Costly monitoring may lead to credit rationing in equilibrium in an economy without any adverse selection or moral hazard problems. Given the widespread phenomenon of government intervention in credit markets in developing and developed countries, the natural question then is, How effective are these government programs? I incorporate government loan programs in a simple, closed, pure exchange economy with borrowing and lending. Intermediation of funds is facilitated in credit markets characterized by a costly state verification problem. I then show that government loan programs (financed with lump-sum taxes) with co-financing can increase credit rationing when the private lender is the prior claimant in the event of a default. Moreover such programs unambiguously decrease the expected utility of both borrowers and lenders. On the other hand, when the government is the prior claimant, such programs decrease credit rationing and increase the expected utility of borrowers. Finally, with proportional repayments there is no effect on credit rationing or expected utility of agents.

JEL classification codes: H81, G33
Key words: credit rationing, co-financing, lenders, borrowers, prior claimant

I. Introduction

Intervention by the government in financial markets is a common occurrence in both developed and under-developed countries. 1 In the United States, for example, the Small Business Administration (SBA) provides loan guarantees up to almost 90 percent of an eligible loan made by financial institutions. 2 Moreover,

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over the years, the volume of such loan guarantees has steadily increased (see Li 1999).

The increasing popularity of Government Credit Programs raises a natural question. Given that credit markets are perfectly competitive, it seems possible that equilibrium outcomes are Pareto efficient. This implies that intervention by the government in perfectly competitive markets would only be welfare decreasing. But if credit markets are operating inefficiently due to imperfect information, leading to credit rationing in equilibrium (see Stiglitz and Weiss 1981; Williamson 1986, 1987; and Boyd and Smith 1997, 1998), can government credit programs improve welfare?

Several papers have addressed this issue by incorporating government credit programs in an economy with credit market frictions. For example Smith and Stutzer (1989) analyze government loan programs, like loan guarantees, direct loans and equity participation loans. In their model, credit is rationed to low risk borrowers due to an adverse selection problem. They show that some types of loan programs, such as loan guarantees issued through lenders, might improve economic efficiency. They also show that the incentive for high risk borrowers to masquerade as low risk borrowers worsens under other forms of loans like direct lending to rationed borrowers. Thus they highlight the significance of conducting model-specific policy analysis. Li (1998) studies loan programs in the presence of financial market frictions caused by moral hazard problems. According to her, credit programs cannot lead to efficiency gains since the government does not have any information or technology advantage over private lenders. So, the paper analyzes the distributional effects of government credit programs.

Williamson (1994) studies credit programs like loan guarantees and direct loans in two types of credit markets: one where frictions are due a costly state verification (CSV) problem, first analyzed by Townsend (1979), and second, where frictions are due to an adverse selection problem. He concludes that direct loans have no effect because government lending simply displaces an equal quantity of private lending. And loan guarantees have no effect if there is no credit rationing prior to the government intervention, and finally, if there is credit rationing, the loan guarantee program have perverse effects: interest rates faced by lenders (borrowers) decrease (increase) and credit rationing becomes worse. In markets where frictions are due to moral hazard, he infers that if government loan interest rates are set appropriately, then the welfare of targeted groups may increase.

My paper deviates from the above papers by analyzing government credit
programs wherein the government is a co-financer along with a financial institution or an intermediary. The motivation for this analysis is that co-financing by the government and financial institutions or intermediaries is widespread. For example, since 1970 the Asian Development Bank (ADB) has arranged a total of 35 billion dollars of co-financing for 557 projects in member countries. This is only slightly less than the amount raised by the ADB in international capital markets and is nearly double the amount ADB obtained from its Asian Development Fund (ADF) donors. Thus the ADB actively promotes co-financing with funds from commercial financial institutions, official funding agencies and export credit agencies. Funding from official sources include official development assistance provided by donor governments from their budget appropriations, usually in the form of grants and loans.

The World Bank also encourages and helps its borrowers to obtain additional financing from other sources for projects assisted by the World Bank in the form of co-financing. The sources are varied, and include government bilateral aid programs, regional development banks, export credit agencies and commercial banks. In fact, almost half of the Bank assisted projects now involve other sources of financing. The European Bank for Reconstruction and Development (EBRD), since its inception in 1991, has also actively promoted co-financing in its member countries.

The second motivation for this analysis is that this paper also attempts to contribute to the growing literature on insolvency systems. Levine, Loayza and Beck (2000), show that financial intermediary development exerts an economically large positive impact on growth.¹ La Porta, Silanes, Shleifer and Vishny (1997, 1998), emphasize the importance of well functioning insolvency systems in influencing the development of financial systems.² Moreover, according to Claessens and Klapper (2002), there is a growing interest in the design of bankruptcy systems with regard to resource allocation and efficiency.³ For example, one of the objectives of the EBRD in recent years has been to help in the development of legal rules in the area of bankruptcy for its countries in transition.⁴ Rowat (1999)

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¹ For examples of theoretical models, see Greenwood and Jovanovic (1990) and Bencivenga and Smith (1991).
² See also Levine, Loayza and Beck (2002).
³ They use data on relative number of bankruptcy filings in 35 countries to investigate which legal, financial and other country characteristics affect the probability of a bankruptcy procedure being used. See also Stiglitz (2001) and Hart (2000).
⁴ See Averch (2002).
notes that while Argentina, Colombia, Costa Rica and Peru have recently updated their bankruptcy laws, many other countries in Latin America, including Brazil and Mexico, are trying to redesign their insolvency laws. And in Asia, the ADB is conducting regional studies, like the Insolvency Law Reform Project, to better comprehend the problems and reforms of bankruptcy laws.

