

Appendix to Making Rules Credible: Divided Government and Political Budget Cycles

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In this appendix, we first prove that condition (7) is sufficient for no optimal borrowing. Then, we prove Lemmas 1, 2, and 3. Finally, we characterize the voter's decision in a constitutional democracy when there is a trade-off between the control and selection motives.

1 No optimal borrowing condition

Consider a randomly selected candidate in period $t = 0$ who remains in office forever. By quasilinear preferences, the marginal utility of consumption is equal to one. If, in expected value, the marginal utility of the public good is equal to the marginal utility of consumption, any extra resources will be optimally used to reduce taxes. Suppose the government resorts to one extra unit of debt in period t to reduce taxes. From expressions (1)-(4) and (6), expected utility increases $\mathbf{E}_t \left(\frac{1}{\theta_t} \right)$ in period t , while utility falls by $[r'(d_t) d_t + 1 + r(d_t)] \mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right)$ in

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period $t + 1$, when the debt is repaid. Thus, the first order condition for optimal borrowing is:

$$\mathbf{E}_t \left(\frac{1}{\theta_t} \right) - \beta [r' (d_t) d_t + 1 + r (d_t)] \mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right) \leq 0,$$

with strict equality if $d_t > 0$. By the no optimal borrowing condition (7), $\mathbf{E}_t \left(\frac{1}{\theta_t} \right) \leq \beta(1 + r(0))\mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right)$, which implies the first order condition is only satisfied for $d_t = 0$.¹ The second order condition is:

$$-\beta [r'' (d_t) d_t + 2r' (d_t)] \mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right) \leq 0,$$

which is satisfied for all d_t .

2 Lemmas 1, 2, and 3

Lemma 1 *Let $Z = h(X, Y)$ be a function of two independent stochastic variables X and Y , with marginal densities $f_x(x)$ and $f_y(y)$. Let $g(x) = \mathbf{E}[Z | x]$ be the expected value of Z conditional on x . Suppose that $g(x)$ is an increasing and concave function of x . Consider a known vector of information variables W that allows to estimate X and call $\hat{x}(w)$ the estimated value of X when W adopts the value w . Then*

$$\mathbf{E}[Z | \hat{x}(w)] \geq \mathbf{E}[Z] \text{ if and only if } \hat{x}(w) \geq \mathbf{E}[X].$$

¹Since $\mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right)$ is an unconditional expectation (of the reciprocal of government competence), while $\mathbf{E}_t \left(\frac{1}{\theta_t} \right)$ is decreasing and convex in ε_{t-1} , applying Lemma 1 one can show that $\mathbf{E}_t \left(\frac{1}{\theta_t} \right) > \mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \right)$ for $\varepsilon_{t-1} < 0$. Condition (7) rules out that debt is optimal even in the worst possible scenario, i.e., $\varepsilon_{t-1} = -1/2\xi$.

Proof: First, since X and Y are independent stochastic variables, $g(x) = \mathbf{E}[Z | x] = \int h(x, y) f_y(y) dy$. Since $g(x)$ is concave, by Jensen's inequality it follows that $g[\mathbf{E}(X)] \geq \mathbf{E}[g(x)]$. Employing the definition of g , the left hand side of the inequality is $\mathbf{E}[Z | \mathbf{E}[X]]$, while the right hand side is $\mathbf{E}_X[\mathbf{E}[Z | X]]$. Therefore, $\mathbf{E}[Z | \mathbf{E}[X]] \geq \mathbf{E}_X[\mathbf{E}[Z | X]]$. By the law of iterated expectations, $\mathbf{E}[Z] = \mathbf{E}_X[\mathbf{E}[Z | X]]$. Hence,

$$\mathbf{E}[Z | \mathbf{E}[X]] \geq \mathbf{E}[Z]. \quad (\text{A.1})$$

Now, consider the vector of information variables W , whose realization w is known. From inspection of (A.1), if $g(x) = \mathbf{E}[Z | x]$ is an increasing function of x , then $\mathbf{E}[Z | \hat{x}(w)] \geq \mathbf{E}[Z | \mathbf{E}[X]]$ if and only if $\hat{x}(w) \geq \mathbf{E}[X]$. ■

Lemma 2 $\mathbf{E}_t[c_{t+1} + \alpha \ln(g_{t+1}) | \varepsilon_t]$ is an increasing and concave function of ε_t .

