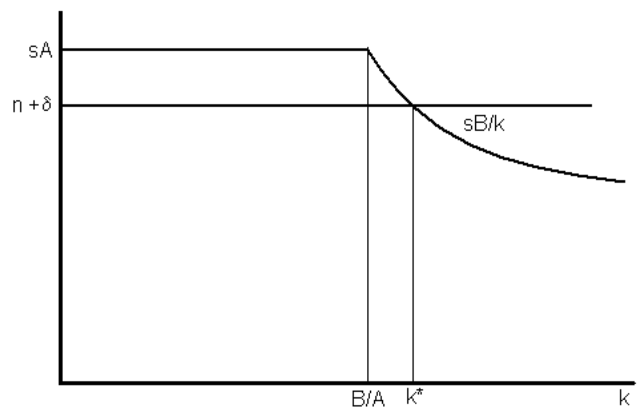
- This now needs to be written as

$$(1+n)\,(k_{t+1}-k_t) = \begin{cases} sAk_t - (n+\delta)\,k_t & \text{if } k < B/A \\ sB - (n+\delta)\,k_t & \text{if } k \geq B/A \end{cases}$$

- or as

$$(1+n)\,\gamma_t = \begin{cases} sA - (n+\delta)\,k_t & \text{if } k < B/A \\ sB/k_t - (n+\delta)\,k_t & \text{if } k \geq B/A \end{cases}$$





Harrod-Domar Model (three possible equilibria: 1)

- equilibrium is at $k = 0$

Harrod-Domar Model (three possible equilibria: 2)

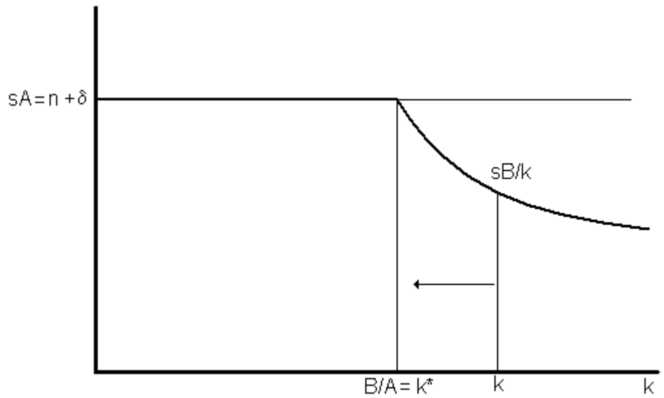
- stationary state at k^* but not all capital is utilized

Harrod-Domar Model (three possible equilibria: 3)

- Knife edge equilibrium

Harrod-Domar Model (possibility 3)

- If initial k is above B/A, the economy goes to $k^* = B/A$
- if initial k is below B/A, the economy stays there (no growth)



- Harrod and others at the University of Cambridge argued
  - the savings rate is not constant
  - with higher growth, the savings rate grows
  - workers have different savings rate = marginal propensity to save than capitalists
  - as economy grows, more income goes to the workers and the savings rate declines
  - think about what this means