In the theoretical literature, Hausch and Ramachandran (1999) develop a market based ACCORD scheme - Auction-based Creditor Ordering by Reducing Debts - where creditors form a queue, and are serviced in sequence. Creditors bid for their position in the queue. They then determine equilibrium bidding strategies and show that economic resources are better used under this scheme, and also increases the firm’s managers incentive to manage the firm more efficiently. My paper addresses the question of insolvency when the government is a creditor along with a private lender. I show that the order in which the creditors, in this case the private lender and the government, are serviced has economic implications. Specifically, when the private lender is the prior claimant, the default probability increases, which causes economic inefficiency. On the other hand when the government is the prior claimant, the default probability decreases. Finally, when repayment is proportional to the amount of the loan, there is no change. The relevance of such a model is supported by real world evidence. For example, in the United States, a creditor may have a priority interest which arises through a statutory law. The implication is that a creditor with a priority must be paid his debt before other creditors can be paid. Congress has granted priority to debts owed to the Federal government.

To carry out the analysis, I consider a closed pure exchange economy where some agents (lenders) are endowed with investment goods, while some agents (borrowers) are endowed with a linear technology that can convert investment goods into consumption goods. Credit markets are characterized by frictions caused by the presence of a CSV problem. Williamson (1986, 1987) showed credit rationing can arise endogenously in equilibrium in such an economy. A natural question then is: Does government intervention reduce the number of borrowers who are denied credit? And what are the distributional effects of such programs in the presence of credit market frictions? And, in the event of a bankruptcy, what is the most efficient way to allocate resources to creditors?

In this paper, I follow the approach of Williamson (1986, 1987) and Boyd and

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7 Those accepting higher reductions in their claimants are placed ahead of those willing to let go smaller proportions of their claims.
Smith (1997, 1998), and assume that frictions in credit markets take the form of a CSV problem. Project returns are assumed to be unobservable except to the project owner. Any other agent can observe the outcome only after incurring a fixed monitoring cost. Monitoring becomes necessary when the project owner defaults, that is, declares bankruptcy. Due to the fact that there are now co-financiers, three possible scenarios can be visualized. One, where the private lender is the prior claimant; two, where the government is the prior claimant; and three, when repayment is proportional to the amount of the loan. I assume that the loan programs are funded by imposing a lump-sum tax on lenders. I conclude that efficiency gains and distribution effects differ in the two scenarios, reinforcing Smith’s and Stutzer’s claim that policy analysis must necessarily be model specific. Specifically, when the private lender is the prior claimant in the event of a default, credit rationing increases. Moreover, the expected utility of all agents, borrowers and lenders, decrease. On the other hand, if the government is the prior claimant, credit rationing decreases, the expected utility of borrowers increase, while the expected utility of lenders may or may not increase. Finally, the proportional repayment policy has neutral effects.

The rest of the paper is organized as follows. Section II describes the benchmark model, that is, the equilibrium outcome when there is no intervention. Section III analyzes credit programs where the government is a co-financier along with a private lender. I consider all three cases in turn: one where the private lender is the prior claimant, two where the government is the prior claimant while the private lender is the residual claimant, and three when the repayment is proportional to the amount of the loan. Finally, Section IV concludes.

II. Model

I consider an environment where there is an infinite sequence of two period lived agents. At each date a continuum of young agents of unit mass is born. Time is discrete and is indexed by $t = 0, 1, 2, 3, \ldots$

Agents are of two types, namely borrowers and lenders. I assume that a proportion $\alpha$ of the population are lenders while a proportion $1 - \alpha$ are borrowers. Borrowers and lenders differ from each other in only two respects. Lenders are endowed with $y$ units of the investment good in period one, while borrowers receive no such endowments. Lenders are endowed with $y$ units of the investment good in period one, while borrowers receive no such endowments. Secondly, borrowers have access to a linear stochastic technology which can convert investment goods into consumption goods. This technology is unavailable to lenders.
This linear stochastic technology operates as follows. Investing $q$ units of the investment good in period one yields $zq$ units of the consumption good in period two. Thus, this technology is indivisible, such that, to initiate any project, at least $q$ units must be invested. This assumption of indivisibility follows Williamson (1986, 1987), Bernanke and Gerther (1989) and Boyd and Smith (1997, 1998). The variable $z$ is an i.i.d. (both across agents and periods) random variable, with probability density function given by $p(z)$ and the cumulative distribution function denoted by $P(z)$. I assume that $P(z)$ is continuous and twice differentiable and the density function $p(z)$ has support $[0, z]$. The mean is denoted by $\hat{z}$ and is given by $\int_0^z z p(z) \, dz$.

The realization of $z$ on any investment project is costlessly observable only by the project owner. Any other agent can observe the value of $z$ only after incurring a fixed monitoring cost of $\gamma$ units of the consumption good. This assumption is standard in models that are characterized by a CSV problem. See Boyd and Smith (1997, 1998). Although $z$ can be costlessly observed by the project owner, all agents know the density function $p(z)$.

Finally, I assume that borrowers and lenders are identical with respect to preferences. Both care about consumption in old age only and they are both risk neutral. Therefore, all young period endowment is saved and invested in the linear technology via intermediation.

A. Credit markets

All young lenders receive $y$ units of the investment good as endowments. Since agents are concerned only with old age consumption, all of the endowment is saved. This is inelastically supplied in credit markets in period one. Borrowers have no endowments, so they need to borrow $q$ units to initiate their project. These transactions can be thought of as being facilitated via banks or intermediaries. That is, any lender or group of lenders can form an intermediary. In addition, by appealing to the law of large numbers, I can infer that monitoring the intermediary is unnecessary since they face a non-stochastic return on their portfolio. Finally, I assume that the proportion of borrowers exceeds that of lenders, so that the following is true:

$$\alpha y < (1 - \alpha) q$$

That is, there is not enough credit to finance all potential projects.

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8 In Boyd and Smith (1997) units of capital are used in monitoring.

9 This assumption has been made for tractability.
To obtain external finance, borrowers announce loan contracts of the form described by Williamson (1986). These contracts are then evaluated by lenders or intermediaries and are either accepted or rejected on the basis of the expected return they offer. I let $r$ denote the expected market rate which will be endogenously determined in the model, but is treated as a parameter by all agents.