Proof: Replace c_{t+1} and g_{t+1} , then replace γ_{t+1}^u and π_{t+1}^u , apply the conditional expected value operator, and use $\mathbf{E}_t\left(\frac{1}{\theta_{t+1}} | \varepsilon_t\right) = \mathbf{E}_{t+1}\left(\frac{1}{\theta_{t+1}}\right)$:

$$\mathbf{E}_t[c_{t+1} + \alpha \ln(g_{t+1}) | \varepsilon_t] = y - \alpha - (1 + r)d_t \mathbf{E}_t\left(\frac{1}{\theta_{t+1}} | \varepsilon_t\right) + \alpha \mathbf{E}_t\left[\ln\left(\frac{\alpha \theta_{t+1}}{\mathbf{E}_{t+1}\left(\frac{1}{\theta_{t+1}}\right)}\right) | \varepsilon_t\right].$$

Expected utility in $t+1$ is increasing in ε_t because of a lower expected burden of outstanding debt, a higher expected competence in the provision of the public good, and a higher expenditure on the public good:

$$\frac{\partial \mathbf{E}_t [c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t} = (1+r)d_t \mathbf{E}_t \left(\frac{1}{\theta_{t+1}^2} \mid \varepsilon_t \right) + \alpha \left[\mathbf{E}_t \left(\frac{1}{\theta_{t+1}} \mid \varepsilon_t \right) + \frac{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^2} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}} \right)} \right] > 0.$$

As to the second derivative of $\mathbf{E}_t [c_{t+1} + \alpha \ln(g_{t+1}) \mid \varepsilon_t]$, first

$$\frac{\partial^2 \mathbf{E}_t (c_{t+1} \mid \varepsilon_t)}{\partial \varepsilon_t^2} = -2(1+r)d_t \mathbf{E}_t \left(\frac{1}{\theta_{t+1}^3} \mid \varepsilon_t \right) = \frac{(-2)(1+r)d_t(\bar{\theta} + \varepsilon_t)}{\left[(\bar{\theta} + \varepsilon_t)^2 - \left(\frac{1}{2\xi} \right)^2 \right]^2} < 0.$$

Since debt may be zero, for expected utility to be concave in ε_t , the second derivative of the public good must be negative. Using $\mathbf{E}_t \left(\frac{1}{\theta_{t+1}^2} \mid \varepsilon_t \right) = \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^2} \right)$,

$$\frac{\partial^2 E_t [\ln(g_{t+1}) \mid \varepsilon_t]}{\partial \varepsilon_t^2} = \frac{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^2} \right) \left\{ \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^2} \right) - \left[\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}} \right) \right]^2 \right\} - 2\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^3} \right) \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}} \right)}{\left[\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}} \right) \right]^2}.$$

Since $\left\{ \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^2} \right) - \left[\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}} \right) \right]^2 \right\}$ is second order in relation to the following term, this derivative is negative. ■

Lemma 3 Suppose that party A controls E in period t , then the difference $D(A, B) = \mathbf{E}_t [c_{t+1}(A, j, A, A) + \alpha \ln g_{t+1}(A, j, A, A) \mid \hat{\varepsilon}_t^A] - \mathbf{E}_t [c_{t+1}(A, j, A, B) + \alpha \ln g_{t+1}(A, j, A, B) \mid \hat{\varepsilon}_t^A]$ is increasing in $\hat{\varepsilon}_t^A$.

Proof: Applying the properties of operator \mathbf{E} and the definitions of c_{t+1} and g_{t+1} ,

$$\begin{aligned}
D(A, B) &= \frac{(1+r) \left[\hat{\omega}_t(A, j) - \frac{1}{\hat{\omega}_t(A, j)} \right]}{\mathbf{E}_t \left(\frac{1}{\theta_t^A} \right)} \mathbf{E}_t \left[\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} - \frac{1}{\theta_{t+1}^A} \mid \hat{\varepsilon}_t^A \right] + \\
&+ \alpha \mathbf{E}_t \left[\ln \left(\frac{\theta_{t+1}^A}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \mid \hat{\varepsilon}_t^A \right] + \alpha \mathbf{E}_t \left[\ln \left(\frac{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} \right) \mid \hat{\varepsilon}_t^A \right].
\end{aligned}$$

