Economic Growth
Class 7
Endogenous growth

Prof. McCandless
UCEMA

April 28, 2009

Endogenous growth: the idea

- We want a model where economies can keep growing
- Where growth does not need to depend on exogenous technology growth
  - So it is not about moving the stationary state
  - It is about economies that don't have stationary states
- The models have AK as their underlying basis

Paul Romer's model on externalities on capital

- Assume that there are some externalities from producing
- These externalities can come from
  - my capital is more productive based on total capital (or average capital per worker)
  - my capital is more productive if I produced more last period (learning by doing)
  - my capital is more productive if everyone knows more (knowledge spillovers)
- Example of a production function with these characterisitcs

$$Y_t = AK_t^{\theta} H_t^{1-\theta} \kappa_t^{\eta}$$

where κ_t is the externality

Romer's version

- let $\kappa_t = k_t = K_t/H_t$
- the above production function is then

$$\begin{aligned} Y_t &= AK_t^{\theta} H_t^{1-\theta} \kappa_t^{\eta} \\ &= AK_t^{\theta} H_t^{1-\theta} k_t^{\eta} \\ &= AK_t^{\theta} H_t^{1-\theta} \left(\frac{K_t}{H_t} \right)^{\eta} \\ &= AK_t^{\theta+\eta} H_t^{1-\theta-\eta} \end{aligned}$$

- Of special interest is when $\theta + \eta = 1$ or $\theta + \eta > 1$

Romer's version

- Write the model in per worker terms

$$Y_t = AK_t^{\theta+\eta} H_t^{1-\theta-\eta}$$

gives

$$y_t = \frac{Y_t}{H_t} = \frac{AK_t^{\theta+\eta} H_t^{1-\theta-\eta}}{H_t} = AK_t^{\theta+\eta} H_t^{-\theta-\eta} = Ak_t^{\theta+\eta}$$

or

$$y_t = Ak_t^{\theta+\eta}$$

Romer's version

- Assume a Solow type model where

$$\begin{aligned} (1+n) k_{t+1} &= (1-\delta) k_t + i_t \\ s_t &= i_t \\ s_t &= sy_t \end{aligned}$$

and the production function

$$y_t = Ak_t^{\theta+\eta}$$

- These can be combined to get the function

$$(1+n) k_{t+1} = (1-\delta) k_t + sAk_t^{\theta+\eta}$$

- The growth rate of capital per capita in period t, γ_t, is calculated as

$$\begin{aligned} \gamma_t &= \frac{k_{t+1}}{k_t} - 1 = \frac{(1-\delta)}{(1+n)} - 1 + \frac{sA}{(1+n)} k_t^{\theta+\eta-1} \\ &= -\frac{\delta+n}{(1+n)} + \frac{sA}{(1+n)} k_t^{\theta+\eta-1} \end{aligned}$$

.What happens when

- $\theta + \eta < 1$
- The growth rate can be written as

$$\gamma_t = \frac{sA}{(1+n)\, k_t^{1-\theta-\eta}} - \frac{(\delta+n)}{(1+n)}$$

where $1-\theta-\eta > 0$ and

$$\frac{d\gamma_t}{dk_t} = (\theta+\eta-1)\, \frac{sA}{(1+n)\, k_t^{2-\theta-\eta}} < 0$$

- when $\theta + \eta < 1$ the rate of growth declines with capital
- there is a stationary state when

$$(n+\delta)\, \overline{k} = sA\overline{k}^{\theta+\eta}$$

or when

$$\overline{k} = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\theta-\eta}}$$

.What happens when

- $\theta + \eta < 1$
- There are two stationary states: $\overline{k} = 0$ and $\overline{k} = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\theta-\eta}}$
- The model behaves like the standard Solow model between these points
  - grows toward the upper stationary state
  - location of upper stationary state depends on country's value for s, A, n, δ, θ and η