A loan contract must specify the following objects. A set $A$ of project return realizations in which verification takes place and a set $B$ of project return realizations in which verification does not take place. Clearly, verification is necessary in some states. The repayment schedule if verification takes place, that is if $z \in A$, is state contingent and is denoted by $R(z)$. If $z \in B$, then verification does not take place, and the repayment schedule must necessarily be independent of the state. I denote this by some constant $x$ (per unit borrowed), which needs to be determined. The optimal contract, then, is the loan repayment pair $[R(z), x]$ which maximizes the expected utility of the borrower subject to the feasibility, incentive compatibility and participation constraints. The solution to this problem is a standard debt contract given by

$$R(z) = zq$$  \hspace{1cm} (2)

$$A = [0, x].$$  \hspace{1cm} (3)

In other words, if the realization $z$ is such that $z \geq x$, then the borrower pays $xz$ to the lender and no monitoring takes place. On the other hand if $z < x$ the borrower then defaults on her loan repayment and declares bankruptcy. The lender expends $\gamma$ units, monitors the project and gets $zq$ while the borrower receives nothing. Given the optimal debt contract, the expected return to the lender per unit borrowed $q$ is

$$\pi = \frac{1}{q} \left[ \int_0^x (zq - \gamma) p(z) \, dz + \int_x^q qz \, p(z) \, dz \right] = x \int_0^x p(z) \, dz - \int_0^x p(z) \, dz - \int_0^x (zq - \gamma) p(z) \, dz$$

\hspace{1cm} (4)

10 Note that I do not allow for stochastic monitoring. While Mookherjee and Png (1989) show that contracts with stochastic monitoring are optimal, Boyd and Smith (1994) show that gains with stochastic monitoring are trivial with realistic parameter values.

11 See Williamson (1986) for a proof.
using integration by parts and rearranging terms. Thus, the expected return to the lender is a function of the loan repayment \( x \) and \((\gamma(q))\). Following Williamson (1986), I assume that \( \pi_{11} < 0 \) holds, so

\[
p(x) + \left( \frac{\gamma}{q} \right) p'(x) \geq 0 \quad \forall \ x \in [0, \mathcal{Z}]
\]

where \( p'(x) = \frac{dp(x)}{dx} \). When this is true, the function \( \pi \) looks as depicted in Figure 1. Clearly, there is a unique value for \( x \), say \( x^* \), that maximizes the expected return to the lender. Then \( x^* \) solves:

\[
1 - P(x^*) - \left( \frac{\gamma}{q} \right) p(x^*) = 0
\]

(6)

So, as noted by Williamson (1986), credit rationing can easily arise endogenously in equilibrium. By equation (1) total demand for credit exceeds the total supply, so borrowers who are denied credit will try to bid up the interest rate they offer. However, the interest rate can only be bid up to the maximum value \( x^* \), beyond which the expected return to the lender decreases due to increasing monitoring costs. Thus, if equations (1) and (5) hold at all time \( t \), credit will be rationed in equilibrium. I focus on this case throughout the paper.

**III. Government lending program**

The presence of imperfect information between borrowers and lenders leads to an equilibrium in which some borrowers are denied credit, resulting in some investments going unfunded. Can this rationing provide a potential rationale for a welfare improving intervention by the government?

I will consider the case in which the government co-funds a project, along with a private lender or an intermediary. That is, the government will finance a fraction \( \delta \) of the total amount borrowed. In other words, since the borrower needs \( q \) units of the investment good to initiate her project, the government lends \( q\delta \) while private lenders lend \( q(1-\delta) \). I assume that these programs are financed by levying a lump-sum tax of \( r \) units on young lenders. Then, as before, borrowers announce loan contracts, which specify a set of project return realizations for which verification takes place, a set of project return realizations for which verification does not take place and a loan repayment schedule \([x, R(z)]\). Then, if
non-verification state occurs, the private lender receives $q x (1 - \delta)$ while the government receives $x \delta q$ as repayment. However, if verification state occurs, that is the borrower defaults, there are two possible outcomes. In the first, the private lender is the prior claimant while the government is the residual claimant, and in the second, the government is the prior claimant while the private lender is the residual claimant. I consider each of the two cases in order.

**A. Private lender is the prior claimant**

Each project is now jointly co-financed by a private lender and the government. As before, borrowers announce loan contracts to get credit. In this scenario with more than one lender for one borrower, each contract must specify sets $A_p$ and $A_g$ of project return realizations for which verification takes place by the private lender and the government respectively. It must also specify a set $B$ of project return realizations for which verification does not take place. Lastly, the contract must specify the loan repayments $R_p(z)$ and $R_g(z)$ to the private lender and the
government respectively when verification occurs by either of the two parties. Obviously, both repayments can be state contingent. Finally, the contract specifies the repayments \( x^p \) and \( x^g \) to the private lender and the government, respectively, in the event that no verification takes place. Finally, I note that, the sets \( A^p \) and \( A^g \) are mutually exclusive, that the second claimant receives nothing if the first claimant monitors the project, and that if the project is monitored by the second claimant it follows that the first claimant must have received the constant payment \( x^p \).

The loan repayment schedule must be feasible:

\[
0 \leq R^p(z) + R^g(z) \leq zq, \tag{7}
\]

\[
0 \leq x^p + x^g \leq \inf z. \tag{8}
\]

The repayment schedule must be incentive compatible, so the borrower truthfully reveals when a monitoring state has occurred:

\[
R^p(z) + R^g(z) \leq (x^p + x^g)q \quad \forall z \in A^p \cup A^g. \tag{9}
\]

In addition, the expected return to the lender must be at least the market interest rate \( r \):

\[
\int_{A^p} (R^p(z) - \gamma) p(z)dz + q \int_{B \cup A^g} x^p p(z)dz \geq rq(1 - \delta). \tag{10}
\]