Differentiating $D(A, B)$:

$$\begin{aligned}
\frac{\partial D(A, B)}{\partial \hat{\varepsilon}_t^A} &= \frac{(1+r) \left[\hat{\omega}_t(A, j) - \frac{1}{\hat{\omega}_t(A, j)} \right]}{\mathbf{E}_t \left(\frac{1}{\theta_t^A} \right)} \frac{\partial \mathbf{E}_t \left[\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} - \frac{1}{\theta_{t+1}^A} \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} + \\
&+ \alpha \frac{\partial \mathbf{E}_t \left[\ln \left(\frac{\theta_{t+1}^A}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} + \alpha \frac{\partial \mathbf{E}_t \left[\ln \left(\frac{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} \right) \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A}.
\end{aligned}$$

$\hat{\omega}_t(A, j) \geq 1$, because either $\hat{\omega}_t(A, j) > 1$ with PBCs, or $\hat{\omega}_t(A, j) = 1$. As to the first derivative,

$$\frac{\partial \mathbf{E}_t \left[\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} - \frac{1}{\theta_{t+1}^A} \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} = (1-\rho) \mathbf{E}_t \left[\frac{(\theta_{t+1}^B)^2 - \rho (\theta_{t+1}^A - \theta_{t+1}^B)^2}{(\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B)^2 (\theta_{t+1}^A)^2} \mid \hat{\varepsilon}_t^A \right].$$

For $\rho = 1$, this is zero, and for $\rho = 0$, this is positive. When $\rho < 1$, this is also positive, because the second term of the numerator is second order with respect to the first term.

Therefore:

$$\begin{aligned}
\frac{\partial \mathbf{E}_t \left[\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} - \frac{1}{\theta_{t+1}^A} \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} &= 0 \text{ if } \rho = 1, \\
&> 0 \text{ if } \rho < 1.
\end{aligned} \tag{A.2}$$

As to the second derivative,

$$\frac{\partial \mathbf{E}_t \left[\ln \left(\frac{\theta_{t+1}^A}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} = \mathbf{E}_t \left[\frac{(1-\rho) \theta_{t+1}^B}{\theta_{t+1}^A (\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B)} \mid \hat{\varepsilon}_t^A \right] \begin{cases} = 0 & \text{if } \rho = 1, \\ > 0 & \text{if } \rho < 1. \end{cases} \quad (\text{A.3})$$

As to the third derivative,

$$\begin{aligned} \frac{\partial \mathbf{E}_t \left[\ln \left(\frac{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} \right) \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} &= \mathbf{E}_t \left\{ \frac{(1-\rho) \mathbf{E}_{t+1} \left(\frac{1}{(\theta_{t+1}^A)^2} \right) \mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} + \right. \\ &\quad \left. - \rho \frac{\left[\mathbf{E}_{t+1} \left(\frac{1}{(\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B)^2} \right) \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right) - \mathbf{E}_{t+1} \left(\frac{1}{(\theta_{t+1}^A)^2} \right) \mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \right]}{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right) \mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} \mid \hat{\varepsilon}_t^A \right\}, \end{aligned}$$

where the first term in the numerator is positive, and the second term of the numerator is second order (since it is the difference of two products of similar magnitude). Hence,

$$\frac{\partial \mathbf{E}_t \left[\ln \left(\frac{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^A + (1-\rho) \theta_{t+1}^B} \right)}{\mathbf{E}_{t+1} \left(\frac{1}{\theta_{t+1}^A} \right)} \right) \mid \hat{\varepsilon}_t^A \right]}{\partial \hat{\varepsilon}_t^A} \begin{cases} = 0 & \text{if } \rho = 1, \\ > 0 & \text{if } \rho < 1. \end{cases} \quad (\text{A.4})$$

