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ARBITRAGE IN FOREIGN EXCHANGE MARKETS

WITHIN THE CONTEXT OF A TRANSACTIONAL ALGEBRA

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Abstract

This paper sets forth the foundations for a transactional approach for the performance of arbitrage in foreign exchange markets. Firstly, we review both the standard model of financial arbitrage and the so-called covered-interest arbitrage environment, and we also lay bare striking shortcomings in these points of view, mainly grounded on a wide- ranging empirical evidence. Next, we move on to what we have labeled in previous research working papers a transactional algebra, from which we expand on its main tools of analysis, namely differential rates, residual information sets, arbitrage gaps and transaction costs functions. Afterwards, we establish and prove the minimal conditions under which a successful arbitrage can be carried out within a transactional algebra.

JEL: F30, F31, G15

Kew words: transactional algebras, arbitrage, covered-interest arbitrage, differential rates, residual information sets, arbitrage gaps.

Disclaimer

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INTRODUCTION

Before the 70s, current practice and standard books on International Finance had been resorting to a very simple and intuitive framework of analysis to deal with foreign exchange arbitrage.

However, as the 70s evolved, many academics and practitioners were feeling at variance with mainstream opinions and started not only pursuing different points of view but also shaping a new toolbox that included transaction costs, comparative economics, conflicts of interests, the so-called institutional approach, asymmetric information problems and even corporate governance issues.

In this paper, I want to review the old model of foreign currency arbitrage, and to also make a humble contribution to its expansion and upgrading through the notion of transactional algebras.

To begin with, I introduce the standard model of financial arbitrage, which will be followed by the so-called covered interest arbitrage, still widely used to teach and discuss foreign exchange key points arising in trade and MBAs' syllabuses. The description of both standpoints will also include an outline of their shortcomings.

Afterwards, I will introduce a my own standpoint, which derives from the frame of mind that has been in construction since the 70s. To attain our goals, the substantive notions of arbitrage gap, differential rates of return, residual information sets, and transactional algebras are laid bare to reshape the covered interest arbitrage into a sensible picture that entails market institutions and frictions, ultimately leading to the core task of intermediaries: the brokerage of asymmetric information² by means of a set of minimal conditions to be met so as to grant a successful arbitrage within transactional algebras.

Summing up, a non standard approach to arbitrage in International Finance is the central claim of this paper. It certainly furthers our previous research work³ and takes advantage of an outstanding job done by distinguished scholars who are going to be referred in due course.

² As far as our knowledge, the expression "brokerage of asymmetric information" was firstly coined and fully developed in Apreda (2001).

³ See Apreda (2004, 2003, 2001a, 2001b, 2000a, 2000b, 2000c).

1. THE STANDARD FINANCIAL ARBITRAGE MODEL

A middle-of-the-road intuition about what is meant by arbitrage is usually framed this way: "You buy cheap, hold on⁴, and then sell to reap the profit arising from the gap in both prices." We could not deny that for some currently round-off transactions this stands up well as a suitable explanation. Moreover, such way of dealing with this issue seems misleading, to say the least, since from many transactions we are not able to get any profit⁵, in spite of a stark difference in prices. Even worse, when we define transactions that involve buying firstly and selling later, we leave out of the picture the reversal situation, in which we set up good arbitrage processes by selling firstly and repurchasing the good later at a lower price. That is why the standard viewpoint of financial arbitrage calls for a tighter semantics⁶.

Definition 1

Financial Arbitrage is a decision-making process whose main features are:

- i) the trade of a financial asset g₁, at an expected moment t₁ and in a certain market m₁, at the value V(g₁; m₁; t₁);
- ii) the trade of a financial asset g_2^7 , at an expected moment t_2 and in a certain market m_2 , at the value $V(g_2; m_2; t_2)$, with $t_1 \le t_2$;
- iii) making a profit from round-off transactions with g_1 and g_2 , either the long-short or the short-long types, that is to say, the payoff functions $\Pi(.)$ become positive:

 $\begin{cases} \Pi(\text{long-short}) = V(g_2; m_2; t_2; s) - V(g_1; m_1; t_1; l) > 0 \\ \Pi(\text{short-long}) = V(g_1; m_1; t_1; s) - V(g_2; m_2; t_2; l) > 0 \end{cases}$

iv) no investment is required for bringing both transactions to completion;

⁴ If the span of the holding period were very small, then the dealer would usually carry out an "spatial arbitrage", whereas the longer the span the more "temporal" the arbitrage becomes.

⁵ This is attributable to the costs of running a transactional algebra, an expansion of which the reader will find in section 3.

⁶ A perspective that stems from Microeconomics can be found in Varian (1987).

⁷ It is frequent in practice that $g_1 = g_2$; otherwise, close substitutes match the deal. Spatial-like types of arbitrage ask for $m_1 = m_2$, as well.

v) risks of rounding off both transactions are null⁸.

An equivalent statement for condition *iii*) will become useful later⁹:

 $\Pi(\text{long-short}) = V(g_2; m_2; t_2; s) / V(g_1; m_1; t_1; l) > 1$ $\Pi(\text{short-long}) = V(g_1; m_1; t_1; s) / V(g_2; m_2; t_2; l) > 1$

To avoid misunderstandings, definition 1 deserves two further qualifications, one linked with the concept of substitution, the other with the arbitrage process itself:

a) Whereas non-financial arbitrage always requires of an asset g_1 (or two close substitutes, g_1 and g_2) to be mispriced, financial arbitrage takes place within a much more encompassing setting. In fact, to what extent can we regard two financial assets as close substitutes? A sensible approach qualifies two assets or portfolios to be substitutes if the following features hold outright:

- their stream of expected future cash flows makes them indistinguishable from each other,
- some relevant measure of risk, when applied to both stream of cash flows puts them into the same risky category¹⁰.

