Economic Growth
Class 8 a
AK with human capital

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Human capital and the AK model

- Lucas (1988) built model with human capital
- human capital grows based on past human capital and studying
- there are versions that include physical capital and some that leave it out
- We will have physical capital
- Human capital grows according to the rule

$$h_{t+1} = dh_t^{\phi} (1-u),$$

- where u is the fraction of non-leisure time spent working (a constant for us)
- h_t is the amount of human capital per worker

Human capital and the AK model

- Lucas argues that $\phi = 1$
- knowledge is non-rival
  - I can learn at the same time as you learn
  - so that learning can be more efficient than physical capital accumulation
    - if I own physical capital, you can't own the same units of physical capital
- we will use the learning function as

$$h_{t+1} = dh_t (1-u),$$

.Goods production

- workers provide "effective" labor to the firms
- human capital is measured in effective labor units
- If I have 5 units of human capital, I as as productive as 1 "normal" unit of labor
- There are Nt workers in the economy, all of whom have the same amount of human capital
- total labor supplied to the firms is uh_tN_t
- the Cobb Douglas production function is

$$Y_t = AK_t^{\theta}\left(uh_tN_t\right)^{1-\theta},$$

- written in per worker (N_t) terms, this is

$$y_t = \frac{Y_t}{N_t} = \frac{AK_t^{\theta}\left(uh_tN_t\right)^{1-\theta}h_t^{\eta}}{N_t} = Ak_t^{\theta}\left(uh_t\right)^{1-\theta}$$

.Solow part of the model

- Physical capital (in per worker terms) accumulates according to

$$k_{t+1} = (1-\delta)\,k_t + i_t$$

- Equilibrium in the market for capital (in a closed economy) is

$$s_t = sy_t$$

- Putting all this together gives

$$\begin{aligned} k_{t+1} &= (1-\delta)\,k_t + sy_t, \\ &= (1-\delta)\,k_t + sAk_t^{\theta}\left(uh_t\right)^{1-\theta} \end{aligned}$$

.Growth rates

- The (constant) gross growth rate of human capital is

$$\gamma^h = \frac{h_{t+1}}{h_t} = \frac{dh_t\,(1-u)}{h_t} = d\,(1-u)\,.$$

- The gross growth rate of capital is

$$\begin{aligned}\gamma_t^k &= \frac{k_{t+1}}{k_t} = \frac{(1-\delta)\,k_t + sy_t}{k_t} = (1-\delta) + s\frac{y_t}{k_t},\\ &= (1-\delta) + s\frac{Ak_t^{\theta}\,(uh_t)^{1-\theta}}{k_t},\\ &= (1-\delta) + sAu^{1-\theta}\left(\frac{h_t}{k_t}\right)^{1-\theta}.\end{aligned}$$

Growth rates

- The gross growth rate of output is

$$\begin{aligned}\gamma_t^y &= \frac{y_{t+1}}{y_t} = \frac{Ak_{t+1}^{\theta}\,(uh_{t+1})^{1-\theta}}{Ak_t^{\theta}\,(uh_t)^{1-\theta}} = \left(\frac{k_{t+1}}{k_t}\right)^{\theta}\left(\frac{h_{t+1}}{h_t}\right)^{1-\theta}\\ &= \left(\frac{k_{t+1}}{k_t}\right)^{\theta}(d\,(1-u))^{1-\theta}\\ &= \left(\gamma_t^k\right)^{\theta}(d\,(1-u))^{1-\theta}\end{aligned}$$

Balanced growth path

- Along a balanced growth path output growth is a constant: $\gamma_t^y = \gamma^y$
- for output growth to be a constant, from

$$\gamma_t^y = \left(\gamma_t^k\right)^{\theta}(d\,(1-u))^{1-\theta}$$

  we need to have capital growth a constant: $\gamma_t^k = \gamma^k$
- to have physical capital growth a constant we need

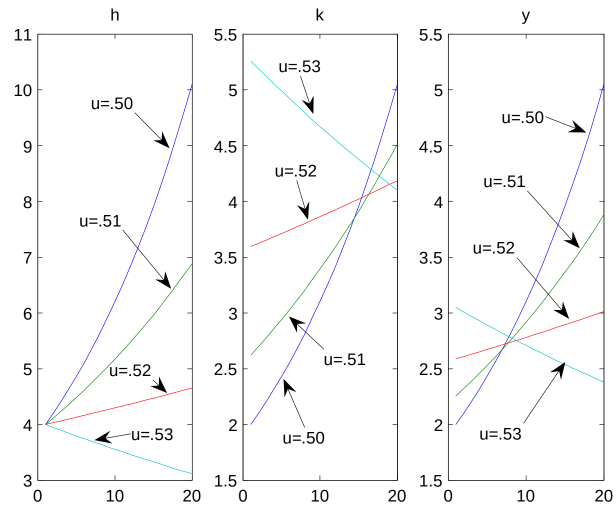
$$\gamma_t^k = (1-\delta) + sAu^{1-\theta}\left(\frac{h_t}{k_t}\right)^{1-\theta}.$$

  to be a constant, which happens only if h_t/k_t is a constant

Balanced growth path

- However, h_t/k_t is a constant only when k_t grows at the same rate as h_t
- therefore, along a balanced growth path, $\gamma_t^k = \gamma^k = d\,(1-u)$
- This also means that output grows at that same rate since

$$\begin{aligned}\gamma_t^y &= \left(\gamma_t^k\right)^{\theta}(d\,(1-u))^{1-\theta}\\ &= (d\,(1-u))^{\theta}(d\,(1-u))^{1-\theta}\\ &= d\,(1-u)\end{aligned}$$



- To find the balanced growth path, we need to find the h_t/k_t ratio that causes physical capital to grow at the same rate as human capital