Summing up, (A.2)-(A.4) imply that Lemma 3 is satisfied. ■

3 Voter's decision in a constitutional democracy

The voter's decision is a dynamic programming problem. Let $W(i, j)$ be the expected utility of the voter in electoral period t given that currently (that is, before elections) party i leads E , and party j , V . The voter's problem has a recursive structure which leads to the following Bellman equation, where $\hat{\varepsilon}_t^i$ is estimated using information set $\mathfrak{I}_t = (g_t, p_t, \varepsilon_{t-1}^E, \varepsilon_{t-1}^V, \hat{\omega}_t)$ and the control variables are $i', j' \in \{A, B\}$, which refer to the voter's choice of parties to play roles E and V :

$$W(i, j \mid \hat{\varepsilon}_t) = \max_{i', j' \in \{A, B\}} \left\{ \beta_t \mathbf{E}_t [c_{t+1}(i, j, i', j') + \alpha \ln g_{t+1}(i, j, i', j') \mid \hat{\varepsilon}_t] \right. \\ \left. + \beta_t^2 \mathbf{E}_t [c_{t+2}(i', j') + \alpha \ln g_{t+2}(i', j') + W(i', j')] \right\},$$

$$\text{where } c_{t+1}(i, j, i', j') = y - \frac{\frac{\alpha}{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^{i'} + (1-\rho) \theta_{t+1}^{j'}} \right)} + (1+r) \frac{\alpha (\hat{\omega}_t(i, j) - \frac{1}{\hat{\omega}_t(i, j)})}{\mathbf{E}_t \left(\frac{1}{\theta_t^i} \right)}}{\rho \theta_{t+1}^{i'} + (1-\rho) \theta_{t+1}^{j'}},$$

$$\ln g_{t+1}(i, j, i', j') = \ln \left(\frac{\rho \theta_{t+1}^{i'} + (1-\rho) \theta_{t+1}^{j'}}{\mathbf{E}_{t+1} \left(\frac{1}{\rho \theta_{t+1}^{i'} + (1-\rho) \theta_{t+1}^{j'}} \right)} \right) + \ln \alpha,$$

$$c_{t+2}(i', j') = y - \frac{\alpha}{\hat{\omega}_{t+2}(i', j') \theta_{t+2}^{i'} \mathbf{E}_{t+2} \left(\frac{1}{\theta_{t+2}^{j'}} \right)},$$

$$\ln g_{t+2}(i', j') = \ln \left(\frac{\hat{\omega}_{t+2}(i', j') \theta_{t+2}^{i'} \alpha}{\mathbf{E}_{t+2} \left(\frac{1}{\theta_{t+2}^{j'}} \right)} \right) + \varphi(i', j') \alpha \ln \rho^2,$$

$$\hat{\omega}_t(i, j) = \begin{cases} 1 & \text{if } i \neq j \\ \omega_t^u & \text{otherwise} \end{cases}, \quad \varphi(i', j') = \begin{cases} 1 & \text{if } i' \neq j' \\ 0 & \text{otherwise} \end{cases}.$$

Let $\Phi(i, j, \hat{\varepsilon}_t^i)$ denote the policy function that solves the voter's decision problem. We make the following conjecture, where i is the party currently in charge of E , and j is in charge of V :

$$\Phi(i, j, \hat{\varepsilon}_t^i) = \begin{cases} (i, i) & \text{if } \varepsilon_H^{i,j} \leq \hat{\varepsilon}_t^i \leq \frac{1}{2\xi}, \\ (i, \sim i) & \text{if } 0 \leq \hat{\varepsilon}_t^i < \varepsilon_H^{i,j}, \\ (\sim i, i) & \text{if } -\varepsilon_L^{i,j} < \hat{\varepsilon}_t^i < 0, \\ (\sim i, \sim i) & \text{if } -\frac{1}{2\xi} \leq \hat{\varepsilon}_t^i \leq -\varepsilon_L^{i,j}. \end{cases} \quad (\text{A.5})$$

The symbol $\sim i$ indicates the opposition (there are only two parties). The higher and lower limits are not symmetrical, depending also on whether the starting point is unified government ($i = j$) or divided government ($i \neq j$).² We now verify this cut point strategy.

As to the vote for the executive, by Corollary 1 the median voter prefers to reelect the incumbent party $i \in \{A, B\}$ for executive office if and only if $\hat{\varepsilon}_t^i \geq 0$, since the voter never receives information about the current shock of the opposition party, whether it is out of office or leads V .