It is when the foregoing remarks are carried on to their full swing that financial economists will regard such assets as equivalent and following the law of one price¹¹. A trading-off between expected return and some risk measure underpins the substitution issue¹².

b) We can also perform arbitrage successfully by selling the asset at a certain moment, waiting for a while, and purchasing it later at a lower price. If we have the asset, that means opening a short position. If we do not, we open a short-selling position, a procedure seldom used in financial

⁸ In real practice, it is enough to ask for a negligible level of risk, even in a model soconstrained as the standard one.

⁹ Mainly in definition 3 and through section 4.

¹⁰ More background on cash flows valuation in Ross (1978), and Elton-Gruber (1995) comes in handy for financial assets and portfolios.

¹¹ Froot et al. (1995) review the performance of the law of one price over 700 years, while Rogoff (1996) deals with the purchasing power parity puzzle. On the other hand, Isard (1977) remains a classic on the topic.

¹² By all means, this is a key issue, of which more details will follow in section 2.

markets since there are many restrictions to the activity, albeit short-selling comes up as a crucial assumption in many Financial Economics models¹³ and classroom settings.

In financial environments, risk can only sparingly be dispensed with, and we say that in such cases a pure arbitrage is under way. But coping with risk is just a fact of life in capital markets. Moreover, whenever the purchase of a security at date t_1 involves its selling at a later date t_2 , the arbitrage is risky, because at the starting point we do not know the value

V (g₂; s₂; t₂)

that the underlying financial asset \mathbf{g}_2 will have at date \mathbf{t}_2 – unless we held some assets like zero-coupon bonds or had eventually hedged our position with derivatives.

1.1 SHORTCOMINGS IN THE STANDARD FINANCIAL ARBITRAGE MODEL: EMPIRICAL EVIDENCE

It is hardly surprising that intensive research has been undertaken to uncover drawbacks of such oversimplified world like the one depicted in the standard financial arbitrage model. On this side of empirical research, let us highlight some contributions worthy of record and comment.

a) In practice, arbitrageurs are able to draw up arbitrage opportunities through the exchange of substitutes that convey a legal right to acquire or dispose of, and conversely an obligation to provide or accept, securities identical with an existing one under known terms and contracts. Henderson and Martine (1986) studied this issue, mainly through the following examples:

- Asset substitutes by risk grades, term to maturity, coupons, duration and convexity.
- Convertible bonds and new issues of ordinary shares.
- Options and futures.
- ADRs and US Treasury Strips.
- Risk arbitrage: by opening positions in stock from companies in the threshold of mergers or acquisitions, to profit later arbitraging between them.

¹³ The fact is that in many countries such mechanism is forbidden. But dealers can attain similar outcomes by carrying out repos and reverses so as to synthesize short-selling positions (an introductory grounding in Blake, 2000).

b) In real-life environments, it is very difficult to carry out a complete arbitrage with financial assets, because of quantities, terms to maturity, regulations, marginal accounts balances linked with leverage and shortselling positions, as it was proved by Larkman (1986).

c) Bralsford (1986), working with both currencies and interest rate swaps, highlights some facts that enhance discrepancies between prices and returns, namely mismatching of regulations, foreign exchange policies, minimal sizes required for any trade. That is to say, the sort of features that hinge upon market microstructure.

d) Metcalf (1990) has done econometric research on departures from pure arbitrage. In the United States, there is a tax exemption granted to municipal and state bonds that has allowed municipal and state governments to profit from arbitrage opportunities in a consistent and lasting way. Accordingly, they issue tax-exempted bonds at a rate r(m) and invest the proceeds at taxable rate r(t). This is a clear example of how much influential microstructure, transaction costs and creative accounting could become in the capital markets. Although the practice is illegal, the Internal Revenue Service has not been able to prevent state and local governments from earning arbitrage windfalls. Even the so-called Gram-Rudman-Hollings Law (that deals with balanced budgets) has proved very difficult of being enforced. In fact, Metcalf singles out two kinds of arbitrage:

- *Financial Arbitrage*: like the one above-mentioned.
- Savings Arbitrage: by raising taxes to invest the proceedings in securities with higher returns. The interest from investments is paid to taxpayers under the guise of lower taxes expected in the future.

e) There is an interesting case of the law of one price violation in the bonds market, provided in an empirical research by Daves y Ehrhardt (1993). Interest coupons and principal coupons are not perfect substitutes, since when reconstructing them after the moment they had entered the market as strips, there is a positive differential in favor of the principal coupon. By the same token, they are not equivalent as regards liquidity concerns. Dealers rebuild the bond and sell it a higher price than the one they could get by adding up the prices of single coupons. Furthermore, the spread at the selling side is not the same as the spread at the buying side; in general, it is higher.

f) As it was argued by Shleifer and Summers (1990), arbitrage is risky and therefore limited. This approach helps to explain available and noteworthy anomalies in the efficient markets model, and also key patterns of market behavior such as trading volumes and actual investment strategies. They singled out two types of economic agents in the capital markets, namely arbitrageurs (also labeled smart money, rational speculators) and noise traders (liquidity traders, irrational agents)¹⁴. The former are exceedingly proficient in building-up portfolios on fully rational expectations about financial assets returns. The latter, on the contrary, are subject to systematic biases, acting on psychological impulse (noise). It is for the arbitrageurs to bring prices towards fundamentals. Ultimately, two types of risk actually limit arbitrage in real markets:

- *Fundamental risk:* selling short to repurchase later at a lower price may fail whenever expectations on that asset improve; that is to say, prices may never revert to fundamental values eventually.
- Unpredictability of the future resale price: at the time of liquidating his position in the future, the arbitrageur would bear the risk for the asset of being overpriced. This type of risk is financial by nature because, even if prices converge to fundamentals, the path could be uncertain and bumpy.