Balanced growth path

- for a balanced growth path, we want

$$\gamma_t^k = (1-\delta) + sAu^{1-\theta}\left(\frac{h_t}{k_t}\right)^{1-\theta} = d\,(1-u)$$

- after a bit of algebra, that gives the human capital - physical capital ratio along a balanced growth path of

$$\overline{h/k} = \left[\frac{d\,(1-u)-(1-\delta)}{sAu^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

- when human capital per worker is h_t and the physical capital per worker is $h_t = h_t/\overline{h/k}$, then the balanced growth path has $\gamma_t^y = \gamma_t^k = \gamma_t^h = d(1-u)$

Balanced growth path
For an economy with $A = 1$, $\theta = .4$, $\delta = .1$, $s = .15$, $d = 2.1$, and $h_1 = 4$
Dynamics

- What happens if $h_t/k_t \neq \overline{h/k}$
- There are two possibilities $h_t/k_t > \overline{h/k}$ or $h_t/k_t < \overline{h/k}$

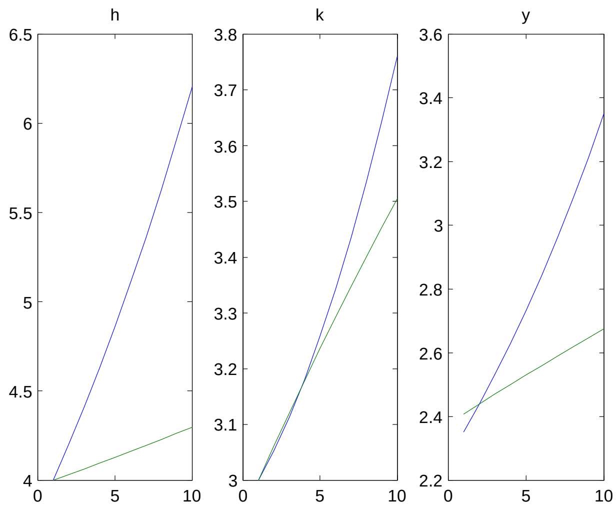
- if $h_t/k_t > \overline{h/k}$, then the economy has less physical capital than it needs to be on the balanced growth path
  - then
    $$\gamma_t^k = (1-\delta) + sAu^{1-\theta}\left(\frac{h_t}{k_t}\right)^{1-\theta} > (1-\delta) + sAu^{1-\theta}\left(\overline{h/k}\right)^{1-\theta}$$
    and physical capital grows faster than human capital and the economy goes on the balanced growth path
- if $h_t/k_t < \overline{h/k}$ then the economy has more physical capital than it needs to be on the balanced growth path
  - then
    $$\gamma_t^k = (1-\delta) + sAu^{1-\theta}\left(\frac{h_t}{k_t}\right)^{1-\theta} < (1-\delta) + sAu^{1-\theta}\left(\overline{h/k}\right)^{1-\theta}$$
    and physical capital grows slower than human capital and the economy goes to the balanced growth path

.Why we call this model an AK model

- The production function is
  $$Y_t = AK_t^{\theta}\left(uh_tN_t\right)^{1-\theta}.$$
- With constant population, one can write the production function as
  $$Y_t = A\left(\frac{Nk_t}{uh_tN}\right)^{\theta} uNh_t.$$
- Since human capital grows at the same rate as physical capital, the parameter
  $$\overline{A} = A\left(\frac{k_t}{uh_t}\right)^{\theta} uN = A\left(u\overline{h/k}\right)^{-\theta} uN$$
  is a constant along the balanced growth path.
- The production function can be written as
  $$Y_t = \overline{A}h_t.$$

.What happens if u changes

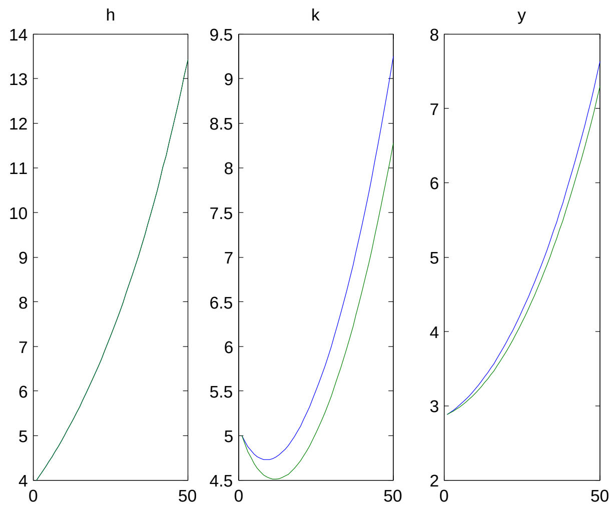
- Assume that an economy moves along a balanced growth path. At some date t, the government convinces people to increase the fraction of time they work. What does this do to the path of the economy?



- Economy has a new balanced growth path
- that path will have
  - slower growth of human capital
  - initially more output, since labor is working more
  - initially more physical capital, since there is higher output and savings is a fixed fraction of that output
- Why this might be important for thinking about poorer economies

.What happens if u changes
Here u goes up from .50 to .52, otherwise same economy as earlier example
.What happens if s is higher

- A higher savings rate, s, gives a smaller balanced growth $\overline{h/k}$, as seen from
$$\overline{h/k} = \left[\frac{d\left(1-u\right)-\left(1-\delta\right)}{sAu^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$
- that means that the physical capital stock will be higher for each h_t
- with higher physical capital, output will be higher, via
$$y_t = Ak_t^{\theta}\left(uh_t\right)^{1-\theta}$$
- the balanced growth path will have the same growth rate (that of human capital) but will have a greater output for each h_t on that path



.What happens if s is higher

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