Crecimiento Economico
Class 11
Research and Development

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.A model of Research and Development

- Competitive economies and R&D
- Standard theory indicates that in a perfectly competitive economy
  - There is NO research and development
- Inventor cannot benefit from invention: it is co-opted by the rest of the economy
- Output goes up for all producers (if technological improvement)
- Learning by doing is possible
  - those who produce get more efficient
  - only learn by doing, so those who don't do – don't learn

.Technological improvements

- What are they?
- A better way of combining factors to produce goods
  - Changing the way factors are mixed (farm example)
- Different types of goods
  - Hedonistic preferences
  - Am interested in the characteristics of the good
  - Many goods have different mixes of the same characteristics
  - New goods can have better mixes of the characteristics

  - Or can contain more characteristics in a single good (computer, for example)

Rival and non-rival goods

- A rival goods is one that when one person consumes it, no one else can
- My act of consuming makes it so that no one else can consume the good
- Examples
  - Food products (once I have eaten a steak, no one else can)
  - Driving a car, no one else can (but some people can go with me)
  - Clothes that I am wearing
  - A fish that I catch (unless I throw it back)
- Some goods are not completely rivals
- Examples
  - reading a book: while I am reading it no one else can, only after I finish
  - or a book can be photocopied (THIS IS ILLEGAL and against the 10 commandments)
  - music that has been bought on a CD

Non-rival goods

- Others can consume it while I am
- Examples
  - National defense
  - Air (although there are issues with air polution)
  - A sunset
  - Driving on a highway on off peak hours
    * During peak hours a highway becomes a rival good
  - transmission on cable television
  - transmission on open air tv channels

Excludable and non-excludable goods

- These are different from rival and non-rival
- A good is excludable is you can keep others from consuming it

- whether you consume it or not

- Private property is the making something excludable
  - I get to choose how to use my labor (unless I am a slave)
  - I can keep others out of my house
  - By eating a steak, I am excluding others from consuming it
  - Excludable goods among animals (territory)
- Food can be excludable or not
- The kiosk owner keeps me from eating the candy (until I pay him)
  - Note that a restraurant is different: consume then pay (can restrict entry)
- Robbery is making a normally excludable good non-excludable

Excludable and non-excludable goods

- Easily excludable goods: examples
  - food you are eating (although my some is sometimes difficult to exclude)
  - clothing you are wearing
  - use of a cable television line
- More difficult to exclude
  - Books and recorded music (because they can be copied)
  - computer programs (same reason)
  - From restraurants, clubs etc
  - people for living on public land
- Difficult to exclude
  - From fishing in the sea, hunting on public land
  - Using an air television signal (or radio signal)
  - air for breathing and for industrial processes

Tragedy of the commons

- What is a "commons"
- Land that could be used by all members of the village or town to graze their animals

- The main idea was a milk cow or a pig or chicken for meat (among peasants)
- Commons tended to be overused and over-eaten
- A "commons" is a non-excludable good from which people get rival benefits
- Commons tend to be over used and used up
  - rivers
  - water sources
  - air
  - The amazon jungle (for oxygen for all of us)
  - Environmental issues often deal with problems of commons
- Coase: Solution is giving people property rights over common goods (examples)

Rival and excludable goods

	rival	**non-rival**
easily excludable	food, clothes	cable tv
parcially excludable	books, seats in theaters	computer programs
difficult to exclude	fish in the sea	defense, ideas

So why are we talking about this?

- Problem of ideas
- Ideas are non-rival and difficult to exclude
- Need legal protection (or secrecy) to keep others from using your ideas
- Patents
  - legal monopoly on the use of an invention or idea
  - hold the right for a certain number of years
    - general goods in US: 18 years
    - medicines: 8 years
    - Artist products (books, music) very long time
- Firms tend to patent goods and keep processes secret
  - When you get a patent, you give away the secret (in the patent application)

Importance of patents

- Some economists argue that the industrial revolution began only after patents were invented
- Not quite true (but could be an important incentive)
- There were substantial (and frequently underappreciated) inventions during the medieval ages
- The British navy offered a very large prize for a good way of telling longitude
- Many mechanical processes were developed during this search
- Solution was an accurate clock (a cronometer)
- This required an ability to machine metal very accurately
- These tools made the steam engine and mechanical loom possible
- (story about Lowell)