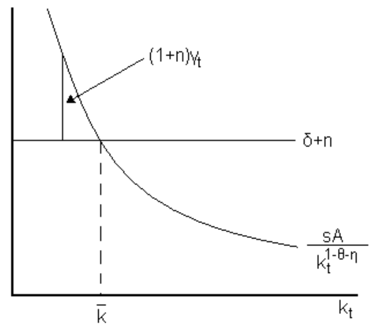
.What happens when

- $\theta + \eta < 1$
- The growth rate equation is

$$(1+n)\, \gamma_t = \frac{sA}{k_t^{1-\theta-\eta}} - (\delta+n)$$

- This gives the graph

.What happens when



- $\theta + \eta = 1$
- From above calculations, growth is

$$\gamma_t = \frac{sA}{(1+n)\, k_t^{1-\theta-\eta}} - \frac{(\delta+n)}{(1+n)}$$

- but in this case, $1 - \theta - \eta = 0$,
- so growth is simply

$$(1+n)\,\gamma_t = sA - (\delta+n)$$

- The result of this model is the same as an AK model

.What happens when

- $\theta + \eta = 1$
- There is no stationary state
- growth rate is constant at all capital stock levels

.What happens when

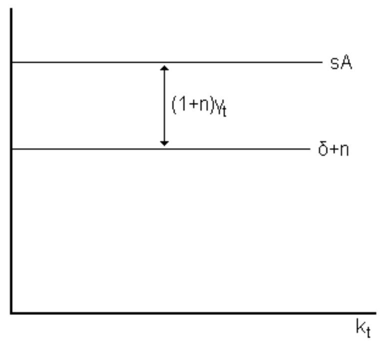
- $\theta + \eta > 1$
- From above calculations, growth is

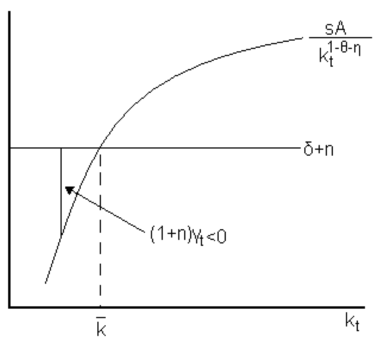
$$\gamma_t = \frac{sA}{(1+n)\, k_t^{1-\theta-\eta}} - \frac{(\delta+n)}{(1+n)}$$

- but $1 - \theta - \eta < 0$, so the growth rate can be written as

$$\gamma_t = \frac{sAk_t^{\theta+\eta-1}}{(1+n)} - \frac{(\delta+n)}{(1+n)}$$

where $\theta + \eta - 1 > 0$





- This means that the growth rate grows with the capital stock

.What happens when

- $\theta + \eta > 1$
- the stationary state is a repeller
- economies below $\overline{k}$ shrink and those above $\overline{k}$ grow every faster
- This is NOT observed in the data

.An alternative formulation

- Suppose that instead of $\kappa = k$, the per worker capital, the externality was based on $\kappa = K$, the stock of capital
- To keep things simple, assume that population is constant: $n = 0$
- Write the production function as

$$
\begin{aligned}
Y_t &= AK_t^{\theta} H_t^{1-\theta} \kappa_t^{\eta} \\
&= AK_t^{\theta} H_t^{1-\theta} K_t^{\eta} \\
&= AK_t^{\theta+\eta} H_t^{1-\theta}
\end{aligned}
$$

- Dividing by H_t, we get
$$y_t = Ak_t^\theta K_t^\eta$$
- K_t^η can be written as $K_t^\eta = k_t^\eta H_t^\eta$
- the per worker production function can be written as
$$y_t = Ak_t^{\theta+\eta} H_t^\eta$$

An alternative formulation

- Use the rest of the model
$$\begin{aligned} k_{t+1} &= (1-\delta)\, k_t + sy_t \\ &= (1-\delta)\, k_t + sAk_t^{\theta+\eta} H_t^\eta \end{aligned}$$
- to get
$$\gamma_t = \frac{k_{t+1}}{k_t} - 1 = sAk_t^{\theta+\eta-1} H_t^\eta - \delta$$
- In the AK case where $\theta + \eta = 1$, we get
$$\gamma_t = sAH_t^\eta - \delta$$
- H_t is the number of workers in the country, so bigger countries (China, India, USA, Mexico, Brazil) should grow faster than smaller ones (Hong Kong, Singapore, Luxemburg). This is a model with a *scale effect*.