Then the optimal contract is the loan repayment schedule \([ (R^p(z), x^p), (R^g(z), x^g) ] \) which maximizes the expected utility of the borrower subject to the feasibility, incentive compatibility and participation constraints. That is,

\[
\max \int_{A^p} (zq - R^p(z) - R^g(z)) p(z)dz + \int_{A^p} (zq - x^p q - R^g(z)) p(z)dz + \int_B (z - x^p - x^g) p(z)dz \tag{11}
\]

subject to the constraints (7), (8), (9) and (10). Then, as before, the optimal repayment
The objective function of the borrower is the same as before. In addition, the feasibility and the incentive compatibility constraints are also the same. The participation constraint, however, is different. Monitoring is costly for lenders. The intuition underlying the Williamson result is that the optimal contract is the one that minimizes the monitoring cost. In other words, the contract that maximizes the objective function of the borrower is the one that minimizes the monitoring cost. In this case, as well, the standard debt contract achieves that.
Again, following Williamson (1986), I assume that the following holds:

\[
\pi_1 = x - \frac{1}{1 - \delta} \int_0^{x(1 - \delta)} P(z)dz - \frac{\gamma}{q(1 - \delta)} P(x(1 - \delta)) \tag{17}
\]

Again, following Williamson (1986), I assume that the following holds:

\[
\frac{\partial^2 \pi_1}{\partial x^2} < 0 \tag{18}
\]

Next, I need an expression for the expected return to the government. The government is the residual claimant, so monitoring by the government occurs if and only if \(x(1 - \delta) \leq z < x\). In such a scenario, the government expends \(\gamma\) units on monitoring while receiving \(zq - xq(1 - \delta)\) units. Finally, the government receives nothing if \(z \leq x(1 - \delta)\), and receives \(x\delta q\) if \(z \geq x\). Mathematically, the expected return to the government is given by

\[
\Pi^G = \int_{x(1 - \delta)}^x (z - x(1 - \delta))\gamma p(z)dz + xq\delta[1 - P(x)] - \gamma \int_{x(1 - \delta)}^x p(z)dz \tag{19}
\]

Dividing by \(q\delta\) and rearranging terms, yields the expected return per amount borrowed

\[
\pi^G = x - \frac{1}{\delta} \int_{x(1 - \delta)}^x P(z)dz - \frac{\gamma}{q\delta} [P(x) - P(x(1 - \delta))] \tag{20}
\]

Clearly, it is evident that the convex combination of the expected returns to the private lender and the government is equal to the expected return without government intervention, where the weights are \(1 - \delta\) and \(\delta\) respectively. So,

\[
\pi = (1 - \delta)\pi^P + \delta\pi^G \tag{21}
\]

Again, due to assumption (18), \(\pi^P\) looks as depicted in Figure 2. A unique loan repayment value, \(\tilde{x}\) exists that maximizes the expected return to the private lender. The following proposition then shows how \(\tilde{x}\) changes with increases in \(\delta\).

**Proposition 1.** The loan repayment \(\tilde{x}\) and the expected return to the lender is greater under government intervention than without it. Moreover, they are both increasing in \(\delta\).

Proof in appendix.

The expected return to the private lender is positively related to the loan
The return to the prior claimant is higher than the return to the residual claimant. Therefore, at each value of the loan repayment rate $x$, the expected return is higher. In order to obtain credit, borrowers try to bid up the interest rate. Because the expected return is higher, the interest rate can be bid up to a value above $x^*$.\footnote{The expected return schedule of the government lies below the expected return schedule of the private lender. Therefore, the government is receiving a rate of return that is lower than the market rate. In addition, the return is also lower than $\pi(x^*)$, the market rate without intervention.}

I next examine how the expected return to the government varies as $\delta$ varies. Clearly this is important from the lenders’ point of view, since high expected returns to the government may translate into lower lump-sum taxes for lenders. Using result (A2) in Appendix by which $\pi^P_1 (\tilde{x}) = \pi(x^*) / (1 - \delta)$ in (21) yields:
Now, since $\bar{x} > x^*$, and $\pi' (x^*) = 0$, it must be true that, $\pi' (\bar{x}) < 0$ at $x = \bar{x}$. From Proposition 1, $\frac{\partial \bar{x}}{\partial \delta} > 0$, and so it follows that $\delta \pi^G$ is decreasing in $\delta$. This is summed up in the next proposition.

**Proposition 2.** The expected return to the government decreases with increases in $\delta$. Moreover, the total return $q \delta \pi^G (\bar{x})$ also falls as the proportion of the government loan program $\delta$ increases.

To carry out these loan programs, the government needs revenue. I assume that each lender has to pay $\tau$ units of their endowment as lump-sum tax when young.\(^{14}\) This revenue is then utilized by the government to co-finance the investment projects of the borrowers. The proportion of lenders in the economy is $\alpha$, each of whom receive $y$ units of the investment good as endowment, so the total revenue collected by taxation is $\alpha \tau$. By assumption (1) and (18), credit rationing arises in equilibrium. Let $\theta$ denote the proportion of borrowers who receive credit.

The proportion of borrowers in the economy is $(1- \alpha)$, so a proportion $(1- \alpha) \theta$ of borrowers operate projects. Total government expenditure is then simply $(1- \alpha) \theta q \delta$. However, the government also receives $\pi^G$ from its loan programs, so the government budget constraint is

$$\alpha \tau = (1- \alpha) \theta q \delta - (1- \alpha) \theta q \delta \pi^G, \quad (23)$$

which yields, rearranging terms,

$$\alpha \tau = (1- \alpha) \theta q \delta (1- \pi^G). \quad (24)$$

One other aggregate condition remains in order to complete the description of equilibrium of the economy. Since lenders receive $y$ units as endowment and pay a lump-sum tax of $\tau$ units, sources of funds equal uses of funds when

$$\alpha (y- \tau) = (1- \alpha) q (1- \delta) \theta \quad (25)$$

\(^{14}\) This case can be trivially extended to proportional taxes without changing the main results.
Equation (25) can be used to plug in for \( \tau \) in the aggregate budget constraint. This gives \( \theta \) as a function of \( \delta \pi^G \).

\[
\theta = \frac{\gamma \alpha}{(1-\alpha)q(1-\delta \pi^G)}
\]  

(26)

From Proposition 2, \( \frac{\partial \delta \pi^G}{\partial \delta} < 0 \), so it is obvious that \( \frac{\partial \theta}{\partial \delta} < 0 \). Proposition 3 then follows immediately.