Although risk makes arbitrage ineffective, pervasive constraints in actual arbitrage are also to be found in the imperfect knowledge about fundamental values, and even in the ability to detect price changes that reflect deviations from fundamentals. That is to say, there are problems with mispricing identifications as well as with the risk of betting against them. News alone does not fully explain price changes, because uninformed changes in demand also have a word along the process. Finally, capital requirements mean that money cannot be indefinitely raised, neither costlessly nor unfettered by regulations.

g) Whenever both investors and securities are subject to differential taxation, there might be a lack of referential prices to rule out tax arbitrage (Dammon and Green, 1987). We should bear in mind that in ideal modelsThe existence of "no-tax-arbitrage" prices ensures the existence of equilibrium prices. Identical securities that contribute to taxable income to different degrees will, in general, be valued differently and equilibrium will fail to exist unless short-sale restrictions are imposed that prevent investors from exploiting such arbitrage opportunities.

¹⁴ In spite of the labels, they are both rational actors. In the case of liquidity traders, their rationality is contestant of the one on the side of arbitrageurs. They have adversarial information sets, and take advantage of asymmetric information (a development around these topics in Apreda (2001a))

h) Mitchell and Pulvino (2000) followed a sample of 4,750 stock-swap and cash mergers, cash mergers and cash tender offers during 1963 – 1998, so as to single out the risk and return in risk arbitrage processes, finding out excess returns of 4 % per year. The risk arbitrage came out from the difference between the target's stock price and the offer price, that is to say, a distinctive arbitrage spread. Those results suggest that financial markets exhibit systematic inefficiency in the pricing of firms involved in mergers and acquisitions.

i) Arbitrage is very active between spot and future markets, with offsetting positions so that the law of one price holds. In the real world, however, mispricings are not infrequent and market frictions prevent arbitrage from taking place. Kempf (2001) did research on short-selling restrictions and early unwinding opportunities. He found out a major impact on the mispricing behaviour, mainly on index arbitrage trading by using the German stock index DAX and the DAX futures. The paper showed that

- arbitrageurs faced trading and holding costs, and market impact costs as well;
- there was a pervasive influence of asymmetric holding costs and early unwinding.

j) Mitchell, Pulvino and Stafford (2001) studied hindrances to arbitrage in equity markets between 1985 and 2000, any time when the market value of a company is less than the sum of its publicly traded parts. There were arbitrage opportunities, they persisted, and prices did not converge to fundamental values. Moreover, in 82 situations they found out that cross holdings had brought about this environment in which transaction costs stemmed from imperfect information and actual trades.

k) The late 1990s wave of mergers and acquisitions was intended to consolidate industries. It can be explained as a response to market misvaluation of potential acquirers, potential targets and their combinations (Shleifer and Vishny, 2001). This approach sees managers as completely rational, conversant with market inefficiencies, and ready to arbitrage to their profit.

I) The activity of arbitrageurs impact prices, and they back their trading with their own capital (capital adequacies for trading books of banks and securities firms, the margin levels imposed by clearing brokers, margins on futures and on leveraged equity accounts). On this issue, Attari and Mello (2001), conclude that "persistent price deviations occur and then they are the result of arbitrageurs being financially constrained."

m) With costly arbitrage and active noise traders, assets prices will be at variance of fundamental values, as Gemmill and Thomas (2002) proved in connection with a huge number of closed-end funds, which performed with an average discount to fundamental values in the long run, mainly because managers set high charges eventually.

2. THE STANDARD COVERED INTEREST ARBITRAGE MODEL

Without loss of generality, let us suppose that we make an educated guess that it is in the foreign market where a chance may arise for grabbing up a good net trading profit.

Stage 1 There is a local alternative to consider by which, at valuation date \mathbf{t} , we buy a security in the domestic market. Let us assume that it is a term deposit with maturity at \mathbf{T} , when we will sell the security at that date¹⁵.

Formally, if we plan to invest V(t, \$) dollars in the local market **D**, at an annual nominal rate of interest equal to **TNA**_D, then the effective rate of interest for the horizon (t; T) is given by¹⁶

r_{D} (t, T) = TNA $_{D}$. (T – t) / Year-basis $_{D}$

Therefore, the final value V(T, \$, D) in dollars will be

V(t, \$, D) (1 + r_D (t, T)) = V(T, \$, D)

Stage 2 On the other hand, there is also a path of action that involves buying the term deposit in a foreign market. Let us imagine, for the sake of argument, that we choose London. If we were to follow this choice, we should firstly purchase pounds, at valuation date **t**. Let us assume that at that date, the selling quotation for pounds¹⁷ provided by a forex dealer is

¹⁵ In point of fact, when we make a term deposit we buy it spot from an authorized bank and, at the same moment, we sell it forward to the issuing bank. This round-trip exchange involved in the process is a plain swap of cash-flow positions.

