1 Models AK and Human capital

Suppose that the production function uses physical capital and human capital of the form

$$Y_t = BK_t^{\theta} H_t^{1-\theta}$$

and suppose that the rates of depreciation of both physical and human capital are the same, $\delta^f = \delta^h = \delta$. Suppose that we assume that in terms of accumulation of physical and human capital, they are perfect substitutes and that it costs one unit of time t production to produce one extra unit of time t+1 human or physical capital. We can write the budget constaint as

$$K_{t+1} + H_{t+1} = w_t H_t + r_t K_t + (1-\delta) H_t + (1-\delta) K_t.$$

From perfectly competitive factor markets, we get the result that

$$w_t = (1-\theta) BK_t^{\theta} H_t^{-\theta} = (1-\theta) \frac{Y_t}{H_t}$$

and

$$r_t = \theta BK_t^{\theta-1} H_t^{1-\theta} = \theta \frac{Y_t}{K_t}.$$

Given that individuals can choose between accumulation human or physical capital, the returns on the two must be the same in equilibrium, so that

$$w_t = r_t$$

or

$$(1-\theta) \frac{Y_t}{H_t} = \theta \frac{Y_t}{K_t}$$

and in equibrium,

$$H_t = \frac{(1-\theta)}{\theta} K_t.$$

Putting this into the production function, one has that in equilibrium

$$\begin{aligned} Y_t &= BK_t^{\theta} H_t^{1-\theta} = BK_t^{\theta} \left(\frac{1-\theta}{\theta} K_t \right)^{1-\theta} \\ &= B \left(\frac{1-\theta}{\theta} \right)^{1-\theta} K_t \\ &= AK_t \end{aligned}$$

where the constant A is simply

$$A = B \left(\frac{1-\theta}{\theta} \right)^{1-\theta}.$$