.Our model of Research and development

- Assume that products from research and development have a patent with infinite life
- We don't believe this but it makes the math simplier
- Some other models that we don't do
  - vintage models: new inventions replace old ones
  - some processes or goods become obsolete
  - creative destruction (new goods cause old to be left behind:
    - cars and horse
    - computer and many things
    - complete change in photography (the man who fell to earth)
    - complete change in book publication

.Model of research and development

- Goods production: production function

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

- capital is an aggregate of intermediate goods with a CES production function of

$$K_t = \left(\sum_{j=1}^{N_t} x_{jt}^{\alpha} \right)^{\frac{1}{\alpha}}$$

- where N_t is the number of intermediate inputs at date t and x_{jt} is the amount of intermediate good j that is bought and used by the firms in period t.

- Putting this into the production function we get

$$Y_t = AL_t^{1-\alpha} \sum_{j=1}^{N_t} x_{jt}^{\alpha}.$$

Model of research and development

- Goods production has decreasing returns in each intermediate good
- but constant returns to scale in labor and aggregate capital
- All N_t goods are similar, so $x_{jt} = x_t$
- Production function becomes

$$\begin{aligned} Y_t &= AL_t^{1-\alpha} N_t \left(x_t\right)^{\alpha} \\ &= AL_t^{1-\alpha} \left(N_t x_t\right)^{\alpha} N_t^{1-\alpha}. \end{aligned}$$

.

- Goods firm present value equation

$$\sum_{i=0}^{\infty} \left(\frac{1}{r}\right)^i \left(AL_{t+i}^{1-\alpha} \sum_{j=1}^{N_t} x_{jt+i}^{\alpha} - w_{t+i} L_{t+i} - \sum_{j=1}^{N_t} p_{jt+i} x_{jt+i}\right).$$

- Here we let the interest rate be constant as it will be in a stationary state
- Given that production is an in-period problem, we can max

$$AL_t^{1-\alpha} \sum_{j=1}^{N_t} x_{jt}^{\alpha} - w_t L_t - \sum_{j=1}^{N_t} p_{jt} x_{jt}.$$

.

- First order condtions are

$$w_t = (1-\alpha) AL_t^{-\alpha} \sum_{j=1}^{N_t} x_{jt}^{\alpha}$$

  and

$$p_{jt} = \alpha AL_t^{1-\alpha} x_{jt}^{\alpha-1}$$

- the last can be written as

$$x_{jt} = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t p_{jt}^{-\frac{1}{1-\alpha}}$$

Intermediate capital goods

- Inventer has two problems to solve
- should he build a new good or not
- once invented, how much should he charge for it each period
- once invented, he will produce it every period
- Assume that the cost of producing an already invented intermediate good has a marginal cost of 1 unit of the final good

$$MC = 1$$

Intermediate capital goods

- For a good invented in some period in the past, the inventor chooses to maximize the value of the firm (the present value of discounted profits)

$$\sum_{i=0}^{\infty} \left(\frac{1}{r}\right)^i \pi_{jt+i} = \sum_{i=0}^{\infty} \left(\frac{1}{r}\right)^i (p_{jt+i} - MC) \, \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_{t+i} p_{jt+i}^{-\frac{1}{1-\alpha}}$$

- Given that the firm choose p_{jt}, the first order conditions are (after replacing MC with 1)

$$0 = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t p_{jt}^{-\frac{1}{1-\alpha}} + (p_{jt} - 1) \left(-\frac{1}{1-\alpha}\right) \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t p_{jt}^{-\frac{1}{1-\alpha}-1}$$

or

$$\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t p_{jt}^{-\frac{1}{1-\alpha}} = (p_{jt} - 1) \left(\frac{1}{1-\alpha}\right) \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t p_{jt}^{-\frac{1}{1-\alpha}-1}$$

Intermediate capital goods

- A lot cancels out and we get

$$p_{jt} = \frac{1}{\alpha}.$$

- Putting this into the demand function for good x_{jt} gives

$$x_{jt} = \alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} L_t \left(\frac{1}{\alpha}\right)^{-\frac{1}{1-\alpha}} = A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2}{1-\alpha}}.$$

- If we put this into the production function for the final good, we get

$$
\begin{aligned}
Y_t &= AL_t^{1-\alpha}\left(N_t A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2}{1-\alpha}}\right)^{\alpha} N_t^{1-\alpha} \\
&= A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L_t N_t .
\end{aligned}
$$

- Output in linear in the number of intermediate goods used in production

.