¹⁶ In most markets the year-basis equals 360 days, although in some others 365 is a frequent input. We are assuming that the same year-basis convention is used in both markets. Otherwise, an easy adjustment would give us the equivalent rate of interest that matches our target.

¹⁷ "Offer" or "asked" are here used as synonyms. Besides, the "direct quotation" format is the one we use. That is to say, the quotation that tells us how many units in our own domestic currency are needed to buy or sell a unit of foreign currency.



Exhibit 1

Covered- interest arbitrage

FX (t, \$ / £, a)

that means how many dollars we need to buy from the broker a unit of pound sterling.

Hence, our starting capital in dollars could be translated to pounds the following way:

V(t, \$, D) / FX (t, \$ / £, a)

We intend to illustrate through Exhibit 1the standard case for a covered arbitrage of interest rates between a domestic and a foreign market, from which we could make a riskless benefit¹⁸.

Stage 3 After buying the currency, we proceed to purchase the security issued in the foreign market, matching the same class of risk and maturity that conveys the one we could have chosen in the domestic market.

If the foreign money market is offering for term deposits an annual nominal rate of interest equal to TNA $_{\rm F}$, then the effective rate of interest for the investment horizon will be

r_F (t, T) = TNA F. (T - t) / Year-basis F

At maturity, the amount of foreign currency brought about by the deposit adds up to¹⁹

$(V(t, \$, D) / FX(t, \$ / \pounds, a)) . (1 + r_F(t, T)) = V(T, \pounds, F)$

Stage 4 Lastly, we should sell this foreign currency. This leads to two distinctive settings: either we wait till maturity to sell, or we cover our ex ante position with some derivative, mainly through a forward contract to sell the foreign currency. As ours is an ex ante decision-making process, we proceed to cover.

¹⁸ An earlier attempt to handle this issue is to be found in Frankel-Levich (1973); for a unified approach to covered arbitrage of interest rates, see Apreda (1993). An excellent outline of arbitrage and speculation with financial assets can be read in Blake (2000). From another line of analysis, Spraos (1959) provides a helpful development about speculation and arbitrage with foreign currency.

¹⁹ Bear in mind that D or F should be stressed as we did in the expressions above, since we could do a transaction in dollars outside the United States. One thing is the currency adopted as benchmark, quite another the market where we are going to use it.

Hence, at date ${\bm t}$ the buying quotation for future dollars to sell at date ${\bm T}$ is given by^{20}

FX (T, \$ / £, b)

therefore, the amount to collect in dollars will be equivalent to

$(V(t, \$, D) / FX(t, \$ / \pounds, a)) . (1 + r_F(t, T)) . FX(T, \$ / \pounds, b) =$

= V(T, \$, F)

Stage 5 The decision-making process to invest the money in any of these markets is very simple. We are going to agree that the best market is the one that offers the higher amount of money to us.

For instance, the foreign market will be more profitable whenever it holds that

V(t, \$, D) (1 + r_D (t, T)) <

< (V(t, \$, D) / FX (t, \$ / \pounds , a)). (1 + r_F (t, T)). FX (T, \$ / \pounds , b)

Stage 6 As we can see, V(t, \$, D) can be left out from both sides of the inequality, so as to get

$(1 + r_D(t,T)) <$

< (FX (T, $\$ / \pounds, b$) / FX (t, $\$ / \pounds, a$)) . (1 + r_F (t, T))

Now, if we give heed to the first expression within square brackets on the right side of the latter inequality, we realize that it entails a return to be reaped for holding pounds (at **t** we purchase the currency, and after holding it along the horizon, we sell it at **T**, and this amount to a swap of cash flow positions in currencies, dollars-to-pounds-to-dollars). This return is usually denoted as the swap return, r SWAP. All in all,

$(FX(T, \frac{1}{2}, \frac{1}{2}, b) / FX(t, \frac{1}{2}, \frac{1}{2}, a)) = (1 + r_{SWAP}(t, T))$

In conclusion, and by running stages 1 through 6, we have already proved the following statement.

 $^{^{20}}$ If no coverage is achieved, we might have written this quotation as **E(FX (T, \$ / £, b)**).

Lemma 1

$(1 + r_D(t,T)) < (1 + r_F(t,T)) . (1 + r_{SWAP}(t,T))$

Otherwise, the domestic market will provide the arbitrage opportunity.

Once the arbitrage opportunity fades away, it is said that the market becomes arbitraged; in such an extreme case, the outcome turns out to be

$(1 + r_D(t,T)) = (1 + r_F(t,T)) . (1 + r_{SWAP}(t,T))$

which is usually known as **covered-interest arbitrage** (or parity theorem)²¹.

Finally, and for the sake of clarity, it's worth giving heed to the source of the swap return. To begin with, we should wonder about the nature of the expected return arising from the process of purchasing, holding and selling foreign currency. When we handle foreign currency that way, it amounts to the holding of any financial asset, which usually provide a return that can be broken down into a holding return on the one hand, and a financial return²² on the other hand. The latter component is missing with currencies; therefore, they provide a holding return, positive or negative, as a matter of course.

If we firstly recall how to work out the return of any financial asset, next we could attempt to assess the return of holding currencies. By the rate of return r(t, T), for certain financial asset and throughout a planned investment horizon H = (t; T), is meant the following expression:

r(t, T) = (V(T) - V(t) + I(t, T)) / V(t)

where **V(T)** stands for the value of the asset at date **T**, **V(t)** for the one at date t, **I(t, T)** for any cash flow to reward the investor under the guise of interest, dividends, and the like.