**Proposition 3.** *When the government co-finances a project with a private lender and is the second claimant in the event of a bankruptcy, the proportion of borrowers who are denied credit increases, as compared to the case without government intervention.*

Because of intervention, the loan repayment rate is higher. This increases the probability of default, and, consequently, total monitoring costs also increase. As more resources are expended in monitoring, less resources are available for funding projects. Thus, intervention distorts equilibrium, leading to increased inefficiency. Finally, I will show how the welfare of lenders and borrowers are affected by this government loan program. For lenders, the expected return on loans increases, but they also pay a lump-sum tax. Lenders are only concerned with second period consumption, and so all of their first period endowment minus the tax is saved. This saving earns an expected return of \( \pi(x^*)/(1-\delta) \). Therefore, the expected utility of the lender, \( EU_l \), can be written as

\[
EU_l = \frac{\pi(x^*)}{1-\delta} (y-\tau).
\]  

(27)

Plugging in for \((y-\tau)\) from (25) and (26) and simplifying yields

\[
EU_l = \frac{\gamma \pi(x^*)}{(1-\delta \pi^G)}.
\]  

(28)

So, from Proposition 3 and equation (28) it is clear that \( \frac{\partial EU_l}{\partial \delta} < 0 \). Thus the program is welfare decreasing for lenders. Expected return is greater; however, this increased return is offset by the lump-sum tax that lenders are required to pay in order to fund
the government program. How does this program affect the expected utility of borrowers? Let $EU_b$ denote the expected utility of borrowers who receive funding. The expected utility of borrowers in general then is simply $\theta EU$. Borrowers receive total output from the project minus expected return to both lenders and total monitoring costs. Again, borrowers are primarily concerned with second period consumption; therefore, expected utility of funded borrowers is

$$EU_b = \hat{z}q - q(1 - \delta)\pi_1^P(\hat{x}) - q\hat{q}\pi_2^G(\hat{x}) - \gamma P(\hat{x}) = \hat{z}q - q\pi(\hat{x}) - \gamma P(\hat{x}).$$  \hspace{1cm} (29)$$

Then, using (4),

$$\frac{\partial EU_b}{\partial \delta} = -q[1 - P(\hat{x})] \frac{\partial \hat{x}}{\partial \delta} < 0,$$

since $\frac{\partial \hat{x}}{\partial \delta} > 0$. Thus, expected utility of funded borrowers decreases in equilibrium. This is so because, borrowers are now paying a higher loan repayment rate or a higher interest rate on their loans.

What about the expected utility of borrowers in general? Clearly, since $\theta < \theta < \theta$, $\theta < \theta$. That is, the expected utility of all borrowers decreases in equilibrium. This leads to the final result in this section.

**Proposition 4.** The government loan program makes everyone worse off in the sense that the expected utility of lenders, as well as the expected utility of both funded and unfunded borrowers decreases.

Both agents are worse off because of intervention. Lenders are worse off because the government is levying a lump-sum tax. And, borrowers are worse off because of two reasons. One, because of the higher loan repayment rate, and two, because more borrowers are now denied credit in equilibrium. It is easy to produce numerical examples to show that intervention with private lender as prior claimant increases credit rationing. Here is one example that does so.

**Example 1:** Let $p(z) = 1/z$. Then if $z = 2$, $\alpha = 0.7$, $q = 0.5$, $\gamma = 0.7$ and $\gamma = 0.1$, then it can be easily verified that $x^* = 0.6$, $\pi = 0.09$ and $\theta = 0.47$. So, expected return to the lender is $EU_l = 0.009$ and expected return to the borrower is $EU_b = 0.009$. If the government co-finances loans with $\delta = 0.1$ then, $\hat{x} = 0.67$, $\pi_1^P = 0.1$, $\pi_2^G = -0.12$, and $\theta = 0.46$. Finally, expected return to the lender is $EU_l = 0.0089$ and expected return to the borrower is $EU_b = 0.731$. 

In short, the government loan program, with the private lender as the prior claimant, and the government as the residual claimant, is detrimental to the economy. Expected utility of lenders is lower, while more borrowers are credit rationed. Moreover, borrowers pay a higher loan repayment rate. In this sense, this program works very similar to a loan guarantee program. The reasoning behind the results is simple. The return to the prior claimant is always higher than the return to the second claimant, because for some values of the project realization $z$ the residual claimant receives nothing. The private lender seeks to maximize her own returns. As before, interest rates will be bid up as borrowers try to obtain credit. The interest rate will be bid up as long as returns to the lender are increasing. Because she is now the prior claimant, her total monitoring costs are lower. As a result, expected returns will be increasing even at $x = x^*$. So the equilibrium loan repayment rate is now going to be bid up to a higher rate. This in turn implies that default probability is higher, leading to higher monitoring costs. Hence less funds are now available for investment, consequently increasing credit rationing. Moreover, low returns for the government implies that taxes have to be higher in order to fund these loan programs. The private lender fails to take this into account when maximizing her return from lending investment projects. Finally, the expected utility of borrowers also declines because of the higher loan repayment rate.

B. Government is the prior claimant

I turn to the case where the government is the prior claimant and the private lender is the residual claimant. Again, I assume that the government finances its loan program by imposing a lump-sum tax of $\tau$ units on the young lenders.

As before, the government and the private lender co-finance projects, with the government lending a fraction $\delta$ of the total amount $q$, and the private lender lending the remaining $(1-\delta)q$. Borrowers then announce loan contracts of the form described in the previous section. That is, contracts must specify sets $A^p$ and $A^g$ of project return realizations for which verification takes place by the private lender and the government, respectively. It must also specify a set $B$ of project return realizations for which no verification takes place. Lastly, the contract must specify the loan repayments $R^p(z)$ and $R^g(z)$ to the private lender and the government.

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Note that I am using the same notation for convenience, but these values are clearly different from the previous case where the private lender is the prior claimant.
respectively, when verification occurs by either one of the two parties. Finally, the contract specifies the repayments $x^p$ and $x^g$ to the private lender and the government respectively, for instances when no verification occurs.