- Putting this price into the profit function gives

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i} \pi_{jt} = \sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i}\left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

- since the right hand side is a constant, this gives

$$\pi_{jt} = \pi = \left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L = \left(\frac{1}{\alpha}-1\right)x$$

- assuming that the interest rate stays constant

$$\sum_{i=0}^{\infty}\left(\frac{1}{r}\right)^{i} \pi_{jt} = \pi \frac{r}{r-1}$$

.Cost of inventing and present value of profits

- Assume free entry and exit of firms
- A firm pays the invention cost in period t but only starts earning profits in period t+1
- Free entry means that in equilibrium, invention costs, Φ, will be equal to

$$\Phi = \sum_{i=1}^{\infty}\left(\frac{1}{r}\right)^{i} \pi = \pi \frac{r}{r-1} - \pi = \frac{\pi}{r-1}$$

- Notice that the sum is from 1 to ∞ and not from 0 to infinity as before
- Cost of inventing could go up or down with number of inventions (in theory)
- We assume here that the cost of inventing is fixed and exactly equal to

$$\Phi = \frac{\pi}{r-1}$$

.Consumers

- Consumers maximize

$$\sum_{i=0}^{\infty} \beta^i \frac{c_t^{1-\theta} - 1}{1-\theta}$$

- subject to the budget constraint

$$b_{t+1} + c_t = w_t + r b_t$$

- where b_t are the bonds the households at the beginning of the period

.Consumers

- First order conditions from the consumer maximization problem give

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = r\beta$$

- With a constant growth rate of consumption

$$\gamma_c = \frac{c_{t+1}}{c_t} - 1 = (r\beta)^{\frac{1}{\theta}} - 1$$

.Consumers

- Using the condition on free entry into inventing from above,

$$r = \frac{\pi}{\Phi} + 1,$$

- the growth rate of consumption can be written as

$$\gamma_c = \left(\beta\left(\frac{\pi}{\Phi} + 1\right)\right)^{\frac{1}{\theta}} - 1$$

- and substituting in the profits to get

$$\gamma_c = \left(\beta\left(\frac{\left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}{\Phi} + 1\right)\right)^{\frac{1}{\theta}} - 1.$$

.Equilibrium in the financial market

- Consumers hold b_t bonds at the beginning of period t
- The only other asset in the economy are the stream of future profits that the intermediate goods firms are going to receive.
- Some of the lending can be used to finance new inventions

- All that implies that, in equilibrium,

$$b_t = N_t \Phi$$

- This can be put into the consumer's budget constraint along with the definition of wages as

$$w_t = (1-\alpha)\frac{Y_t}{L_t}$$

to get

$$\Phi N_{t+1} = (1-\alpha) Y_t + r\Phi N_t - c_t.$$

.Growth rate of number of goods

- Subtract ΦN_t from both sides of the budget constraint to get

$$(N_{t+1} - N_t)\Phi = (1-\alpha) Y_t + (r-1)\Phi N_t - c_t,$$

where $(N_{t+1} - N_t)\Phi$ is the investment being made in inventing new goods in this period.

- Using

$$(r-1)\Phi = \pi,$$

- one can rewrite the expression as

$$(N_{t+1} - N_t)\Phi = (1-\alpha) Y_t + \pi N_t - c_t.$$

.Growth rate of number of goods

- Since

$$\pi = \left(\frac{1}{\alpha} - 1\right) x$$

- this becomes

$$\begin{aligned}(N_{t+1} - N_t)\Phi &= (1-\alpha) Y_t + \left(\frac{1}{\alpha} - 1\right) x N_t - c_t \\ &= Y_t - x N_t + \frac{x N_t}{\alpha} - \alpha Y_t - c_t.\end{aligned}$$

- It is fairly easy to show that $\frac{xN_t}{\alpha} - \alpha Y_t = 0$, so the equation gives

$$(N_{t+1} - N_t)\Phi = Y_t - x N_t - c_t.$$

.Growth rate of number of goods

- From the equation

$$(N_{t+1} - N_t)\Phi = Y_t - x N_t - c_t.$$

one can see that production is used for

- paying of investment in new goods
- producing intermediate goods for production
- consumption

