Economic Growth
Class 7
Learning by doing

Prof. McCandless
UCEMA

April 28, 2009

1 Learning by doing

Idea of learning by doing

- Based on a model by Robert Lucas
- Data from WW2 in the US: "Liberty Ships, cargo ship produced throughout the war
- 2458 ships produced in 14 different shipyards
- All built on standardized plans
- During the three years these were produced, output per man-hour grew by 40% per year

Model

- Consider a simple growth model where output today is a function of the capital stock and of the output produced the period before, output for firm j is
$$y_t(j) = Ak_t(j)^{\theta} y_{t-1}(j)^{\alpha},$$
- the law of motion for capital is
$$k_{t+1}(j) = (1-\delta)k_t(j) + sy_t(j)$$
where s is the constant fraction of output that is saved.

Stationary state

- Look for a stationary state, (using the previous equations)

$$y(j)^{1-\alpha} = Ak_t(j)^{\theta},$$

and

$$\frac{\delta}{s}k(j) = y(j)$$

- that gives

$$k_t(j) = \left(\frac{A}{\left(\frac{\delta}{s}\right)^{1-\alpha}}\right)^{\frac{1}{1-\theta-\alpha}},$$

and

$$y(j) = \frac{\delta}{s}k(j).$$

- For the case where $\theta + \alpha < 1$, one can find the stationary state for the economy as above.

AK version

- However, if $\theta + \alpha = 1$, the behavior of the model is different, it behaves as if it were an AK type model, converging to a constant growth rate path. Let γ be the constant growth rate of output

$$\begin{aligned}\gamma &= \frac{y_t(j)}{y_{t-1}(j)} = \frac{Ak_t(j)^{\theta} y_{t-1}(j)^{\alpha}}{Ak_{t-1}(j)^{\theta} y_{t-2}(j)^{\alpha}} = \left(\frac{k_t(j)}{k_{t-1}(j)}\right)^{\theta}\left(\frac{y_{t-1}(j)}{y_{t-2}(j)}\right)^{\alpha} \\ &= \left(\frac{k_t(j)}{k_{t-1}(j)}\right)^{\theta}\gamma^{\alpha}\end{aligned}$$

- This implies (because $1 - a = \theta$) that the growth rate of the capital stock is

$$\frac{k_t(j)}{k_{t-1}(j)} = \gamma$$

as well.

AK model

- Using this, we get

$$\begin{aligned}\gamma &= \frac{k_{t+1}(j)}{k_t(j)} = (1-\delta) + s\frac{y_t(j)}{k_t(j)} = (1-\delta) + sA\frac{y_{t-1}(j)^{\alpha}}{k_t(j)^{1-\theta}} \\ &= (1-\delta) + sA\frac{y_{t-1}(j)^{\alpha}}{k_t(j)^{\alpha}}\end{aligned}$$

- so

$$\frac{y_{t-1}(j)}{k_t(j)} = \left(\frac{\gamma-(1-\delta)}{sA}\right)^{\frac{1}{\alpha}}$$

- or

$$\frac{y_t(j)}{k_t(j)} = \gamma \left(\frac{\gamma - (1-\delta)}{sA} \right)^{\frac{1}{\alpha}}$$

AK model

- Using

$$\gamma = \frac{y_t(j)}{y_{t-1}(j)} = A \left(\frac{k_t(j)}{y_{t-1}(j)} \right)^{\theta} = A \left(\frac{sA}{\gamma - (1-\delta)} \right)^{\frac{\theta}{\alpha}}$$

- or

$$\gamma^{1+\frac{\alpha}{\theta}} - (1-\delta)\,\gamma^{\frac{\alpha}{\theta}} = sA^2$$

- since, in this case, $\theta = 1 - \alpha$,

$$\gamma - (1-\delta)^{1-\alpha}\,\gamma^{\alpha} = \left(sA^2\right)^{1-\alpha}$$

- This can be solved for the balanced growth rate, γ