In dealing with foreign currency, the equation above turns out to be

²¹ More on this topic can be followed in Blake (2000), in particular the international Fisher effect and the purchasing power parity. A related article in the New Palgrave Dictionary of Money and Finance proves to be useful to practitioners.

²² Often, this is conveyed through payment of interest, dividends or the like to the investor, who had bought any asset in the financial market, from where it derives the label of "financial asset".

$r_{SWAP}(t, T) = (FX(T, \$/\pounds, b) - FX(t, \$/\pounds, a)) / FX(t, \$/\pounds, a))$

that is equivalent to

$(1 + r_{SWAP}(t, T)) = (FX(T, $ / £, b) / FX(t, $ / £, a))$

2.1 DRIVING FORCES AND CONSTRAINTS BEHIND REAL MARKETS

The standard model of covered interest arbitrage disregards the actual constraints behind real markets. It is true that the dynamics provided by supply and demand, on the one hand, and the adjustment process provided by arbitrage, on the other, point to real market forces. But this only entails a first level of understanding for the market dynamics; indeed, this seems to be the standard approach arising from the neoclassical standpoint, of which Lemma 1 is a direct heir.

As a matter of fact, a second level of understanding for the market dynamics, and a deeper one by all intents and purposes, tracks down the underpinnings of forces that the first level has rather neglected, namely the institutional arrangements (rules of the game, distinctive laws and regulations, law enforcement and transparency); transaction costs (on which we are going to expand further in section 3.1); different endowment of information claimed by economic agents (asymmetric information); opportunistic behavior, and the essential role intermediaries perform in the markets²³. By choosing such level of research and analysis, we take advantage of a positive approach we have called "transactional algebras" that c omes in handy to bring about arbitrage in down-to-earth settings.

3. TRANSACTIONAL ALGEBRAS²⁴

We are going to introduce the toolbox required to handle our proposal, namely differential rates, residual information sets, arbitrage gaps, transactional algebras and enlarged transaction costs.

3.1 Differential Rates and Residual Information Sets

For the time being, let us assume that the information set^{25} claimed by a decision-maker ${\bf k}$ at date ${\bf t}$

²³ We have dealt with this issue in depth elsewhere (Apreda, 2001).

²⁴ To the extent of our knowledge, the expression "transactional algebra" was firstly coined in Apreda (2003).

Ω(†;k)

allows him to handle a rate of return linked to any distinctive economic variable²⁶ that comes defined along the investment horizon H = (t; T), contingent upon such information set. If we could assess another information set $\Omega_1(t; k)$ such that it would hold

$\Omega_1(\mathbf{t};\mathbf{k}) \subseteq \Omega(\mathbf{t};\mathbf{k})$

and which would explain reasonably well the value of a rate of return

$r_1(t, T, \Omega_1(t; k))$

then, we could also attempt to measure up for how much of the starting rate value it still remains unexplained by this new one²⁷.

This is the logic behind the complementary notions of differential rates of return and residual information sets. We are going to provide a simplified definition of both concepts, but good enough for the purposes of this paper. But the reader should keep in mind that when we try to deal with more than two differential rates and residual information sets, the mathematics of the subject become more intricate and we have to deal with algebras of sets, a complete development of which was provided elsewhere (Apreda, 2000a, 2004).

Definition 2 Differential Rates and Residual Information Sets

The rate of return g(.) that solves the following equation (1 + $r(t, T, \Omega(t; k))$) =

$= (1 + r_1(t, T, \Omega_1(t; k))) \cdot (1 + g(t, T, \Omega_R(t; k)))$

is called the differential rate of r(.) given $r_1(.)$, whereas the underlying information set for this rate

²⁵ That is to say, all the information available to the decision-maker. Whereas the conventional models assume all the economic actors share the same information (symmetric information), as from the 70s the stress is laid upon different endowments of information (asymmetric information).

²⁶ It could be, for instance, either a price, a rate of return, or the value of a certain market index.

²⁷ In the case of a financial asset, we mean here the rate of return of that financial asset. But there are other natural examples, as in the foreign exchange market when dealing with the appreciation or depreciation of the foreign currency against the domestic one. Also we could point to the rate that measures the change undergone by certain index.

 $\Omega_R(t;k)$

gets the label of **residual**.

3.2 Arbitrage Gaps

At this juncture, I introduce a concept that will help us to keep the further discussion within operational bounds²⁸. We are speaking about the arbitrage gap, which intends to measure up the expected return from an arbitrage opportunity, in nominal terms.

Definition 3

By **Arbitrage Gap** is meant the rate of return brought about either in longshort or short-long arbitrage processes, springing from the following equations:

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\begin{cases} 1 + r(arbitrage; long-short) = V(g_2; m_2; t_2; s) / V(g_1; m_1; t_1; l) \\ 1 + r(arbitrage; short-long) = V(g_1; m_1; t_1; s) / V(g_2; m_2; t_2; l) \end{cases}
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For any arbitrage to be successful both rates must be positive. The problem arises when the transaction costs and microstructure features of the market are not taken into account, therefore preventing the economic agent from eventually seizing the arbitrage opportunity.

3.3 Transactional Algebras

The concept of a transactional algebra summons not only a framework for the analysis of financial trading, including arbitrage processes within an institutional context, but also two matching tools: firstly, a suitable transaction costs function and, secondly, the concept of residual information sets.