.Growth rate of number of goods

- Use the previous equation to write out the growth rate of the number of goods as

$$\begin{aligned}\gamma_N \Phi &= \frac{(N_{t+1} - N_t)}{N_t}\Phi = \frac{Y_t}{N_t} - x - \frac{c_t}{N_t} \\ &= \frac{AL^{1-\alpha}(N_t x)^{\alpha} N_t^{1-\alpha}}{N_t} - x - \frac{c_t}{N_t} \\ &= AL^{1-\alpha}x^{\alpha} - x - \frac{c_t}{N_t}.\end{aligned}$$

- everything on the right hand side is a constant except c_t/N_t which also needs to be a constant to get a constant growth rate of number of goods. That happens only when the growth rate of consumption is the same as that for the number of goods

.Comparing the market equilibrium to a social planner

- A social planner maxes utility subject to the resource constraint
- That is: max

$$\sum_{i=0}^{\infty} \beta^i \frac{c_t^{1-\theta} - 1}{1-\theta}$$

- subject to

$$Y_t = AN_t x^{\alpha} L^{1-\alpha} = c_t + xN_t + \Phi(N_{t+1} - N_t)$$

- The Lagrangian is

$$\mathcal{L} = \sum_{i=0}^{\infty} \beta^i \left[\frac{c_t^{1-\theta} - 1}{1-\theta} + \lambda_t \left(AN_t x^{\alpha} L^{1-\alpha} - c_t + (\Phi - x) N_t - \Phi N_{t+1}\right)\right]$$

.Comparing the market equilibrium to a social planner

- The first order conditions can be solved to give

$$1 = \alpha A x^{\alpha-1} L^{1-\alpha}$$

and

$$\frac{1}{\beta}\Phi = \left(\frac{c_t}{c_{t+1}}\right)^{\theta} \left(Ax^{\alpha} L^{1-\alpha} + \Phi - x\right)$$

- and we have the budget constraint

$$AN_t x^\alpha L^{1-\alpha} = c_t + xN_t + \Phi\left(N_{t+1} - N_t\right).$$

.Comparing the market equilibrium to a social planner

- The first first order condition says that in the social planner economy

$$x_{sp} = A^{\frac{1}{1-\alpha}} L \alpha^{\frac{1}{1-\alpha}}$$

- Which we can compare to the amount of each intermediate good used in the market solution

$$x_m = A^{\frac{1}{1-\alpha}} L_t \alpha^{\frac{2}{1-\alpha}}$$

- Since $\alpha < 1$,

$$\alpha^{\frac{2}{1-\alpha}} < \alpha^{\frac{1}{1-\alpha}}$$

- The amount of each intermediate good used in the social planner economy is larger than in the market economy

.Comparing the market equilibrium to a social planner

- The growth rate of consumption in the social planner economy is found from the second first order condition as

$$\begin{aligned} \gamma_c^{sp} &= \frac{c_{t+1}}{c_t} - 1 = \left[\frac{\beta\left(Ax^\alpha L^{1-\alpha} + \Phi - x\right)}{\Phi}\right]^{\frac{1}{\theta}} - 1 \\ &= \left[\frac{\beta\left(A^{\frac{1}{1-\alpha}} L\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) + \Phi\right)}{\Phi}\right]^{\frac{1}{\theta}} - 1 \end{aligned}$$

- Recall tat the growth rate of consumption in the market economy was

$$\gamma_c^m = \left(\beta \frac{\left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + \Phi}{\Phi}\right)^{\frac{1}{\theta}} - 1.$$

.Comparing the market equilibrium to a social planner

- We conpare the two growth rates

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$$\gamma_c^m - 1$$

$$\left(\beta \frac{\left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + \Phi}{\Phi}\right)^{\frac{1}{\theta}}$$

$$\beta \frac{\left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L + \Phi}{\Phi}$$

$$\alpha^{\frac{2}{1-\alpha}}$$

5cm

$$\gamma_c^{sp} - 1$$

$$\left(\beta \frac{\left(\frac{1}{\alpha} - 1\right) A^{\frac{1}{1-\alpha}} L \alpha^{\frac{1}{1-\alpha}} + \Phi}{\Phi}\right)^{\frac{1}{\theta}}$$

$$\beta \frac{A^{\frac{1}{1-\alpha}} L \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{1}{1-\alpha}} + \Phi}{\Phi}$$

$$\alpha^{\frac{1}{1-\alpha}}$$

- Since $\alpha < 1$, growth in the social planner economy is greater than in the market economy