The loan repayment schedule must continue to satisfy the feasibility and incentive compatibility constraints so that equations (7), (8) and (9) hold. In addition, the expected return to the lender must be at least the market interest rate, so that the following equation is true:

$$\int_{A^p} (R^p(z) - \gamma) p(z) dz + q \int_B x^p p(z) dz \geq rq(1-\delta)$$  \hspace{1cm} (31)$$

Then the optimal contract is the loan repayment schedule $[\{R^p(z), x^p\}, \{R^g(z), x^g\}]$ which maximizes the expected utility of the borrower subject to the feasibility, incentive compatibility and participation constraints. That is, the optimal contract solves,

$$\max \int_{A^p} (zq - R^g(z) - R^p(z)) p(z) dz + \int_{A^p} (zq - x^g q - R^p(z)) p(z) dz + q \int_B (z - x^g - x^p) p(z) dz$$  \hspace{1cm} (32)$$

subject to the constraints (7), (8), (9) and (31). Then, as before, the optimal repayment schedule is the standard debt contract given by

$$R^p(z) = zq - xq\delta \hspace{0.5cm} \text{for} \hspace{0.5cm} x\delta \leq z < x,$$  \hspace{1cm} (33)$$

$$A^p = [x\delta, x),$$  \hspace{1cm} (34)$$

$$R^g(z) = zq \hspace{0.5cm} \text{for} \hspace{0.5cm} 0 \leq z < x\delta,$$  \hspace{1cm} (35)$$

$$A^g = [0, x\delta).$$  \hspace{1cm} (36)$$

If $z \in B$, then no verification is necessary and the government receives $q\delta x$ while the private lender receives $q(1-\delta)x$. On the other hand, if $z \notin B$, then verification is necessary. But now, if $0 \leq z < x\delta$, then private lenders receive nothing, while the
government expends $\gamma$ units on monitoring and claims $zq$ units. However, if $x \delta < z < x \delta$, then the government receives its share of $x \delta q$, while the private lender is now the residual claimant, receiving $zq - x \delta q$, after expending $\gamma$ units on monitoring.

From the above contract, the expected return to the private lender can now be written as

$$\Pi^p_2 = \int_{x \delta}^{x} (z - x \delta)q p(z)dz + xq[1 - P(x)] - \gamma \int_{x \delta}^{x} p(z)dz.$$  \hspace{0.5cm} (37)

Dividing by $q(1 - \delta)$ and rearranging terms yields the expected return per amount borrowed

$$\pi^p_2 = x - \frac{1}{1 - \delta} \int_{x \delta}^{x} P(z)dz - \frac{\gamma}{q(1 - \delta)} \{P(x) - P(x \delta)\}. \hspace{0.5cm} (38)$$

Again following Williamson (1986), I assume that the following holds:

$$\frac{\partial^2 \pi^p_2 (\cdot)}{\partial x^2} < 0. \hspace{0.5cm} (39)$$

Similarly, the expected return to the government is given by

$$\Pi^G = \int_{0}^{x \delta} zqp(z)dz + xq[1 - P(x \delta)] - \gamma \int_{0}^{x \delta} p(z)dz.$$  \hspace{0.5cm} (40)

Dividing by $q \delta$ and rearranging terms yields the expected return per amount borrowed,

$$\pi^G = x - \frac{1}{\delta} \int_{0}^{x \delta} P(z)dz - \frac{\gamma}{q \delta} P(x \delta). \hspace{0.5cm} (41)$$

Again, as before, the convex combination of the expected return to the government and the private lender is equal to the expected return to the lender without government intervention, where the weights are $\delta$ and $(1 - \delta)$, respectively. So, the following:
\[ \pi = (1 - \delta)\pi^P + \delta\pi^G, \]  
\[ \text{holds for all } \delta \in (0, 1). \]

Proposition 5 is then immediate.

**Proposition 5.** When the government is the prior claimant and the private lender the residual claimant, the optimal loan repayment \( \bar{x} < x^* \) and \( \pi(\bar{x}) < \pi(x^*) \).

Proof in Appendix.

The effect on the expected return schedule of the private lender is now the exact opposite. She is now the residual claimant, so at each loan repayment rate \( x \), the expected return schedule is lower as depicted in Figure 3. Here, with intervention, the expected return schedule is lower. Thus, the loan repayment rate is also lower.

**Figure 3. The expected return functions with and without intervention**
Again, to carry out these loan programs, the government needs revenue. As in the previous section, I assume that the government resorts to taxing each of the young lenders, an amount of \( \tau \) goods. As before, assumptions (1) and (39) hold, leading to credit being rationed in equilibrium. Thus, only a proportion \( \theta \) of borrowers receive credit, while a proportion \( 1-\theta \) are denied credit. It then follows that the government budget constraint given by equation (24) continues to hold. In addition, lenders receive \( y \) units of the investment good as endowment and are required to pay a lump-sum tax of \( \tau \) units. Thus, the aggregate budget constraint given by equation (25) holds. Consequently, equation (26) will also hold. Proposition 6 then follows immediately.

**Proposition 6.** When the government co-fines a project with a private lender, and is the prior claimant, the proportion of borrowers who are denied credit decreases.

Proof in Appendix.

From Proposition 5, the loan repayment rate is lower. Clearly then, the probability that the borrower will default on her loan repayment is also lower. A lower default probability then translates into lower monitoring costs. Because of this increased efficiency, more borrowers can be funded in equilibrium.

As long as the expected return to the private lender is increasing, borrowers will continue to bid up the interest rate in order to obtain credit. Because the government is the prior claimant, the expected return schedule of the government is above the expected return schedule of the private lender. In this case, the return to the government is greater than the market return.