Optimal taxes where government expenditures enter the production function

- Suppose government expenditures, G_t, enter the production function as
$$Y_t = AK_t^\theta G_t^{1-\theta}$$
- Disposible income (which is what matters to households) is
$$Y_t^d = (1-\tau)\, Y_t = (1-\tau)\, AK_t^\theta G_t^{1-\theta}$$
where τ is the tax rate on output.
- Assume that the government's budget is balanced: taxes equal expenditures, so
$$\tau Y_t = G_t$$

Optimal taxes

- Writing everything in per worker terms (dividing by H_t) and defining $g_t = G_t/H_t$, one gets
$$y_t^d = (1-\tau)\, Ak_t^\theta g_t^{1-\theta}$$

- The law of motion of capital is

$$\begin{aligned}(1+n)\,k_{t+1} &= (1-\delta)\,k_t + s y_t^d \\ &= (1-\delta)\,k_t + s\,(1-\tau)\,A k_t^{\theta} g_t^{1-\theta}\end{aligned}$$

- The growth rate of capital is

$$(1+n)\,\gamma_t = (1+n)\left[\frac{k_{t+1}}{k_t} - 1\right] = s\,(1-\tau)\,A k_t^{\theta-1} g_t^{1-\theta} - (n+\delta)$$

.Optimal taxes

- Using the government's budget constraint, one gets

$$g_t = \tau y_t = \tau A k_t^{\theta} g_t^{1-\theta}$$

which can be solved to get

$$g_t = \tau^{\frac{1}{\theta}} A^{\frac{1}{\theta}} k_t$$

- Putting this into the growth rate equation gives

$$\begin{aligned}(1+n)\,\gamma_t &= s\,(1-\tau)\,A k_t^{\theta-1}\left(\tau^{\frac{1}{\theta}} A^{\frac{1}{\theta}} k_t\right)^{1-\theta} - (n+\delta) \\ &= s\,(1-\tau)\,A A^{\frac{1-\theta}{\theta}} \tau^{\frac{1-\theta}{\theta}} - (n+\delta) \\ &= s\,(1-\tau)\,A^{\frac{1}{\theta}} \tau^{\frac{1-\theta}{\theta}} - (n+\delta)\end{aligned}$$

.Optimal taxes

- The optimal tax rate can be found from $\frac{d(1+n)\gamma_t}{d\tau} = 0$, or

$$0 = -sA^{\frac{1}{\theta}} \tau^{\frac{1-\theta}{\theta}} + \frac{1-\theta}{\theta} s\,(1-\tau)\,A^{\frac{1}{\theta}} \tau^{\frac{1-\theta}{\theta}-1}$$

- or

$$\frac{\theta}{1-\theta} = \frac{(1-\tau)}{\tau}$$

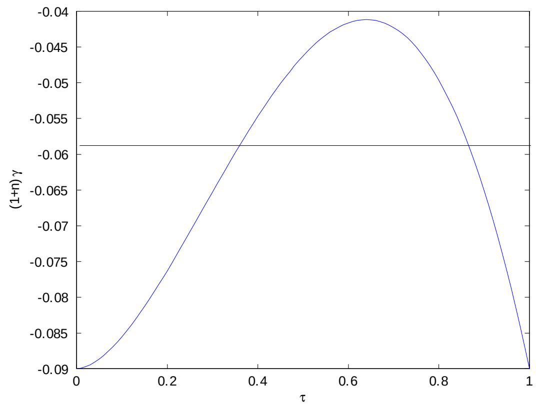
- or

$$\tau^* = 1-\theta$$

.Optimal taxes

- For an economy with $\theta = .36, A = 1, s = .3, \delta = .08, n = .01$, the growth rate as a function of the tax is
- max is at $\tau = .64$

.Homework



- Consider a model of learning by doing where the output of period t is a function of the output of period $t-1$ of the form

$$y_t = A k_t^{\theta} y_{t-1}^{\eta}$$

- Describe the economy when $\theta + \eta < 1$
  - find the stationary states
- Describe the economy when $\theta + \eta = 1$
  - find the constant growth rate for an example economy (you might need to use Matlab)