Definition 4

By a **Transactional Algebra** it is meant a complex structure whose distinctive features are:

²⁸ Actually, it has been introduced implicitly when, after definition 1, a remark was made about condition iii of such definition.

i) Existence of one or more markets where financial assets can be traded either in public or private settings.

ii) The institutional framework and the market microstructure are set up by means of

- trading and regulatory institutions;

- intermediaries, investors, and regulators;

- enforceable laws and rules of the game;

- contractual arrangements about the property rights attached to each transaction.

iii) A total transaction costs function is explicitly given²⁹ which includes all relevant and computable enlarged transaction costs.

iv) The metrics for this structure comes out of differential rates, which are conditional upon their respective residual information sets

3.4 Enlarged transaction costs: the costs of running transactional algebras

Transaction costs have been usually neglected for decades. Even worse, as it is stated in some quarters, most transaction costs would become negligible as communication devices improve. To say the least, this belief is misplaced because transaction costs are the costs of running nothing less than a transactional algebra, as next remarks intend to bring to light:

- Firstly, what it is customarily meant by transaction costs points only at some particular types of trading costs, mainly linked with purchasing and selling securities. Although in some markets trading costs are being curbed, in other places they are not. The sensible question to elicit is about the structure of those trading costs, which is not so simple as it seems at first sight.
- Secondly, enlarged transaction costs encompass a broad variety of items³⁰, which refer to manifold sources of distinctive costs:

- *intermediation (INT),* which stem from the actual trading stages of purchasing or selling financial assets;

²⁹ Section 3.4 labors this issue in length.

³⁰ More background on this subject can be found in Apreda (2000a, 2004).

- *microstructure (MICR),* which stand for those costs stemming from market regulations, dealers trading arrangements to meet their intermediary roles; and restrictions on capital movements;

- *information (INF),* which involves the search of all kinds of information needed by investors and dealers; also the costs of drafting, implementing, monitoring and improving the underlying contracts to enforce transactions (Flood et al., 1998);

- taxes (TAX);

- *financial costs (FIN)* associated with the purchase, selling or holding securities or foreign currencies.

Although these five categories are neither exhaustive nor the only ones to work out, we believe that they pave the way for a sensible assessment of the transaction costs rate denoted by³¹

TC $(t_1; t_2; m_1; m_2; x)$

which is a construct with the following features:

- a. it comes by the side of every single transaction;
- b. it amounts to a rate of change that may be expressed in percentage points;
- c. and it may be framed out of the functional relationship provided by a multiplicative model:
- $< 1 + TC(t, T, \Omega^{TC}) > = < 1 + int(t, T, \Omega^{INT}) > . < 1 + micr(t, T, \Omega^{MICR}) > .$

 $. < 1 + tax(t, T, \Omega^{TAX}, t) > . < 1 + inf(t, T, \Omega^{INF}, t) > . < 1 + fin(t, T, \Omega^{FIN}, t) > .$

with the restriction

 $\Omega^{k}_{t} \subseteq \Omega^{TC}_{t}$ for k : int, micr, inf, fin, tax

³¹ In the format that follows, a vectorial frame is used by which t₁ and t₂ stand for starting and maturity dates for the transaction; m₁ and m₂ for the markets in which the involved transactions can be carried out at those dates; x for long or short position at the current date. It goes without saying that if we worked out costs at the starting date and in certain market, we should drop the remaining date and market, as we are going to do below.

Bearing in mind definition 4, we can see that the transaction cost rate gives account not only of any distinctive cost to be attributed to transactions and trades, but also their contractual costs and those that arise from underlying institutional settings. In fact, the role and essence of intermediation can be explained out of transaction costs (Benston and Smith, 1976).

We are interested here in applying this functional relationship to arbitrage processes, distinguishing long from short positions, as shown next³².

 $\begin{cases} \text{ short position: } 1 + \text{TC}(t_2; m_2; s) = \\ (1 - \text{int}(s)) . (1 - \text{micr}(s)) . (1 - \tan(s)) . (1 - \inf(s)) . (1 - fin(s)) \\ \text{ long position: } 1 + \text{TC}(t_1; m_1; l) = \\ (1 + \text{int}(s)) . (1 + \text{micr}(s)) . (1 + \tan(s)) . (1 + \inf(s)) . (1 + fin(s)) \end{cases}$

When selling, costs lessen the cash flows to be finally collected. When purchasing, they add to incurring outflows³³.

Before concluding this section, we need to take a step further and embed each transaction cost rate pertaining the short and long position into a comprehensive differential rate that might account for the costs of running transactional algebras.

Definition 5

In a transactional algebra environment, let us denote with

diff TC (long-short)

³² See footnote 20.

³³ Although decision-making in (1) requires an ex ante setting, we leave out the expectations operator symbol, E(.), for ease of notation. Each component has its own functional structure, which does not come up as linear, necessarily. In fact, non-linearity is customary and useful in latest research, which take advantage of piece-wise linear functions, or still better, the so-called simple or step functions, so as to approximate more complex relationships. (For instance, Levy-Livingston (1995) on portfolio management; Day (1997) in nonlinear dynamics applied to economics; Apreda (1999) on chaotic patterns for the arbitrage gap in capital markets)

the differential transaction costs rate that solves the equation

```
1 + diff TC(long-short) =
```

$= (1 + TC(t_2; m_2; s)) / (1 + TC(t_1; m_1; l))$

In other words, the differential transaction costs rate measures up the whole impact of the costs of running the transactional algebra involved with an arbitrage. It will prove functional to the main statements conveyed by Lemmas 2 and 3, which are going to be proved in next section.