Finally, the change in welfare of borrowers and lenders as a result of conducting this government credit program needs to be addressed. Lenders now face lower interest rates, and in addition pay a lump-sum tax of \( \tau \) units. Lenders are concerned with second period consumption, so using result (A8) in Appendix their expected utility is

\[
EU_I = \frac{1}{1-\delta}[(\pi(\bar{x})-\pi(x\delta)](y-\tau)
\]

(43)

Mutatis mutandis, plugging in for \((y-\tau)\) from (25) and (26) and simplifying yields

\[
EU_I = \frac{y[\pi(\bar{x})-\pi(x\delta)]}{1-\delta\pi_I^0}.
\]

(44)
so that \( EU_j \leq y\pi(x^*) \) or \( EU_j \geq y\pi(x^*) \). The effect on the expected utility of lenders may increase or decrease. The expected return to the lender is less than the expected return without intervention. If \( \pi^G \) is high enough, the government may make a profit on the program. In particular, if \( \pi^G_i > 1 \), \( \tau \) becomes a subsidy. In this case, expected utility of lenders may increase in equilibrium.

Next, I turn my attention to borrowers. As before, the expected utility of borrowers is the return from operating the project, minus the expected return to both the lenders and expected monitoring costs. Borrowers have no endowment and pay no taxes, so their expected utility is simply

\[
EU_b = \bar{z} - q(1-\delta)\pi^1_b(\bar{x}) - q\delta\pi^G (\bar{x}) - \mathcal{P}(\bar{x}) = \bar{z} - q\pi(\bar{x}) - \mathcal{P}(\bar{x}) > 0
\]

(45)

So, expected utility of borrowers who obtain credit increases. The expected utility of borrowers in general is given by \( EU_b \). Clearly, since, \( \theta \) has increased, the expected utility of borrowers in general also increases. Borrowers are better off because they are now paying a lower loan repayment rate. I sum this up in the last result in this section.

**Proposition 7.** When the government is the prior claimant in the event of a default, co-financing leads to an increase in expected utility of borrowers, while it may or may not increase the expected utility of lenders.

The government loan program, in which the government and the private lender are co-financeurs, and the government is the prior claimant, leads to the funding of more projects in the economy. The reasoning behind the results is simple. The return to the prior claimant is always higher than the return to the residual claimant, because for some values of \( z \) the residual claimant receives nothing. The private lender seeks to maximize her own returns. Again, as before, interest rates will be bid up as long as the expected return to the private lender is increasing. Expected return to the private lender is lower, because her monitoring cost is now higher. As a result, the loan repayment rate can now be bid up to a lower value. This means that default probability is lower, leading to lower monitoring costs.

The return to the government is positive. Note that, in the previous case, the return was negative. From equation (26), it is clear that this decreases credit rationing in equilibrium. But, are resources simply being redistributed from lenders to borrowers without any increase in efficiency? The key to the answer lies in the fact
that monitoring costs are lower when the government is the prior claimant. So, more resources are available in order to fund projects. In other words, total output net of monitoring costs is higher with intervention with government as the prior claimant.

Again its easy to produce examples to show that credit rationing decreases when the government is the prior claimant. Here is one that does so.

Example 2: Let \( p(z) = 1/z \). Then if \( z = 2, \alpha = 0.7, q = 0.5, \gamma = 0.7 \) and \( y = 0.1 \), then it can be easily verified that \( x^* = 0.6, \pi = 0.09 \) and \( \theta = 0.47 \). Expected return to the lender is \( EU_l = 0.009 \) and expected return to the borrower is \( EU_b = 0.781 \). If the government co-finances loans with \( \delta = 0.1 \), then \( x = 0.54, \pi^G = 0.08, \pi^G = 0.16 \) and \( \theta = 0.48 \). Finally, expected return to the lender is \( EU_l = 0.0075 \) and expected return to the borrower is \( EU_b = 0.80 \).

In this case, expected utility to the lender is lower while expected utility of the borrower increases. Why does credit rationing decrease in equilibrium? The answer has to do with the lower loan repayment rate, which results in lower monitoring costs. To see this more clearly, note that total output net of monitoring costs without intervention is \( zq - \gamma p(x^*) = 0.79 \) and with intervention is \( zq - \gamma p(x) = 0.81 \). Finally, lenders are worse off because of the lower return and higher taxes.

I do not study the determination of the optimal share \( \delta \). To find the optimum \( \delta \), we have to differentiate \( \partial_1 \) with respect to \( \delta \) and equate the resulting equation to zero, that is, \( \frac{\partial \delta \partial_1}{\partial \delta} = \frac{\partial \delta \partial_1}{\partial \delta} \left( x + \delta \frac{\partial \left( x \right)}{\partial \delta} \right) = 0 \). The optimum \( \delta \) depends on the distribution of \( p(z) \). For example, if \( z \) is uniform, it can be easily demonstrated that \( \frac{\partial \delta \partial_1 (x)}{\partial \delta} > 0 \forall \delta \).

C. Repayments are proportional to loans

In this case, I assume that repayments and monitoring costs are proportional to the amount of the loan. As before, the government and the private lender finance projects, with the government lending a fraction \( \delta \) and the private lender lending the remaining fraction \( 1 - \delta \) of the project. However, in this case both the monitoring cost and repayments are proportional to the amount of the loan. That is, in the
verification state, the private lender receives \((1 - \delta)R_i(z)\) after incurring a cost \((1 - \delta)\gamma\) on monitoring, while the government receives \(\delta R_i(z)\) as loan repayment after incurring a cost \(\delta\gamma\) on monitoring. Finally, the contract specifies the repayments \(x_i\) in the no verification state, of which \((1 - \delta)x_i\) goes to the private lender and \(\delta x_i\) goes to the government.