Is there vote splitting? Let party A control E in period t . If $\hat{\varepsilon}_t^A \geq 0$, the median voter picks $i' = A$, and in the Bellman equation only the controls $j' = A, B$ must be considered (if $\hat{\varepsilon}_t^A < 0$, the median voter favors party B instead and similar arguments apply). If the median voter chooses divided government in period t , the impacts on the Bellman equation

²The difference of starting with either unified or divided government is the burden of the debt in $t + 1$. Divided government imposes an expected loss in competence, and reduces the expected variance in competence. The two factors have opposite effects on the expected burden of the debt. When the expected loss in competence prevails, unified government becomes more attractive.

can be broken down into three welfare effects.

The first welfare effect is $\mathbf{E}_t [c_{t+1}(A, j, A, A) + \alpha \ln g_{t+1}(A, j, A, A) \mid \hat{\varepsilon}_t^A] - \mathbf{E}_t [c_{t+1}(A, j, A, B) + \alpha \ln g_{t+1}(A, j, A, B) \mid \hat{\varepsilon}_t^A]$. For $\hat{\varepsilon}_t^A = 0$, the difference is second order and has to do with the effects on variance: with unified government, shock $\hat{\varepsilon}_t^A$ is known in equilibrium, whereas with divided government $\rho \hat{\varepsilon}_t^A + (1 - \rho) \varepsilon_t^B$ has a expected value of zero but a positive variance; on the other hand, in the next period, expected competence is the same, but variance is lower with divided government, since $\rho \varepsilon_{t+1}^A + (1 - \rho) \varepsilon_{t+1}^B$ has the same expected value (zero) but less dispersion than ε_{t+1}^A . These two risk effects have opposite signs. However, as $\hat{\varepsilon}_t^A$ increases, there is a competence effect that clearly favors unified government: by Lemma 3, for $\rho < 1$, $\mathbf{E}_t [c_{t+1}(A, j, A, A) + \alpha \ln g_{t+1}(A, j, A, A) \mid \hat{\varepsilon}_t^A] - \mathbf{E}_t [c_{t+1}(A, j, A, B) + \alpha \ln g_{t+1}(A, j, A, B) \mid \hat{\varepsilon}_t^A]$ is increasing in $\hat{\varepsilon}_t^A$.

As to the second welfare effect, expectations about period $t + 2$ are not conditional on the current competence shock, so $\mathbf{E}_t [c_{t+2}(A, A) + \alpha \ln g_{t+2}(A, A)] - \mathbf{E}_t [c_{t+2}(A, B) + \alpha \ln g_{t+2}(A, B)] = \mathbf{E}_t \left[\frac{\alpha \left(1 - \frac{1}{\omega_{t+2}^u} \right)}{\theta_{t+2}' \mathbf{E}_{t+2} \left(\frac{1}{\theta_{t+2}'} \right)} + \alpha \ln \left(\frac{\omega_{t+2}^u}{\rho^2} \right) \right]$, which is nonnegative because $\omega_{t+2}^u \geq 1$ and $\rho \leq 1$. In period $t + 2$ there will be an efficiency loss with divided government due to the break down in coordination between E and V . Furthermore, there will be no cycle under divided government. Both effects tends to reduce utility in period $t + 2$ compared to a situation with unified government (no PBCs imply more taxes and less public goods in period $t + 2$).

The third welfare effect is $\mathbf{E}_t [W(A, A)] - \mathbf{E}_t [W(A, B)] < 0$. Voter prefers to begin with divided government because there is no debt to repay in the future. Formally, the result follows from a direct inspection of the Bellman equation.

While the second and third welfare effects are fixed costs and benefits, by Lemma 3 the

first welfare effect is increasing in $\hat{\varepsilon}_t^A$. Hence, if for some $\hat{\varepsilon}_t^A \geq 0$ the median voter prefers unified government (A, A) to divided government (A, B) , then for $\hat{\varepsilon}_t^{A'} > \hat{\varepsilon}_t^A$ the voter will also prefer (A, A) to (A, B) ; and if for $\hat{\varepsilon}_t^A \geq 0$ the median voter prefers (A, B) to (A, A) , then for $0 \leq \hat{\varepsilon}_t^{A'} < \hat{\varepsilon}_t^A$ the voter will also prefer (A, B) to (A, A) . Thus, the policy function is a cut point strategy as conjectured in (A.5).