By the same token, if the transaction is of the type short-long, the equation in the definition should become

1 + diff TC(short-long) =

```
= (1 + TC(t_2; m_2; l)) / (1 + TC(t_1; m_1; s))
```

4. ARBITRAGE WITHIN TRANSACTIONAL ALGEBRAS

Rates of return can be broken down into the cost components on the one hand, and a return netted from them, on the other hand. Two lemmas are needed to cope with this matter.

Lemma 2 In a transactional algebra, and for every arbitrage opportunity, it holds true that:

- *i)* there is an arbitrage return net of transaction costs.
- *ii) there are diferential rates to translate each type of transaction costs arising from the rounding-off trades;*

Proof:

Using (1), the money to collect when selling the asset (all-in-cost basis) would amount to

V(s) . (1 - int(s)) . (1 - micr(s)) . (1 - tax(s)) . (1 - inf(s)) . (1 - fin(s)) =

= V(s). (1 + TC(t₂; m₂; s))

and when purchasing the asset (all-in-cost basis):

V(1).(1 + int(1)).(1 + micr(1)).(1 + tax(1)).(1 + inf(1)).(1 + fin(1))

= $V(I) \cdot (1 + IC(t_1; m_1; I))$

The return from this arbitrage yields (either trading or financial, and in nominal terms) follows from

V(s) / V(l) = 1 + r(arbitrage)

When taking into account transaction costs arising from the rounding-off exchanges we can put forth the equation

(2)

V(s). $(1 + TC(t_2; m_2; s)) / V(1) . (1 + TC(t_1; m_1; 1)) =$

= 1 + r net (arbitrage)

and solving for **r** net (arbitrage) we ultimately have a measure of the return in an arbitrage process that embodies transaction costs.

Furthermore, by resorting to definition 5, the differential transaction costs rate follows from

1 + diff TC(long-short) =

therefore, the relationship (2) can now be rewritten in a much more compact format:

$$(V(s) / V(1))$$
. $(1 + diff TC(long-short)) = 1 + r_{net}$ (arbitrage)

and, last of all, we get

(1 + r(arbitrage)). $(1 + diff TC(long-short)) = 1 + r_{net}$ (arbitrage)

ii) Let us substitute now the labels t $_k$ (k: 1, 2, 3, 4, 5) for the enlarged transaction costs labels trad, micr, tax, inf, and fin, respectively. Then,

$$1 + g_k = (1 - t_k(s)) / (1 + t_k(l))$$

where $\mathbf{g}_{\mathbf{k}}$ performs as a differential rate drawn out of $\mathbf{t}_{\mathbf{k}}$ (s) and $\mathbf{t}_{\mathbf{k}}$ (l).

On the other hand, we can replace in (1) to get the equivalence:

1 + diff TC(long-short) = $(\prod (1+g_k))$

= (1 + TC(t₂; m₂; s)) / (1 + TC(t₁; m₁; l)) (END)

Whereas in the standard financial arbitrage model arbitrage opportunities can be grabbed once the conditions of definition 1 are met, in a transactional algebra structure this cannot be warranted. In fact, arbitrage will only be feasible if the arbitrage gap overrides the constraints of the transactional algebra, as the following lemma makes clear.

Lemma 3 In a transactional algebra, the fulfillment of the standard financial arbitrage conditions does not grant that an arbitrage opportunity remains profitable.

Proof:

Whenever an investor takes advantage of arbitrage opportunities, he tries to lock in a sure profit that follows from definition 3:

 $1 + r(arbitrage; long-short) = V(g_2; m_2; t_2; s) / V(g_1; m_1; t_1; I)$

 $1 + r(arbitrage; short-long) = V(g_1; m_1; t_1; s) / V(g_1; m_2; t_2; l)$

Let us analyze each relationship at a turn.

a) the long-short type of arbitrage

If we included transaction costs, according with Lemma 1, we would get a net arbitrage return:

$1 + r_{net}$ (arbitrage) = (1 + r(arbitrage)).(1 + diff TC(long-short))

For the arbitrage to become successful, it must hold:

```
(1 + r (arbitrage; long-short)) =
```

```
= (V(g_2; m_2; t_2; s) / V(g_1; m_1; t_1; l)) > 1
```

but this is not a sufficient feature, because differential transaction costs could lead to

1 + r_{net} (arbitrage; long-short) =

= (1 + r(arbitrage); long-short) . (1 + diff TC(long-short))) < 1

thus yielding a negative return on the net rate of return for the arbitrage.

b) the short-long type of arbitrage

By the same procedure as in a) we would get that is not enough the fulfillment of

$(V(g_1; m_1; t_1; s) / V(g_2; m_2; t_2; I)) > 1$

because the transaction costs structure could bring about the following outcome:

1 + r_{net} (arbitrage; short-long) =

= (1 + r(arbitrage); short-long) . (1 + diff TC(short-long)) < 1

giving forth a negative return on the net rate of return for the arbitrage. ($\ensuremath{\mathsf{END}}\xspace$)

It is from Lemma 2 that we can frame a definition of what is meant by financial arbitrage within a transactional algebra³⁴.

Definition 6

Financial Arbitrage within a transactional algebra is a decision making process whose main features are:

- i. the trade of a financial asset g_1 , at an expected moment t_1 , in a certain market m_1 , at the value $V(g_1; m_1; t_1)$;
- ii. the trade of a financial asset g_2 , at an expected moment t_2 , in a certain market m_2 , at the value $V(g_2; m_2; t_2)$, with $t_1 \le t_2$;
- iii. making a sure profit from round-off transactions with g_1 and g_2 , either the long-short or the short-long types, that is to say, the payoff functions $\Pi(.)$ are positive:

³⁴ See footnotes 7 and 8.