Setting up the problem as before, and solving it, clearly demonstrates that the expected return per unit lent to the private lender is

\[
\pi^p = x_i - \int_0^{x_i} P(z)dz - (\gamma / q_i)P(x_i)
\] (46)

Comparing equations (4) and (46), it is clear that the expected return is the same. This in turn implies that there is no change in the loan repayment value, and consequently, the probability of default and monitoring cost is also unchanged. Thus, there is no effect on credit rationing. In addition borrowers are neither worse nor better off. This is the outcome, because the loan repayment rate is the same. How does this affect the expected utility of lenders? To evaluate this effect, I first need to calculate the expected return to the government. Again one can clearly demonstrate that the expected return to the government is given by

\[
\pi^G = x_i - \int_0^{x_i} P(z)dz - (\gamma / q_i)P(x_i) = \pi^p.
\] (47)

From the above equation, it is clear that the expected utility is also unchanged.\(^{16}\)

To see this more clearly, note that if the government returns from this policy are given back to the old lenders as subsidy, the expected return to the lenders is given by

\[
(y - \tau)\pi^p + \tau \pi^G = y\pi^p
\] (48)

Clearly, from the above equation, there is no change in the expected utility of lenders because of the policy. This outcome occurs because, when repayments and monitoring costs are proportional to loans, from the borrowers' point of view,\(^{16}\)

\(^{16}\) In this case, the return to the government is positive. But unlike in the case where the government is the prior claimant, this is merely due to a redistribution of resources. There is no change in monitoring costs, and hence no change in efficiency.
both the government and the formal private lender effectively behave as one lender. Hence the government loan program with proportional repayments essentially has no effect. This further demonstrates that the seniority of the debt is the key to deciphering the effects of loans in a credit rationed economy.

IV. Conclusions

As demonstrated by several authors, credit market frictions in the form of asymmetric information may lead to endogenously arising credit rationing. How effective, then, are government credit programs given that such frictions are a common phenomenon? I analyze one type of government credit programs, namely co-financing of loans in a closed economy with no production in which a CSV problem in credit markets leads credit to be rationed in equilibrium. I show that co-financing of loans, along with a private lender, is detrimental to the economy when the private lender is the prior claimant in the event of a bankruptcy. Credit rationing increases in equilibrium and the expected utility of all agents, lenders and borrowers decrease. On the other hand, when the government is the prior claimant, the proportion of borrowers who are denied credit actually decreases. Moreover, expected utility of borrowers increase, while that of lenders may increase under certain conditions. Since co-financing by the government and a financial institution or intermediary is a common occurrence, these results have policy implications.

These results have been derived under the simplifying assumptions of a closed pure exchange economy. For future research this model can be extended for the case in which neither of the above conditions are true. For example it might be interesting to examine capital inflows and outflows in an open economy when one country conducts these credit programs. Clearly, given that the seniority of debt repayment is important, it might also be interesting to look at a subordinated private lending structure. In this case with two private lenders, the expected return of the prior claimant is going to be higher than the expected return of the residual claimant. The natural question then is: Will this program reduce credit rationing? Again, as before, the answer would depend on total monitoring costs. If total monitoring costs are lower, then credit rationing will improve in equilibrium.

Finally, the efficacy of these credit programs has been analyzed under the implicit assumption that credit is rationed in equilibrium. As an extension, it will be useful to see how the above results change when credit is no longer rationed in
equilibrium. Also, given that credit rationing as well as government intervention is widespread in developing countries, it might be interesting to analyze situations where such credit programs are financed by seigniorage revenue or by taxing borrowers.

Appendix

A. Proof of Proposition 1

Differentiating $\pi^p_1$ with respect to $x$ and equating the result to zero yields

$$1 - P(\bar{x}(1 - \delta)) - \left(\frac{\gamma}{q}\right)p(\bar{x}(1 - \delta)) = 0.$$ 

Clearly from (5), it is obvious that

$$\bar{x} = \frac{x^*}{(1 - \delta)} > x^*\quad \text{and}\quad \frac{dx}{d\delta} > 0. \quad (A1)$$

To prove that the return to the lender is greater with government intervention, plug in $\bar{x} = \frac{x^*}{(1 - \delta)}$ in $\pi^p_1$,

$$\pi^p_1 = \frac{x^*}{(1 - \delta)} - \frac{1}{(1 - \delta)} \int_0^{x^*} P(z)dz - \frac{\gamma}{q(1 - \delta)} P(x^*) \quad (A2)$$

From (A2) it is clear that $\pi^p(\bar{x}; \delta) > \pi(x^*) > 0$ for $\delta < 1$.

B. Proof of Proposition 5

From equation (42) and using the fact that the return to the prior claimant is greater than the residual claimant, it follows immediately that
The expected return per amount borrowed to the private lender is

\[ \pi^p_x = \frac{1}{1-\delta} \int_0^x (z-x\delta) p(z) \,dz + x[1-P(x)] - \frac{\gamma}{q(1-\delta)} \int_0^x p(z) \,dz \]  
(A4)

Rearranging terms yields

\[ \pi^p_x = \frac{1}{1-\delta} [\pi(x) - \pi(x\delta)]. \]  
(A5)

Let \( \bar{x} \) denote the optimal loan repayment value. Then \( \bar{x} \) solves

\[ F(x) = \pi'(x) - \pi'(x\delta) = 0. \]  
(A6)

Now at \( x = \bar{x} = x^* \) (\( x^* \) is the optimal value of \( x \) without intervention), \( \pi(x^*) = 0 \) and \( \pi(x^* \delta) > 0 \). This implies \( F(x^*) < 0 \). So, clearly, it follows that \( \bar{x} < x^* \) and, from (A3), \( \pi^p_{\bar{x}} (\bar{x}) < \pi(x^*) \).

C. Proof of Proposition 6

The expected return to the government is given by

\[ \pi^G_x = \frac{1}{\delta} \int_0^{\delta x} p(z) \,dz + x[1-P(x\delta)] - \frac{\gamma}{q\delta} \int_0^{\delta x} p(z) \,dz \]  
(A7)

\[ = x - \frac{1}{\delta} \int_0^{\delta x} P(z) \,dz - \frac{\gamma}{q\delta} P(x\delta) \]

Multiplying both sides by \( \delta \) yields

\[ \delta \pi^G_x (\bar{x}) = \pi(\bar{x}\delta) > 0. \]  
(A8)
From equation (26), mutatis mutandis,

\[
\theta = \frac{y\alpha}{(1-\alpha)q(1-\pi^{\alpha})} > \frac{y\alpha}{(1-\alpha)q}.
\]

(A9)

References