 Π (long-short) = 1 + r(arbitrage; long-short) = = $V(g_2; m_2; t_2; s) / V(g_1; m_1; t_1; l) > 1$ Π (short-long) = 1 + r(arbitrage; short-long) = = $V(g_1; m_1; t_1; s) / V(g_1; m_2; t_2; l) > 1$

iv. no investment is required for setting up both transactions;

risks of rounding off both transactions are null. V.

vi. it meets the following boundary conditions

1 + r_{net} (arbitrage; long-short) =

= (1 + r(arbitrage); long-short). (1 + diff TC(long-short)) > 1 1 + r_{net} (arbitrage; short-long) = = (1 + r(arbitrage); short-long). (1 + diff TC(short-long)) > 1

5. **COVERED INTEREST ARBITRAGE** WITHIN A TRANSACTIONAL ALGEBRA

Henceforth, I assume that the trade of financial assets and currency will proceed between a domestic and a foreign market within the context of a transactional algebra. It's worth giving heed to what this fact actually amounts to, by noticing the following remarkable issues:

- a) There is, on the one hand, an institutional setting consisting of the rules of the game (laws, regulations and conventions) as well as exchange arrangements. On the other hand, there are watchdogs that make for the fairness of the whole game (Central Banks and Securities Exchange Commissions).
- b) Investors, dealers, brokers and international banks, work out transaction cost functions³⁵.

 $^{^{35}}$ Although this may be done either approximately or as an educated guess, the key point is that transaction costs are embedded in the entirety of the analysis.

- c) Asymmetric information is the name of the game, and hand in hand with this feature, we may face opportunistic behavior with guile on the side of the players.
- d) Lemma 1 amounts to a valid statement in a world of perfect competition and symmetric information, with a boundless arbitrage activity. Therefore, it should be only regarded as a benchmark that provides technical and helpful approximations.
- e) Broadly speaking, Lemma 2 holds by and large, granting that arbitrage opportunities are meaningful only when transactions costs are embedded in the analysis, and become translated as differential rates.
- f) Lemma 3 also holds by and large, pledging that arbitrage opportunities as discovered by applying Lemma 1 might not ultimately lead to net profits eventually.

Being this the last section of this chapter, let us move on to give due regard to transaction costs, stemming both from the foreign exchange markets and the securities markets.

However, we know that enlarged transaction costs must also include those arising from the market microstructure, the financing of positions in foreign currency, taxes and information costs. With such a broad viewpoint, we are going to trace the round-trip involved with this arbitrage process.

Domestic securities market: When buying, holding and selling a security in the domestic market, the return can be broken down into a net rate of return, and a differential rate

g _□ (t, T)

that stands for transaction costs. Hence:

$(1 + r_D(t,T)) = (1 + net r_D(t,T)) . (1 + g_D(t,T))$

Foreign securities market By the same token, when buying, holding and selling a security in the security market, we get

(4)

(3)

Foreign exchange marketsLast of all, buying foreign currency, holding
it under the guise of a security, and selling
it eventually, leads to a return
decomposable in a net rate of return and
a differential rate of transaction costs(1 + r swap (t, T))= (1 + net r swap (t, T)). (1 + g swap (t, T))

The preceding line of analysis has paved the way for a simple but powerful outcome.

Lemma 4 In a transactional algebra, and conditional upon an information set $\Omega_{t,}$, covered interest arbitrage against the domestic exchange in favor of the foreign exchange requires the fulfillment of the following inequality:

$$(1 + net r_D (t, T)) <$$

< (1 + netr_F (t, T)). (1 + netr_{SWAP} (t, T)). (1 + g(t, T))

where g(t, T) translates the round-trip transaction costs gap.

Proof: By plugging (3), (4) and (5) into the main outcome of lemma 1, the arbitrage will be granted in favor of the foreign market whenever it holds:

 $(1 + net r_D(t,T)) . (1 + g_D(t,T)) <$ < $(1 + net r_F(t,T)) . (1 + g_F(t,T)) .$. $(1 + net r_{SWAP}(t,T)) . (1 + g_{SWAP}(t,T))$

if we build up the round-trip transaction costs gap this way

(1 + g(t, T)) =

= { $(1 + g_F(t,T))$. $(1 + g_{SWAP}(t,T))$ } / $(1 + g_D(t,T))$

we could grant that the foreign market is better, even when we take transaction costs into account. If the transaction costs gap does not meet the latter inequality, then it will mean that the arbitrage is not advisable because it comes short of covering costs. (END)

CONCLUSIONS

Although the standard financial arbitrage model and the covered-interest arbitrage's proposition provide an extreme framework of analysis, they have proved more of a hindrance than a help in real financial markets.

In line with the new theoretical and empirical viewpoints that have evolved in Financial Economics since the 70s, this chapter puts forth a proposal to overcome the constraints of the standard models by means of the notion of Transactional Algebra, which is a construct that adds to arbitrage the institutions that rule the markets, also the transaction costs structure that stems from such setting, and a new toolbox to address these issues, namely differential rates, residual information sets, arbitrage gaps and the differential transaction cost rate.

The main outcome of this chapter sets up the conditions to be fulfilled in order to successfully carry out an arbitrage with foreign currencies in a transactional algebra.

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